Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Delay in Mobility-Assisted Constant-Throughput Wireless Networks

Vikrant Sharma, Emilio Frazzoli and Petros G. Voulgaris

Abstract— **Mobility has been shown to increase the capacity of wireless networks. As the number of mobile entities in a bounded environment increases, traffic congestion occurs and these entities have to slow down to avoid colliding physically with their neighbors. This increases the time a mobile entity takes to physically carry data from one point in the environment to another. In this paper, we provide a lower bound on the maximum delay that it takes in delivering data in networks where mobility is used to achieve constant throughput per node. We also show that this delay is achievable for some networks.**

I. INTRODUCTION

A wireless network consists of a set of nodes communicating with each other over a wireless channel. When a node is transmitting, it is creating potential interference to all other nodes in its vicinity that might be receiving data from some other sender over the same channel. This limits the number of simultaneous transmissions in a bounded and shared environment. One way to allow many transmissions to happen at the same time is to allow only close range transmissions. This implies that the same data will require several hops in order to reach its distant destination. This relay burden on the network leads to a reduced per node throughput since now nodes have to spend their transmission time sending data of other nodes. It was shown in [1], that for any arbitrary network in a unit area disk, the throughput per node achieved scales as $O(1/\sqrt{n}\bar{L})$, where $\bar{L}$ is the average distance between the source and destination of bits. In [2], authors considered a random network where all the nodes are randomly located in the environment and are moving in a manner that their positions at any time are identically and independently distributed in a uniform manner in the environment. They showed in this paper that if we allowed only close range communications in which data was sent to a node close to the sender and allowed to stay with the recipient till it got close to the intended destination node, one could achieve a constant per node throughput with this scheme. In [5], the authors showed that a constant throughput per node is feasible even with a more restricted mobility model. These works indicated that mobility could be used to reduce the relay burden on the network and also avoid long distance transmissions. This is achieved by letting data travel on mobile nodes to reach close to their destinations before being transmitted to their intended destinations. They did not, however, characterize the delay that the data delivery might take because of the travel time on mobile nodes. The study of delay due to travel on mobile nodes in such situations is particularly important because the data does majority of travel towards its intended destination on such nodes and since their typical speed is much smaller as compared to the speed of electromagnetic propagation, such delay will constitute the major portion of the overall delay in transporting the data.

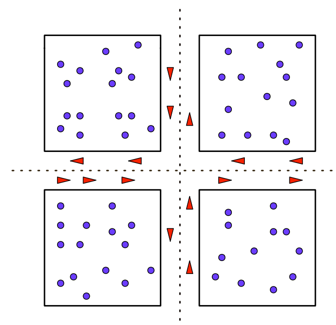

Fig. 1. A figure showing four city blocks with blue dots as being wireless network users and red arrows being mobile vehicles on the roads. A typical situation where mobility can be used to carry data.

In [3], authors later characterized the throughput-delay trade-off for the kind of random network considered in [2]. They scaled down the velocity of the nodes by some factor of n to capture the effect of growing number of mobile nodes in the environment. Similar delay/capacity tradeoff studies were conducted in [6], [4], [8] and [7], but none of them considered the fact that mobile nodes are susceptible to collisions and with increasing number of such nodes in the environment their motion becomes increasingly constrained. In this paper, we present a collision model of mobile nodes where they stay at least a velocity dependent distance away from each other for safety. This rule is exactly what vehicles on roads tend to follow to avoid collisions. Using this model we motivate and precisely quantify the scaling down of mobile agent velocities with increasing number of such agents in the environment and its effect on the delay in data delivery.

In this paper, we consider networks where n static nodes are placed in a square of unit area. The network is required to transport bits over an average distance $\bar{L}$, i.e., the average distance between the source and destination of bits is $\bar{L}$.

This work was supported by the National Science Foundation under ITR Grant No. 0325716

Vikrant Sharma is a PhD student in the Aerospace Engineering Department at the University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-2935 vsharma@uiuc.edu

Emilio Frazzoli is with Faculty of Mechanical and Aerospace Engineering, University of California at Los Angeles, Los Angeles, CA 90095, USA frazzoli@ucla.edu

Petros G. Voulgaris is with Faculty of Aerospace Engineering, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801-2935 voulgari@uiuc.edu

0-7803-9568-9/05/$20.00 ©2005 IEEE

Since the main aim of this work is to consider situations where mobility is used to increase the throughput and hence analyze the delay that the network has to incur due to the use of mobility, we restrict our analysis to situations where $\bar{L} = \omega(1/\sqrt{n})$. There are mobile agents present in the environment, with no throughput requirements of their own, that are allowed to move in the environment in any arbitrarily chosen manner under a no-collision constraint. To avoid collisions, the agents stay at least a velocity dependent distance away from each other. Since mobile agents are also a limited resource, we make an assumption that the number of mobile agents cannot grow faster than some constant times the number of static nodes. The mobile agents have wireless capability and can act as relays for the data generated at the static nodes by receiving transmissions, physically transporting it to locations close to the next hop and then transmitting it to the next hop node or agent. We show that for any such network, to achieve a constant throughput per static node, the bits have to be transported physically by mobile agents at least over an average distance of the same order as $\bar{L}$. We show that the time taken for p mobile nodes to travel between points at an average distance L from each other scales proportional to $\sqrt{p}L$. We show that due to this reason, for any network that has to transport data over an average distance $\bar{L}$ and sustain constant per node throughput, some data will take time $\Omega(\sqrt{n}\bar{L})$ to reach its destination after leaving its origin. We also show that there is a placement of nodes, the mobility pattern of the mobile agents, the selections of data traffic pattern such that the network can transport bits over an average $\bar{L}$ distance and achieve constant per node throughput while ensuring that all bits take $O(\sqrt{n}\bar{L})$ time to reach their destinations.

In Section II we present the basic setup of the problem and also present the model of wireless interference and mobile agent collision. In Section III we present a lower bound on the time it takes for mobile agents to travel between points at a certain average distance from each other. In Section IV we present a lower bound on the maximum delay that the network incurs to achieve constant throughput and show that it is $\Omega(\sqrt{n}\bar{L})$. In Section V we present a scenario where the network achieves a constant per node throughput and incurs a maximum delay of $O(\sqrt{n}\bar{L})$. In Section VI we draw some conclusions and mention future research plans.

II. Models and Definitions

Consider a square environment $\mathcal{Q}$ of unit area. We consider n static nodes arbitrarily located in $\mathcal{Q}$. At least some constant fraction of the nodes wish to send data to their arbitrarily chosen destinations at a constant rate. In other words the network needs to support a constant average rate per node. Each node can select an arbitrary range or power level for each transmission. There are m mobile agents $\mathcal{A}_i, i \in \{1,..,m\}$ in the environment that are allowed to move in an arbitrary manner such that they do not collide with each other. We assume that the number of mobile agents do not grow faster than the some constant times the number of static nodes, i.e., $m \leq \beta n, \beta \geq 1$. Let $v_i(t)$ be the velocity of agent $\mathcal{A}_i$ at time t, and assume its magnitude is bounded by $v_{\max} > 0$.

A. Agent Collision Model

We define an *exclusion zone C*, modeled as a disk centered at the agent's position, and with radius depending affinely on the agent's velocity, i.e.,

$$C_i(t) = \left\{ z \in \Re^2 : \| z - x_i(t) \| \leq r_0 + k \| v_i(t) \| \right\}, \quad (1)$$

for given constants $r_0 \geq 0, k > 0$. In this paper, we assume that each agent is represented as a point in the environment, i.e., $r_0 = 0$. We say that a *conflict* occurs between agents $\mathcal{A}_i$ and $\mathcal{A}_j$ if there exists a time t_c such that:

$$C_i(t_c) \cap C_j(t_c) \neq \emptyset.$$

This collision model was previously proposed in [9] and [10]. Such a rule is required to avoid physical collision when a neighbor takes a worst case action and there is a delay of k seconds to react to the action by the two vehicles involved. Notice, the worst case action is assumed to be a sudden direction change by two vehicles towards each other. We assume in this paper that mobile nodes are aware of the positions of the static nodes close to them and the fact that the static nodes do not move. Hence, they do not require such a buffer to avoid collision with the static nodes. Since each of the mobile agents and static nodes are simply a point, means that the mobile nodes can move arbitrarily close to the static nodes. Hence we will allow the mobile nodes to physically occupy the same position as the static nodes during the discussion in Section V, without losing any generalization.

Each mobile agent has the capability of receiving and transmitting data over the wireless media but they do not have their own data to send, i.e., they can act as relays (both by physically carrying the bits and by sending them through wireless transmissions) for the bits originating at the static nodes. Each node and agent can choose arbitrary range or power level for each transmission. The mobile nodes are constrained to stay inside the environment at all times. In the paper we will use the term nodes to refer to static nodes or mobile agents and will explicitly state when we want to differentiate. We describe when a transmission is received successfully by its intended recipient. We use the protocol model described in [1].

B. The Protocol Model

Suppose node X_i transmits over the cth subchannel to another node X_j. Then this transmission is successfully received by X_j if

$$|X_k - X_j| \geq (1+\Delta)|X_i - X_j| \quad (2)$$

for every other node X_k simultaneously transmitting over the same subchannel. The quantity $\Delta > 0$ models situations where a guard zone is specified by the protocol.

We work with the same definitions of throughput and delay that have previously been presented in [3].

Definition 2.1: A throughput $\lambda > 0$ is said to be feasible/achievable if every node can send at an average rate of λ bits per second to its chosen destination.

Definition 2.2: The delay of a bit in a network is the time it takes the bit to reach its destination after it leaves the source. We do not take queueing delay at the source into account, since our interest is in the network delay. The maximum delay for the network is the maximum delay over all bits transported. In our network, the delay for a packet is the sum of the times spent at each relay.

We write $f(n) = O(g(n))$ if there exists a positive constant c_1 such that $f(n) \leq c_1 g(n)$ for all n large enough. Similarly, we write $f(n) = \Omega(g(n))$ if there exists a positive constant c_2 such that $f(n) \geq c_2 g(n)$ for all n large enough. We say that $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$. We say $f(n) = \omega(g(n))$ if $\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$.

III. Time Complexity of Vehicle Routing

Consider a situation where p agents are at their initial way-points $W_i^{\text{init}}, i \in \{1, .., p\}$ respectively at time $t = 0$. Each agent $\mathcal{A}_i$ has a next way-point W_i^{next} assigned to it that it has to visit. The following Lemma, which is a result of the previous work done in [9] and [10], gives a lower bound on the time it takes for the last agent to visit its corresponding next way-point.

Lemma 3.1: For a set of p agents located at points $W_i^{\text{init}}, i \in \{1, .., p\}$ at time $t = 0$, the time by which all of them have visited their respective next way-point W_i^{next} without colliding is at least $\sqrt{\pi}k\sqrt{p}L/2$, where L is the average distance between the initial and next way-points.

Proof: Let the time at which the last agent reaches its respective next way-point be denoted by T^*. Let us assume that the motion of all the agents can be represented as a set of straight-line motions, over a common, synchronized time schedule of length h. For simplicity, let us assume that each time interval has the common duration $\bar{\tau}$ and $T^* = h\bar{\tau}$. Let us denote by r_i^j the length of the straight-line segment along which the i-th agent moves during the j-th time interval. Obviously, we have $\sum_{j=1}^{h} r_i^j \geq L_i$, where $L_i = |W_i^{\text{init}} - W_i^{\text{next}}|$, and

$$\sum_{i=1}^{p}\sum_{j=1}^{h} r_i^j \geq pL, \tag{3}$$

where $L = \frac{1}{p}\sum_{i=1}^{p} L_i$.

Assuming the velocity of any agent i during a time interval j stays constant and is denoted by v_i^j, we define $\delta_i^j = kv_i^j = kr_i^j/\bar{\tau}$. Hence the exclusion region area at any time during interval j for agent i is given by

$$A_i^j = \pi\left(\delta_i^j\right)^2 = \pi k^2 \left(\frac{r_i^j}{\bar{\tau}}\right)^2$$

The collision avoidance constraint requires that at any time during a time interval, the sum of the areas of the exclusion regions of all agents lying inside the environment $\mathcal{Q}$ cannot exceed the area of the environment. Since the environment chosen is a square, this means that at least one fourth of each exclusion region is within $\mathcal{Q}$. Hence, the sum of the areas of the exclusion regions of all agents at any time during any time interval cannot exceed four times the area of the environment, i.e.,

$$\frac{\pi k^2}{\bar{\tau}^2}\sum_{i=1}^{p}\left(r_i^j\right)^2 = \sum_{i=1}^{p} A_i^j \leq 4.$$

Summing over all intervals in the time schedule, and rearranging, we get

$$\sum_{i=1}^{p}\sum_{j=1}^{h}\left(r_i^j\right)^2 < \frac{4h\bar{\tau}^2}{\pi k^2}. \tag{4}$$

Consider a convex function $f : \Re \to \Re$; Jensen's inequality states that

$$f\left(\frac{1}{P}\sum_{p=1}^{P} x_p\right) \leq \frac{1}{P}\sum_{p=1}^{P} f(x_p).$$

Since the function $x \mapsto x^2$ is convex, we can apply Jensen's inequality to (4) to obtain

$$\left(\sum_{i=1}^{p}\sum_{j=1}^{h} r_i^j\right)^2 \leq hp\sum_{i=1}^{p}\sum_{j=1}^{h}\left(r_i^j\right)^2 < \frac{4h^2\bar{\tau}^2 p}{\pi k^2},$$

that is,

$$\sum_{i=1}^{p}\sum_{j=1}^{h} r_i^j < \left(\frac{4(T^*)^2 p}{\pi k^2}\right)^{\frac{1}{2}}. \tag{5}$$

Thus, from (3) and (5) we get

$$T^* > \left(\frac{\pi k^2}{4}\right)^{\frac{1}{2}}\sqrt{p}L.$$

Since we have made no assumptions on the time schedule, this bound applies in the limit as $\bar{\tau} \to 0$, i.e., for continuous schedules, which proves the result. ■

This Lemma will be used to obtain a lower bound on the maximum delay in data delivery in the next section.

IV. A Lower Bound on the Maximum Delay

We consider the setting on a planar square environment of unit area. We work with the following set of assumptions:

- There are n static nodes arbitrarily located in a square of unit area on the plane.
- The network transports λnt bits over t seconds, where t is chosen large enough and $\lambda > 0$ is the throughput per static node achieved by the network.
- The average distance between the source and destination of bits is $\bar{L}$ and $\bar{L} = \omega(1/\sqrt{n})$, i.e. $1/\sqrt{n}\bar{L} \to 0$ as $n \to \infty$.
- There are $m \leq \beta n$ mobile agents in the environment, which move in an arbitrary manner without colliding. Their motion is such that they stay inside the environment all the time and do not collide.
- Each node and agent can transmit over any subset of C subchannels with capacities W_c bits per second, $1 \leq c \leq C$, where $\sum_{c=1}^{C} W_c = W$. We assume in this paper,

for ease of exposition, that $W_c = w, \forall c \in \{1,..,C\}$. The analysis and proof for the case when we do not make this assumption follows on the same lines.

- Transmissions are slotted into synchronized slots of length $\tau = 1/w$ seconds. This means a transmitter (receiver) can send (receive) exactly one bit on any subchannel in any slot since it takes $1/w$ seconds for the transmission of a bit.

Theorem 4.1: If by using some scheme of transporting bits from their sources to their destinations, the network achieves a constant throughput, then the maximum delay incurred by the bits transported in the network is $\Omega(\sqrt{n}\bar{L})$.

Proof: Consider bit b, where $1 \le b \le \lambda nt$. Let us suppose that it moves from its origin to its destination in a sequence of $h(b)$ hops, where during the hth hop the distance between the sender and the receiver is $r^1_{b,h}$ at the end of the transmission and let $r^2_{b,h}$ be the distance between the point where the receiver is at the completion of bit b's reception and the point where this receiver completes the bit's retransmission during the $h+1$th hop. Notice, $r^1_{b,h}$ is the distance between the sender and the receiver of bit b in its hth hop at the end of the time slot in which this hop happens. Notice $r^2_{b,h}$ can be non-zero only if the receiver is a mobile agent. Then

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}(r^1_{b,h}+r^2_{b,h}) \ge \lambda nt\bar{L} \tag{6}$$

Note that in any slot at most $(\beta+1)n/2$ bits can be transmitted over a subchannel. Hence for any subchannel c and any slot s

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } s) \le \frac{(\beta+1)n}{2}$$

Summing over the subchannels and the slots, and noting that there can be no more than t/τ slots in t seconds, yields

$$H := \sum_{b=1}^{\lambda nt} h(b) \le \frac{(\beta+1)Wtn}{2} \tag{7}$$

Dividing both sides by the total number of bits gives

$$\frac{H}{\lambda nt} \le \frac{(\beta+1)W}{2\lambda} \tag{8}$$

Since λ is a constant, the above inequality suggests that the average hops per bit are less than some constant.

Consider two simultaneous transmissions from node X_i to X_j and X_k to X_l over the same subchannel. The triangle inequality applied to the positions of these nodes at any time during the transmission and (2) gives

$$\begin{aligned} d(X_j,X_l) &\ge d(X_j,X_k) - d(X_l,X_k) \\ &\ge (1+\Delta)d(X_i,X_j) - d(X_l,X_k), \end{aligned}$$

where $d(.,.)$ is a distance function. Similarly,

$$d(X_l,X_j) \ge (1+\Delta)d(X_k,X_l) - d(X_j,X_i)$$

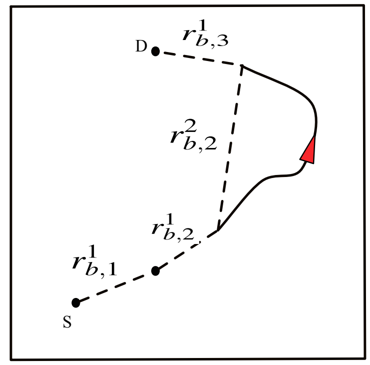

Fig. 2. A figure showing three hops that bit b takes to reach its destination D starting from source S

Adding the two inequalities, we obtain

$$d(X_l,X_j) \ge \frac{\Delta}{2}(d(X_k,X_l)+d(X_i,X_j)).$$

Now since a bit's transmission takes an entire time slot, the above has to be satisfied throughout the transmission including the end of the time slot. This implies that if we place a disk around each receiver of radius $\Delta/2$ times the length of the hop (which is measured as the distance between the receiver and sender at the end of the transmission) centered at their positions at the end of the transmission, the disks should be disjoint for successful transmission under the protocol model. Hence the sum of area of all these disks during a slot s in the cth subchannel lying in the environment is at most the area of the environment. Since at least one-fourth of the area of such disks lies inside the environment, we have

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } s)\frac{\pi\Delta^2}{16}(r^1_{b,h})^2 \le 1$$

Summing over all the subchannels and the slots gives

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}\frac{\pi\Delta^2}{16}(r^1_{b,h})^2 \le Wt \tag{9}$$

Since the quadratic function is convex, we have

$$\left(\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}\frac{1}{H}r^1_{b,h}\right)^2 \le \sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}\frac{1}{H}(r^1_{b,h})^2. \tag{10}$$

Combining (9) and (10) yields

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} r^1_{b,h} \le \frac{4\sqrt{WtH}}{\sqrt{\pi}\Delta} \tag{11}$$

Now substituting (6) in (11) gives

$$\lambda nt\bar{L} - \sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} r^2_{b,h} \le \frac{4\sqrt{WtH}}{\sqrt{\pi}\Delta}.$$

This can be rewritten as

$$\frac{1}{\lambda nt}\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} r_{b,h}^2 \geq \bar{L} - \frac{4}{\Delta}\sqrt{\frac{W}{\lambda n\pi}}\sqrt{\frac{H}{\lambda nt}} \tag{12}$$

The above inequality and (8) along with the facts that λ is a constant and $\bar{L} = \omega(1/\sqrt{n})$ imply that the right hand side can be made arbitrarily close to $\bar{L}$ by choosing n large enough. So one can find a constant c_1 and N such that for all $n > N$ the following holds

$$\frac{1}{\lambda nt}\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)} r_{b,h}^2 \geq c_1\bar{L} \tag{13}$$

The above inequality means that the average distance travelled physically by bits on mobile agents is greater than some constant times $\bar{L}$ for all large enough values of n. This inequality can be rewritten as

$$\frac{1}{\lambda nt}\sum_{s=1}^{tw}\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}\sum_{c=1}^{C} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } s) r_{b,h}^2 \geq c_1\bar{L}.$$

This implies that

$$\frac{tw}{\lambda nt}\max_{s\in\{1,..,tw\}}\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}\sum_{c=1}^{C} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } s) r_{b,h}^2 \geq c_1\bar{L}.$$

Let

$$\tilde{s} = \arg\max_{s\in\{1,..,tw\}}\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}\sum_{c=1}^{C} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } s) r_{b,h}^2.$$

Hence, the following holds

$$\sum_{b=1}^{\lambda nt}\sum_{h=1}^{h(b)}\sum_{c=1}^{C} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } \tilde{s}) r_{b,h}^2 \geq \frac{c_1\bar{L}\lambda n}{w}. \tag{14}$$

Let us define the following

$$H_{\tilde{s},i} = \{b \in \{1,..,\lambda nt\} | \sum_{h=1}^{h(b)}\sum_{c=1}^{C} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } \tilde{s} \text{ to } \mathcal{A}_i) = 1\}.$$

In other words, $H_{\tilde{s},i}$ is the set of bits that are transmitted during the slot $\tilde{s}$ to mobile agent $\mathcal{A}_i, i \in \{1,..,m\}$. Notice a bit can be transmitted only once during a slot since it takes $1/w$ seconds to transmit a bit on any channel and the slot size is $1/w$. Let $M_{\tilde{s}} = \{i \in \{1,..,m\} | H_{\tilde{s},i} \neq \emptyset\}$. Let us define the following for every $b \in H_{\tilde{s},i}$

$$l_{\tilde{s},i}^b = \sum_{h=1}^{h(b)}\sum_{c=1}^{C} 1(\text{The } h\text{th hop of bit } b \text{ is over subchannel } c \text{ in slot } \tilde{s} \text{ to } \mathcal{A}_i) r_{b,h}^2.$$

Now (14) can be rewritten as

$$\sum_{i\in M_{\tilde{s}}}\sum_{b\in H_{\tilde{s},i}} l_{\tilde{s},i}^b \geq \frac{c_1\bar{L}\lambda n}{w}.$$

This inequality implies that the following holds

$$\sum_{i\in M_{\tilde{s}}} |H_{\tilde{s},i}| \max_{b\in H_{\tilde{s},i}} l_{\tilde{s},i}^b \geq \frac{c_1\bar{L}\lambda n}{w}.$$

Since there are C channels and each receiver can receive at most C bits during a slot implies that $|H_{\tilde{s},i}| \leq C$ and hence the following holds

$$\frac{1}{|M_{\tilde{s}}|}\sum_{i\in M_{\tilde{s}}}\max_{b\in H_{\tilde{s},i}} l_{\tilde{s},i}^b \geq \frac{c_1\bar{L}\lambda n}{wC|M_{\tilde{s}}|}. \tag{15}$$

Let

$$b_i = \arg\max_{b\in H_{\tilde{s},i}} l_{\tilde{s},i}^b.$$

Hence (15) becomes

$$\frac{1}{|M_{\tilde{s}}|}\sum_{i\in M_{\tilde{s}}} l_{\tilde{s},i}^{b_i} \geq \frac{c_1\bar{L}\lambda n}{W|M_{\tilde{s}}|}. \tag{16}$$

This equation means that there are $|M_{\tilde{s}}|$ mobile agents where each $\mathcal{A}_i, i \in M_{\tilde{s}}$ has to carry a bit b_i respectively, from a position where it is at the end of the slot $\tilde{s}$ to a position which is at a distance $l_{\tilde{s},i}^{b_i}$ where it will complete the retransmission of this bit and the average of these distances is at least $\frac{c_1\bar{L}\lambda n}{W|M_{\tilde{s}}|}$. By Lemma 3.1 this means that the time taken before all these bits get retransmitted will be $\Omega(\sqrt{|M_{\tilde{s}}|}\frac{c_1\bar{L}\lambda n}{W|M_{\tilde{s}}|})$ or $\Omega(\frac{c_1\bar{L}\lambda n}{W\sqrt{|M_{\tilde{s}}|}})$. Since $|M_{\tilde{s}}| \leq \beta n$ implies that at least one bit takes $\Omega(\frac{c_1\bar{L}\lambda\sqrt{n}}{W\sqrt{\beta}})$ time to reach its destination. ■

V. Achievability Result on the Maximum Delay

We will now show that the order of the lower bound in the previous section is sharp by exhibiting a scenario where it is achieved.

Theorem 5.1: There is a placement of static nodes, choice of data traffic, scheduling of transmissions and a scheme of collision-free mobility of agents such that the network can achieve a constant throughput per static node and the maximum delay is $O(\sqrt{n}\bar{L})$, where $\bar{L}$ is the average distance between the origin and destination of bits.

Proof: Divide the environment $\mathcal{Q}$ into a grid of n identical squares. Place the n static nodes at the centers of these squares. We assume that $\sqrt{n}$ is an even natural number. It can be shown that we do not lose any generality with this. Let us denote the bottom left corner of the environment as the origin of coordinate axes. Then the positions of the n static nodes is given by

$$\left(\frac{1}{2\sqrt{n}} + i\frac{1}{\sqrt{n}}, \frac{1}{2\sqrt{n}} + j\frac{1}{\sqrt{n}}\right)$$

where $i, j = 0,..,\sqrt{n}-1$. Let for every value of $j \in \{0,..,\sqrt{n}-1\}$, the destination of data originating at the static node at location $(\frac{1}{2\sqrt{n}} + i\frac{1}{\sqrt{n}}, \frac{1}{2\sqrt{n}} + j\frac{1}{\sqrt{n}})$ for every $i \in \{0,..,\sqrt{n}/2-1\}$ be the static node at location

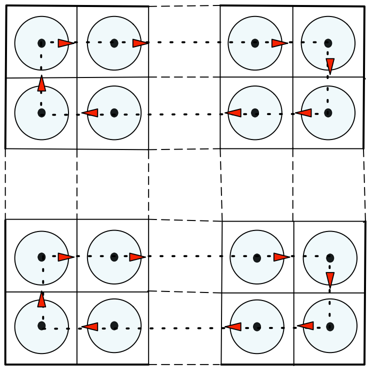

Fig. 3. The figure showing the set up for the upper bound. The dots are the static nodes, the circles around each static node are their safe-transmission zones, the dotted rectangular closed curves are curves along which the agents move and the red arrowheads are the agents moving along these curves.

$(\frac{1}{2\sqrt{n}} + (i+\sqrt{n}/2)\frac{1}{\sqrt{n}}, \frac{1}{2\sqrt{n}} + j\frac{1}{\sqrt{n}})$ respectively and vice-versa. Clearly with this kind of origin destination assignment, the average distance between the source and destination of every bit is $1/2$, a constant. Let us define a safe-transmission zone around each static node to be a circle around the node of radius $r = \frac{1}{(2+\Delta)\sqrt{n}}$. We put n mobile agents in the environment and at time 0, each one of them is located at the location of one exclusive static node.

We generate $\sqrt{n}/2$ closed curves by doing the following for every $j = 0, 2, .., \sqrt{n}-2$: we join the point $(\frac{1}{2\sqrt{n}}, \frac{1}{2\sqrt{n}} + j\frac{1}{\sqrt{n}})$ to the point $(\frac{1}{2\sqrt{n}}, \frac{1}{2\sqrt{n}}+(j+1)\frac{1}{\sqrt{n}})$ with a straight line and then this point with the point $(\frac{1}{2\sqrt{n}}+(\sqrt{n}-1)\frac{1}{\sqrt{n}}, \frac{1}{2\sqrt{n}}+(j+1)\frac{1}{\sqrt{n}})$ with a straight line and then this point with the point $(\frac{1}{2\sqrt{n}} + (\sqrt{n}-1)\frac{1}{\sqrt{n}}, \frac{1}{2\sqrt{n}} + j\frac{1}{\sqrt{n}})$ and then this point back to the first point $(\frac{1}{2\sqrt{n}}, \frac{1}{2\sqrt{n}} + j\frac{1}{\sqrt{n}})$ making a closed rectangular curve. We make each of the agents that are lying on their respective exclusive curve to start moving in a clockwise direction along that curve with a velocity $v = \frac{\eta}{4k\sqrt{n}}, \eta < 1$. This ensures that exclusion regions of no two mobile agents overlap throughout their motion. We construct a transmission schedule now. A static node transmits its own data to a mobile agent on all channels at a total rate of W bits per second when the agent is inside the static node's safe-transmission zone and moving away from the node. A mobile agent transfers data if it has any destined for a static node at the rate W bits per second when it is inside the safe-transmission zone of that node and moving towards it. It can be seen that no interference of transmissions can occur with the choice of the safe-transmission zone radius under the protocol model. The throughput that each node achieves is the fraction of time each static node gets to transmit its data times W, which is $\frac{W}{(2+\Delta)\sqrt{n}}\sqrt{n} = \frac{W}{2+\Delta}$. The maximum time that it can take for the delivery of any bit is the maximum distance between a pair of origin destination of bits, along the clockwise direction on the rectangular curve they lie on, divided by the velocity of a mobile agent, which is $\frac{3}{2}\frac{1}{v} = \frac{6k\sqrt{n}}{\eta}$. ∎

VI. Conclusion

In this paper, we have characterized the maximum delay incurred by any arbitrary network of n static nodes, with constant throughput requirements, that wishes to transport bits to destinations an average distance $\bar{L}$ away using $m \leq \beta n$ mobile agents. The analysis is restricted to cases where the use of mobility is critical to achieve constant throughput by restricting $\bar{L} = \omega(1/\sqrt{n})$. We show that in order to achieve constant per node throughput for such a network, the average distance that bits have to be physically transported on mobile agents grows proportional to $\bar{L}$. Since mobile agents have to stay a velocity dependent distance away from each other for safety, their motion becomes increasingly constrained with increasing number of such agents in a bounded environment. The main contribution of our work is in motivating and quantifying the scaling down of average velocity with the increased number of agents and how this translates into characterizing the maximum delay in the network. We have shown that for the arbitrary networks considered in this paper, the maximum delay is $\Omega(\sqrt{n}\bar{L})$. We also show there are networks for which a constant throughput can be achieved while ensuring that the maximum delay is $O(\sqrt{n}\bar{L})$. In future we intend to conduct the same analysis for random networks.

VII. Acknowledgments

This material is based upon work supported by the National Science Foundation under ITR Grant No. 0325716. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

[1] P. Gupta and P. R. Kumar, The Capacity of Wireless Networks, *IEEE Transactions on Information Theory*, vol. 46, 2000, pp 388-404.

[2] M. Grossglauser and D. Tse, Mobility Increases the Capacity of Ad Hoc Wireless Networks, *IEEE Transactions on Networking*, vol. 10, no. 4, 2002, pp. 477-486.

[3] Abbas El Gamal, James Mammen, Balaji Prabhakar and Devavrat Shah, "Throughput-Delay Trade-Off in Wireless Networks", *in proceedings of IEEE INFOCOM*, Hong-Kong, 2004.

[4] N. Bansal and Z. Liu, "Capacity, Mobility and Delay in Wireless Ad Hoc Networks", *in proceedings of IEEE INFOCOM*, San Francisco, 2003.

[5] S. N. Diggavi, M. Grossglauser and D. Tse, "Even One-Dimensional Mobility Increases Ad Hoc Wireless Capacity", *in proceedings of ISIT 2002*, Laussane, Switzerland, 2002.

[6] M. Neely and E Modiano, Capacity and Delay Tradeoffs for Ad Hoc Mobile Networks, *to appear in IEEE Transactions on Information Theory*.

[7] E. Perevalov and R. Blum, "Delay Limited Capacity of Ad Hoc Networks: Asymptotically Optimal Transmission and Relaying Strategy", *in proceedings of IEEE INFOCOM*, San Francisco, 2003.

[8] X. Lin and N. B. Shroff, The Fundamental Capacity-Delay Tradeoff in Large Mobile Wireless Networks, *submitted to IEEE Transactions on Information Theory, 2004*.

[9] M. Savchenko and E. Frazzoli, "On the Time Complexity of the Multiple-Vehicle Coordination Problem", *accepted in the American Control Conference*, 2005.

[10] V. Sharma, M. Savchenko, E. Frazzoli and P. G. Voulgaris, "Time Complexity of Conflict-Free Vehicle Routing", *submitted to the Robotics: Science and Systems Conference*, 2005.