Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Switching Adaptive Output Feedback MPC for Input-constrained Neutrally Stable Linear Plants

Jung-Su Kim, Tae-Woong Yoon and Hyungbo Shim

Abstract— **This paper presents an adaptive output feedback MPC for an uncertain input-constrained neutrally stable linear plant using control-relevant switching. By employing an input-to-state stabilizing MPC as the multi-controller, switching adaptive output feedback MPC is proposed for the system. Unlike the previous results which consider nominal stability and use the state feedback, our scheme uses the output feedback and can deal with modelling uncertainties. The proposed scheme guarantees global stability and uses finite horizon MPC. Simulation results are given to show the effectiveness of the scheme.**

I. INTRODUCTION

MPC or model predictive control is a receding horizon strategy, where the control is computed via an optimization procedure at every sampling instant. It is therefore possible to handle physical constraints on the input and/or state variables through the optimization. Over the last decade, there have been many stability and robustness results on constrained MPC. See e.g. [1]. Moreover, explicit solutions to constrained MPC are proposed recently [2], [3], [4]. These results reduce on-line computational burden regarded as a main drawback of MPC and extend the applicability of MPC to faster plants as in electrical applications.

In [5], [6], it is proved that global stabilization of input-constrained neutrally stable systems is possible via MPC, in which infinite horizon is used. However, infinite horizon MPC can cause trouble in practice. For implementation, the optimization problem should be reformulated as a finite horizon MPC with a variable horizon, and it is not possible to predetermine a finite upper bound on the horizon in the presence of disturbances. Unlike this infinite horizon approach, a finite horizon MPC is proposed in [7], [8]. In the papers, a non-quadratic Lyapunov function as in [9], [10], [11] is employed as the terminal cost. In addition to global stability, an input-to-state stabilizing MPC is devised for the input-constrained neutrally stable system in [12]. However, these previous results on stabilization of the system via MPC [5], [6], [7], [8], [12] use the state feedback. In this paper, an output feedback finite horizon MPC for the discrete-time input-constrained system is presented; this is the first contribution of this paper.

Recently, switching has been recognized as an effective method in systems and control [13]. Switching in adaptive control context is embodied in switching adaptive control. In other words, adaptive control based on logic-based *"switching"* instead of continuous parameter *"tuning"* has been proposed in order to overcome the limitations and improve performance of the adaptive control. See [14], [15], [16], [17], [18], [19], [20], [21] for the switching adaptive control literature. Historically, one of the origins of MPC is adaptive control [22]. In other words, MPC is popularly employed as the controller in adaptive control [22], [23]. In spite of rich results on stability theory of constrained MPC, adaptive MPC for an input-constrained plant is left as an open problem in the MPC literature. A major obstacle to adaptive MPC is feasibility. Namely, optimization based on the estimated model may be infeasible. This paper proposes a switching adaptive MPC for an uncertain input-constrained system, which guarantees feasibility at all times and closed-loop stability, and uses the output as the measurement. This adaptive MPC can deal with modelling uncertainties unlike the previous results. This is the second contribution of the paper. The proposed switching adaptive MPC is coded using a SQP (Sequential Quadratic Programming) algorithm, and simulation results are given to show the effectiveness of the method.

II. PRELIMINARIES

This section is devoted to introduce two ingredients for developing main results of this paper: input-to-state stability and input-to-state stabilizing MPC.

A. Input-to-State Stability

Consider the following discrete-time nonlinear system with an external input;

$$x(k+1) = f(x(k), w(k)) \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and $w \in \mathbb{R}^m$ an external disturbance.

Definition *1:* [24] The system (1) is ISS if there exist $\alpha, \gamma \in \mathcal{K}_\infty$ and $\beta \in \mathcal{KL}$ such that

$$\alpha(\|x(k)\|) \leq \beta(\|x(0)\|, k) + \gamma(\|w(k)\|). \tag{2}$$

Theorem *1:* [24] The system (1) is ISS if and only if there exists an ISS Lyapunov function $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that V satisfies

$$\alpha_1(\|x(k)\|) \leq V(x(k)) \leq \alpha_2(\|x(k)\|)$$

and

$$V(x(k+1)) - V(x(k)) \leq -\rho(\|x(k)\|) + \gamma(\|w(k)\|) \tag{3}$$

This work was not supported by any organization

Jung-Su Kim and H. Shim are with ASRI, the School of Electrical Engineering and Computer Science, Seoul National University, KOREA jskim@acorn.snu.ac.kr,hshim@snu.ac.kr

Tae-Woong Yoon is with the Department of Electrical Engineering, Korea University, KOREA twy@korea.ac.kr

where α_1, α_2, and $\rho \in \mathcal{K}_\infty$. In the later sections, the $\mathcal{K}_\infty$ function γ is referred to as the gain function.
If the system (1) is ISS, then the system is robust against an external disturbance in the sense that the bounded external input leads to bounded state. It is also implied by ISS that the unforced system is globally asymptotically stable.

B. Input-to-state stabilizing MPC

Consider the following discrete-time neutrally stable plant with input saturation

$$x(k+1) = Ax(k) + B\text{sat}(u(k)) + w(k) \tag{4}$$

where A is neutrally stable, $x \in \mathbb{R}^n$ the state, $u \in \mathbb{R}^m$ the control input, and $w \in l_1$ an external disturbance. The saturation function, denoted by sat$(\cdot)$, is defined as follows:

$$\text{sat}(u) = [\text{sat}(u_1)\ \text{sat}(u_2)\ \cdots\ \text{sat}(u_m)]^T$$

where

$$\text{sat}(u_i) = \begin{cases} u_{\max}, & u_i > u_{\max} \\ u_i, & |u_i| \leq u_{\max} \\ -u_{\max}, & u_i < -u_{\max} \end{cases}$$

and $u_{\max}$ is a positive constant. Then for any L_u satisfying $L_u u_{\max} > 1$, we have

$$\|\text{sat}(u) - u\| \leq L_u u^T \text{sat}(u). \tag{5}$$

In order to stabilize this system, the following MPC is employed:

$$\min_{\mathbf{u}(k)=[u(k|k),\cdots,u(k+N-1|k)]} J(x(k), \mathbf{u}(k))$$

subject to

$$x(k|k) = x(k),$$
$$x(k+i+1|k) = Ax(k+i|k) + Bu(k+i|k),$$
$$u(k+i|k) = \text{sat}(u(k+i|k)), \quad i \in [0, N-1],$$

where

$$J(x(k), \mathbf{u}(k)) = \sum_{i=0}^{N-1} l(x(k+i|k), u(k+i|k)) + \Theta W(x(k+N|k)),$$
$$l(x, u) = x^T Q x + u^T R u, \text{ and}$$
$$W(x) = x^T M_q x + \Lambda (x^T M_c x)^{\frac{3}{2}}$$

with Q and R being positive definite, and M_q and M_c satisfying

$$A^T M_c A - M_c \leq 0$$

and

$$(A - \kappa BB^T M_c A)^T M_q (A - \kappa BB^T M_c A) - M_q = -\varpi I.$$

Also, ϖ is a positive constant as a design parameter, κ and Λ meeting

$$\kappa B^T M_c B < I \text{ and } \Lambda = \frac{2\kappa L_u \sigma_{\max}(A_c^T M_q B)}{\sqrt{\lambda_{\min}(M_c)}},$$

and Θ satisfying

$$\Theta > \lambda_{\max}(Q + \kappa^2 A^T M_c B R B^T M_c A). \tag{6}$$

Note that such M_c and M_q exist from neutral stability of A and global asymptotic stability of $A - \kappa BB^T M_c A$ [12], respectively. The following lemma states that the MPC is input-to-state stabilizing for the system in (4) with sufficiently small external disturbance.

Lemma *1:* [12] There exists a positive constant δ such that the closed-loop consisting of the system in (4) and the MPC with any positive integer N is ISS with respect to $\|w(k)\| < \delta$. ■

This lemma is proved by showing that, under $\|w(k)\| < \delta$, the optimal cost resulting from the MPC satisfies the following ISS characterization;

$$J^*(x(k+1)) - J^*(x(k)) \leq -\lambda_Q \|x(k)\|^2 + \beta \|w(k)\| \tag{7}$$

where $J^*(x(k))$ is the optimal cost yielded by the MPC and λ_Q ($< \lambda_{\min}(Q)$) and β are positive constants. In section III-D, the optimal cost J^* is importantly used in the switching logic.

III. Supervisory MPC for input-constrained neutrally stable plant

In this section, the main result of the paper is presented. The previous results [5], [6], [7], [8] on stabilization of input-constrained neutrally stable linear plants may suffer from taking modeling uncertainties into account. In order to stabilize the plant with modelling uncertainties, a switching adaptive output feedback MPC is proposed in the paper. Figure 1. depicts the general structure of the switching adaptive control. For details of the general architecture of the switching adaptive control, see [14], [16], [20], [17], [18]. Henceforth, the definitions and roles of each subsystem in

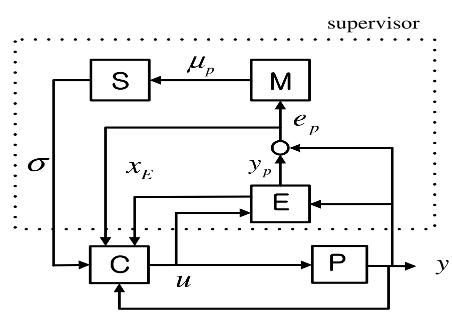

Fig. 1. Supervisory control architecture

the proposed switching adaptive MPC are described.

A. Single input single output input-constrained neutrally stable plant with modelling uncertainties (P)

To deal with the modelling uncertainties, we consider a family of models as follows:

$$x(k+1) = A_p x(k) + B_p \text{sat}(u(k)) \tag{8}$$
$$y(k) = C_p x(k) \tag{9}$$

where $p \in \mathcal{P} := \{p_1, p_2, \cdots, p_m\}$ is a finite index set, $x \in \mathbb{R}^n$ the state, $y \in \mathbb{R}$ the output, and $u \in \mathbb{R}$ the input. In addition, for each $p \in \mathcal{P}$, A_p is neutrally stable, (A_p, B_p) stabilizable, and (C_p, A_p) observable. It is assumed that there exists an index $p^* \in \mathcal{P}$ such that $(A_{p^*}, B_{p^*}, C_{p^*})$ is coincident with the true plant. Without loss of generality, suppose that (A_p, B_p, C_p) is the observer canonical form for each $p \in \mathcal{P}$, in other words, it is of the form

$$A_p := \begin{bmatrix} 0 & \cdots & \cdots & 0 & -a_0 \\ 1 & 0 & \cdots & 0 & -a_1 \\ & \ddots & & & \vdots \\ & & & & \vdots \\ & & & 1 & -a_{n-1} \end{bmatrix}, \ B_p = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ \vdots \\ b_{n-1} \end{bmatrix}$$

$$C_p = \begin{bmatrix} 0 & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

where a_i and b_i $(i=0, \cdots, n-1)$ depend on the index $p \in \mathcal{P}$. To sum up, the plant to be controlled is a SISO uncertain observable neutrally stable system and the objective of the control is to drive the state of the uncertain plant to zero.

B. *Multi-estimator* (**E**)

For the uncertain plant, consider a state-shared multi-estimator given by

$$x_E(k+1) = \begin{bmatrix} A_E & 0 \\ 0 & A_E \end{bmatrix} x_E(k) + \begin{bmatrix} \mathbf{0} \\ b_E \end{bmatrix} \mathrm{sat}(u) + \begin{bmatrix} b_E \\ \mathbf{0} \end{bmatrix} y(k) \quad (10)$$

$$y_p(k) = H_p x_E(k) \quad (11)$$

where $x_E \in \mathbb{R}^{2n}$ is the state of the estimator, $y_p \in \mathbb{R}$ the output from the pth model (A_p, B_p, C_p), A_E a Hurwitz matrix, $\mathbf{0} = [0, 0, \cdots, 0]^T \in \mathbb{R}^n$, $b_E = [0, 0, \cdots, 0, 1]^T \in \mathbb{R}^n$, $H_p = [L_p^T \ B_p^T] \in \mathbb{R}^{2n}$ a parameter vector, and $A_E^T = A_p - L_p b_E^T$ [14], [25]. Such L_p exists since (b_E^T, A_p) has the observer canonical form. In the next subsection, the estimation error defined by $e_p = y_p - y$ plays an important role in finding the best model for control. Note that the estimation error associated with the correct model exponentially goes to zero from the observer theory [14]. Hence, e_{p^*} satisfies

$$\|e_{p^*}(k)\| \le C_1 {\eta^*}^k \quad (12)$$

where $C_1 > 0$ depends on $e_{p^*}(0)$ and $\eta^* = \max|\lambda(A_E)| \in (0, 1)$ with $\lambda(\cdot)$ denoting the eigenvalues. Such a state-shared multi-estimator enables us to obtain outputs generated by a number of models using the only $2n$ dimensional dynamic equation in (10) which is independent of the number of model m.

C. *Multi-controller: MPC* (**C**)

In this subsection, we presents the multi-controller by employing the MPC based on the pth model. Using the feedback interconnection $y = y_p - e_p$, the equation in (10) can be written as

$$\begin{aligned} x_E(k+1) = & \left(\begin{bmatrix} A_E & 0 \\ 0 & A_E \end{bmatrix} + \begin{bmatrix} b_E \\ \mathbf{0} \end{bmatrix} H_p\right) x_E(k) \\ & + \begin{bmatrix} \mathbf{0} \\ b_E \end{bmatrix} \mathrm{sat}(u) - \begin{bmatrix} b_E \\ \mathbf{0} \end{bmatrix} e_p(k) \\ =: & \ M_p x_E(k) + Z \mathrm{sat}(u) - \begin{bmatrix} b_E \\ \mathbf{0} \end{bmatrix} e_p(k) \quad (13) \\ y_p(k) = & \ H_p x_E(k). \quad (14) \end{aligned}$$

The system in (13) is referred to as the pth injected system. It is necessary to keep in mind that the multi-controller is designed for the pth injected system instead of the pth model. Note that

$$M_p := \begin{bmatrix} A_E & 0 \\ 0 & A_E \end{bmatrix} + \begin{bmatrix} b_E \\ \mathbf{0} \end{bmatrix} H_p = \begin{bmatrix} A_p^T & b_E B_p^T \\ 0 & A_E \end{bmatrix},$$

this matrix is also neutrally stable, and (M_p, Z) is controllable due to the observability of (C_p, A_p). Therefore, the MPC using the pth injected system is as follows:

$$\min_{\mathbf{u}(k) = [u(k|k), \cdots, u(k+N-1|k)]} J_p(x_E(k), \mathbf{u}(k))$$

subject to

$$\begin{aligned} & x_E(k|k) = x_E(k), \\ & x_E(k+i+1|k) = M_p x_E(k+i|k) + Z u(k+i|k), \\ & u(k+i|k) = \mathrm{sat}(u(k+i|k)), \quad i \in [0, N-1] \end{aligned}$$

where

$$\begin{aligned} & J_p(x_E(k), \mathbf{u}(k)) \\ & = \sum_{i=0}^{N-1} l(x_E(k+i|k), u(k+i|k)) + \Theta_p W_p(x_E(k+N|k)). \end{aligned}$$

where the design parameters $\Theta_p, W_p(\cdot), M_{q,p}, M_{c,p}$, and Λ_p are defined similarly as in section II-B with (M_p, Z) in (13). $J_p(\cdot)$ denotes the cost function using the pth model. Note that, in this case, the initial value for the prediction is the state of the multi-estimator x_E and that the estimation error $e_p(k)$ in (13) corresponds to the external disturbance $w(k)$ in (4). Consequently, in view of *Lemma 1*, we can show that, for sufficiently small e_p,

$$J_p^*(x_E(k+1)) - J_p^*(x_E(k)) \le -\rho_p(\|x_E(k)\|) + \beta_p \|e_p(k)\| \quad (15)$$

where J_p^* is the optimal cost when using the pth model, $\rho_p(r) = \lambda_Q r^2$, and $\beta_p > 0$. This implies that the interconnected system of the estimator and the MPC is ISS with respect to the estimation error and that the gain function is linear.

D. *Monitoring signal generator* (**M**) *and switching logic* (**S**)

The performance monitoring signal μ_p associated with each $p \in \mathcal{P}$ is defined as follows:

$$\mu_p(k) = \eta \mu_p(k-1) + \|e_p(k)\| \quad (16)$$

where $\eta \in (\eta^*, 1)$. The function of the design parameter η is similar to that of the forgetting factor in classical parameter adaptive control. See [20] for details. Note that the linear function of $\|e_p(k)\|$ in (16) comes from that in (15). Smallness of a monitoring signal implies that the controller designed using the corresponding model may provide satisfactory performance in terms of *certainty equivalence principle*. The switching logic **S** places in the feedback loop the controller designed using the model from an estimator corresponding to the smallest monitoring signal. The switching logic should be designed such that the closed-loop stability is achieved as switching occurs. The output of the switching logic is denoted by $\sigma(k) \in \mathcal{P}$, the value of which is the index of the model selected by the switching logic at time k. In this paper, the control-relevant switching logic ($\mathbb{S}_{LF}$) proposed in [26] is employed. $\mathbb{S}_{LF}$ is given as follows:

Switching logic $\mathbb{S}_{LF}$:

1. Initialize $\sigma(k)$, τ_D; initialize $s(k) := 0$; set θ ($\theta < 1$)
2. Find the best model
$$q := \arg\min_{p\in\mathcal{P}} \mu_p(k) \tag{17}$$
3. If $\sigma(k) = q$,
then $s(k) := 0$, nosw:=nosw+1, and compute τ_D
else if
$$J_q^*(x_E(k)) - J_{\sigma(k)}^*(x_E(k)) \leq \theta\, s(k) \tag{18}$$
then $\sigma(k) := q \quad s(k) := 0$, nosw:= 0, and compute τ_D
4. Compute and apply the control
$$u(k) := u_\sigma(k)$$
5. $s(k+1) := s(k) + \rho_{\sigma(k)}(\|x_E(k)\|)$, nosw:=nosw+1
6. If nosw$\geq \tau_D$
then $s(k) := 0$, $\sigma(k) := q$, compute τ_D, nosw:= 0
7. $k := k+1$ and $\sigma(k+1) := \sigma(k)$ and go to step 2

In the switching logic, τ_D denotes the state-dependent dwell time devised in [27], [26], which is a nonlinear discrete-time version of the dwell time in [14]. What needs to be emphasized is that the dwell time τ_D is adjustable according to the size of the state and is computed at every switching times. After finding the best model leading to the minimum value of the monitoring signal, switching is allowed to take place in $\mathbb{S}_{LF}$ when one of the two conditions is met. The first one is the inequality in (18). From this condition, we can see that switching is not allowed even when there is a better model, if use of this new model increases the value of the Lyapunov function in such a way that violates the condition given in (18). The second one is the dwell time switching condition; if switching does not occur for the dwell time after the previous switching, then switching occurs. Checking the two conditions implies that both estimation and control performance is considered in $\mathbb{S}_{LF}$ and that persistent switching is guaranteed. Note that switching logics considers only estimation performance in [14], [17], [18] and that switching stops in a finite time in [17]. See [26] for more details about $\mathbb{S}_{LF}$.

E. Stability analysis

In this subsection, closed-loop stability of the overall switching adaptive control system previously presented is shown. Before proceeding further, it is important to note that the existence of p^*, the definition of the monitoring signal and the condition in (17) result in the following.

Lemma *2:* [14] There exist a finite switching time k^* and a set $\mathcal{P}^* \subset \mathcal{P}$ containing p^* such that
1. for any switching time $k_i > k^*$, $\sigma(k_i) \in \mathcal{P}^*$,
2. for any $p \in \mathcal{P}^*$
$$\sum_{k=0}^{\infty} \|e_p(k)\| < \infty. \tag{19}$$

The proof of this lemma parallels that of the corresponding lemma for continuous-time linear time-invariant systems[1] given in [14] (Lemma 1), and thus is not given here. ∎

Now, we are in the position to prove the closed-loop stability.

Theorem *2:* Consider the switching adaptive control system consisting of the plant (8) and (9), the multi-estimator (10) and (11), the MPC in subsection III-C, the performance monitoring signal generator in (16), and the switching logic $\mathbb{S}_{LF}$. Then, the state of the uncertain plant converges to zero.

Proof. In the light of *Lemma* 2, $e_p \in l_1$ for $p \in \mathcal{P}^*$, thus $e_p, p \in \mathcal{P}^*$ converges to zero. Moreover, it follows from *Lemma* 1 and *Lemma* 2 that there exists a switching time k^{**} and a constant Δ such that $\| \begin{bmatrix} b_E \\ \mathbf{0} \end{bmatrix} e_p(k)\| < \Delta$ for $k > k^{**}$ and the pth injected system in (13) is ISS with respect to $\| \begin{bmatrix} b_E \\ \mathbf{0} \end{bmatrix} e_p(k)\| < \Delta$. This implies that switching occurs among ISS injected systems for $k > k^{**}$. Now, we can apply *Theorem 6* in [26], which says that if all the injected system is ISS with respect to e_p and if $\mathbb{S}_{LF}$ is used for the switching logic, then the state of the resultant switched systems goes to zero. Therefore, the state of the estimator x_E goes to zero. This, in turn, implies that y_p and the control u go to zero in terms of the equations in (11) and the MPC law in section III-C, respectively. Furthermore, in view of Lemma 2, we can see that e_p converges to zero for $p \in \mathcal{P}^*$. It ensues from the relation $y = y_p - e_p$ that the plant output y goes to zero as well. Therefore, the state of the uncertain plant converges to zero thanks to the observability of the uncertain plant. This completes the proof. ∎

Remark *1:* The proposed MPC can deal with modelling uncertainties and uses the output feedback. This is in sharp contrast with the previous results in [5], [6], [7], [8]; in which nominal stability is proved using the state feedback. Since the terminal cost function is a non-quadratic function, the optimization problem associated with the MPC is not a QP (Quadratic Programming). However, it is still a convex program. To solve it, we employ SQP (Sequential QP). Besides, it should be pointed out that global feasibility ensues from the global stability of the closed-loop.

IV. Simulations

In this section, we show simulation results to illustrate the effectiveness of the proposed switching adaptive MPC. We

[1]Although the plant is nonlinear due to the input-constraints, the multi-estimator has linear dynamics with sat(u) and y being the inputs

consider the following family of models

$$x(k+1) = \begin{bmatrix} 1 & -p \\ 0 & -(1+p) \end{bmatrix} x + \begin{bmatrix} 0 \\ 2(p+1) \end{bmatrix} \text{sat}(u) \quad (20)$$

$$y(k) = [0\ 1]x(k) \quad (21)$$

and $p \in \mathcal{P} = \{0.1, 0.12, 0.14, \cdots, 0.9\}$. Note that all models are neutrally stable and that the true parameter $p^* = 0.3$. Control parameters used in the simulation are listed:

$$A_E = \begin{bmatrix} 0 & 1 \\ -0.56 & 1.5 \end{bmatrix}, \ N = 2, \ \theta = 0.1, \ \eta = 0.85,$$

and $M_{c,p}$ is computed using LMI toolbox. Figures 2 and 3 show that the proposed switching adaptive MPC successfully stabilizes the uncertain input-constrained neutrally stable plant in the sense of the convergence of the state of both the uncertain plant and the estimator. In Figure 4, the circles show the switching time by the dwell time switching condition and the multiplication marks the switching times by the Lyapunov function switching condition in (18). From Figure 4, we can see that the switching logic finds the true parameter as time goes by. In Figures 5 and 6, it is assumed

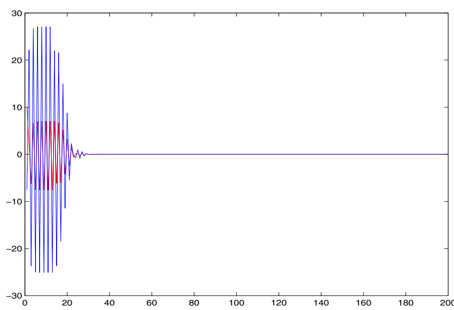

Fig. 2. State of the uncertain plant x

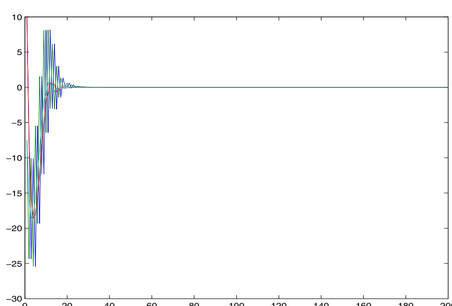

Fig. 3. State of the estimator x_E

that the true parameter $p^* = 0.315$; note that this is not in $\mathcal{P}$. As seen in the figure, the proposed scheme still successfully stabilizes the plant. Simulations were carried out with various initial conditions and the convergence of the state of the uncertain plant to zero was obtained uniformly.

V. Conclusion

This paper proposes a switching adaptive MPC of an uncertain input-constrained neutrally stable linear plant. Unlike

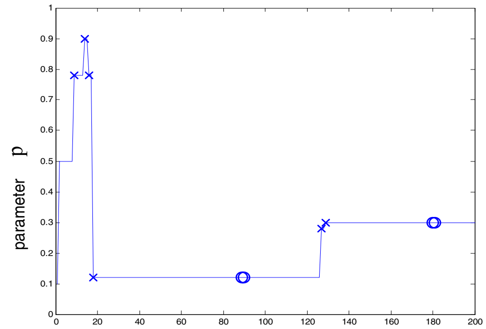

Fig. 4. Selected parameter

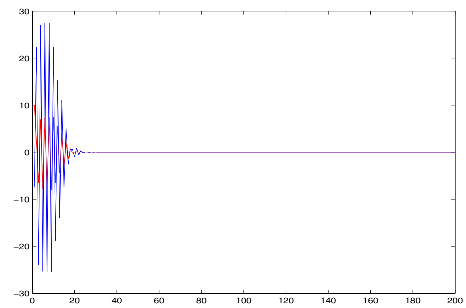

Fig. 5. State

the previous results dealing with nominal case, the proposed scheme can handle modelling uncertainties. It is proved that the state of the uncertain plant goes to zero fulfilling the input constants. The advantage of the proposed scheme is that it uses the output feedback as the measurement; the state is not accessible commonly in practice. Moreover, the proposed MPC takes advantage of a finite horizon as the control horizon and is formulated as a convex program. Since the employed MPC is input-to-state stabilizing for the system, thus global stabilizing, feasibility is always guaranteed. These advantages make the proposed scheme attractive in the sense of implementation.

References

[1] D. Mayne, J. Rawlings, C. Rao, and P. Scokaert, "Constrained model predictive control: stability and optimality," *Automatica, 36*, pp. 789–814, 2000.

[2] M. Seron, G. Goodwin, and J. De Dona, "Characterisation of receding horizon control for constrained linear systems," *Asian J. of Control, 5(2)*, pp. 271–286, 2003.

[3] A. Bemporad, M. Morari, V. Dua, and E. Pistikopoulos, "The explicit linear quadratic regulator for constrained systems," *Automatica, 38(1)*, pp. 3–20, 2002.

[4] T. Johansen, "Approximate explicit receding horizon control of constrained nonlinear systems," *Automatica, 40*, pp. 293–300, 2004.

[5] J. Choi and K.-S. Lee, "Mixed constrained infinite horizon linear quadratic optimal control," *Asian J. of Control, 5*, pp. 412–418, 2003.

[6] D. Chmielewski and V. Manousiouthakis, "Constrained infinite-time linear quadratic optimal control," *Systems & Control Letters, Vol. 29*, p. 121, 1996.

[7] A. Jadbabaie, C. De Persis, and T.-W. Yoon, "A globally stabilizng receding horizon controller for neutrally stable linear systems with input constraints," *In the Proceedings of IEEE CDC*, 2002.

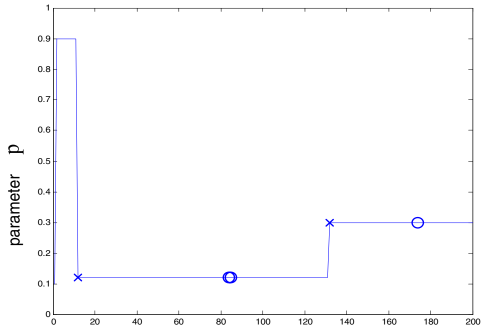

Fig. 6. Selected parameter

[8] T.-W. Yoon, J.-S. Kim, A. Jadbabaie, and C. De Persis, "A globally stabilizing model predictive controller for neutrally stable linear systems with input constraints," *In the proceedings of Int. Conference on Control, Automation and Systems*, 2003.

[9] G. Shi, A. Saberi, and A. Stoorvogel, "On the $L_p(l_p)$ stabilization of open-loop neutrally stable linear plants with input subject to amplitude saturation," *Int. J. of Robust and Nonlinear Control, Vol.13*, 2003.

[10] X. Bao, Z. Lin, and E. Sontag, "Finite gain stabilization of discrete-time linear systems subject to actuator saturation," *Automatica, 36*, pp. 269–277, 2000.

[11] Y. Chitour, W. Liu, and E. Sontag, "On finite gain stabilizability of linear systems subject to input saturation," *SIAM J. Control and Optimization, 34*, pp. 1190–1219, 1996.

[12] J.-S. Kim, T.-W. Yoon, A. Jadbabaie, and C. De Persis, "Input-to-state-stable finite horizon MPC for neutrally stable linear discrete-time systems with input constraints," *To appear in Systems & Control Letters*, 2005.

[13] D. Liberzon, *Switching in Systems and Control*. Birkhauser, 2004.

[14] A. Morse, "Supervisory control of families of linear set-point controllers-part i:exact matching," *IEEE Trans. on Automatic Control*, pp. 1413–1431, 1996.

[15] J. Hochermann-Frommer, S. Kulkarni, and P. Ramadge, "Controller switching based on output prediction error," *IEEE Trans. on Automatic Control, 43(5)*, 1998.

[16] J. Hespanha and A. Morse, "Certainty equivalence implies detectability," *Systems & Control Letters, 36(1)*, pp. 1–13, 1999.

[17] J. Hespanha, D. Liberzon, and A. Morse, "Supervision of integral-input-to-state stabilizing controllers," *Automatica, 38(8)*, pp. 1327–1335, 2002.

[18] C. De Persis, R. De Santis, and A. Morse, "Supervisory control with state-dependent dwell-time logic and constraints," *Automatica, 40(2)*, pp. 269–275, 2004.

[19] K. Narendra and C. Xiang, "Adaptive control of discrete-time systems using multiple models," *IEEE Trans. on Automatic Control, 45(9)*, pp. 1669–1686, 2000.

[20] D. Liberzon, J. Hespanha, and A. Morse, "Control using logic and switching," *Lecture Note for IEEE CDC 2001, Workshop*, 2001.

[21] J. Hespanha, D. Liberzon, and A. Morse, "Hysteresis-based switching algorithms for supervisory control of uncertain systems," *Automatica, 39(2)*, 2003.

[22] D. W. Clarke, C. Mohtai, and P. Tuffs, "Generalized predictive control, parts 1 and 2," *Automatica, 23*, pp. 137–160, 1987.

[23] T.-W. Yoon and D. Clarke, "Adaptive predictive control of the benchmark plant," *Automatica, 30(4)*, 1994.

[24] Z.-P. Jiang and Y. Wang, "Input-to-state stability for discrete-time nonlinear systems," *Automatica, 37*, pp. 857–869, 2001.

[25] I. Mareels and J. Polderman, *Adaptive Systems: An Introduction*. Birkhauser, 1996.

[26] T.-W. Yoon and J.-S. Kim, "New supervisory control using control-relevant switching," *In the Proceeding of IFAC World Congress*, 2005.

[27] J.-S. Kim, T.-W. Yoon, and C. De Persis, "State dependent dwell time switching for discrete-time stable systems," *IEICE Trans. on Fundamentals of ECCS, Vol.E87-A(12)*, pp. 3436–3438, 2004.