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# Noise tolerant iterative learning control for identification of continuous-time systems

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*Abstract*— **The paper is concerned with both iterative learning control (ILC) and identification of continuous-time systems based on sampled I/O data in the presence of measurement noise. First, we propose a new ILC method which achieves tracking for uncertain plants by iteration of trials. The distinguished feature of this method is that (i) the leaning law works in a finite dimensional parameter space rather than the infinite dimensional input space and (ii) it takes account of noise reduction by using I/O data of all past trials efficiently. Second, it is shown how to estimate parameters of a class of linear continuous-time systems based on the proposed ILC method in noisy circumstances. Its effectiveness is demonstrated through numerical examples.**

## I. INTRODUCTION

One of the most important issues in control system design is to obtain an accurate model of the plant to be controlled. Though most of the existing identification methods are described in discrete-time, it would be convenient to have continuous-time models directly from sampled I/O data. Because most design tools are suitable for continuous-time systems and it is easy for us to capture the plant dynamics intuitively in continuous-time rather than discrete-time.

The fundamental difficulty of the direct identification approach is that it inherently requires time-derivative of I/O data in the presence of measurement noise. Therefore, a lot of efforts has been made to cope with this difficulty. A comprehensive survey of these techniques has been first given by [17] and then by [15]. A book has also been devoted to these so-called direct methods [13]. The CONtinuous-Time System IDentification (CONTSID) tool-box has been developed on the basis of these methods [6], [7], [5].

On the other hand, as for one of powerful model-free control methods, iterative learning control (ILC) has attracted much attention for the last two decades (see e.g., [2], [11], [10], [3], [4], [16], [1]). This method yields the desired input which achieves perfect tracking by iteration of trials for uncertain systems. Though ILC does not need any plant models, most ILC approaches need time-derivative of I/O data [14] and it is sensitive to measurement noise. Recently, Hamamoto et al. [8], [9] propose an ILC where learning law works in the prescribed finite-dimensional subspace, showing that even when precise information of the model is not available, time-derivative of tracking error is not required to achieve perfect tracking. Following this work, Sakai et al. [12] has shown that this type of ILC can be used for identification of a class of linear continuous-time systems. Though this method is more robust against measurement noise compared to exiting ILC, it does not seem to be enough for identification. Partly because, though it takes account of noise reduction in one trial, each trial is independent and the method does not use abundant past I/O data in other trials so far. Also, since accurate information on tracking error plays an essential role in any ILC, it is a long-standing critical issue to make ILC robust against measurement noise.

The purpose of this paper is two-hold. One is to propose a new ILC method which is robust against measurement noise. The method utilizes the ILC with a finite-dimensional prescribed I/O space proposed by Hamamoto, and reduces the noise effect by employing the past trial data effectively. Also, a concrete design procedure is given based on I/O sampled-data. The other is to demonstrate that the proposed ILC can be adopted for identification of a class of linear continuous-time systems. This provides an alternative identification methods from an entirely different viewpoint from the conventional one. One advantage is that parameters of the continuous-time system can be identified without the use of time-derivative of I/O data. In addition, since system parameters are identified while performing a tracking control to the reference trajectory, we are able to confirm the identification accuracy by watching the tracking error quantity. Detailed simulation study shows the effectiveness (such as robustness against noise) of the proposed method.

In this paper, the superscript of the variables denotes the trial number of the experiment and the subscript of those denotes the element number of a set or a matrix. Namely the input $u$ of the $k$th trial is denoted by $u^k$ and the $i$th element of the vector $\boldsymbol{x}$ is denoted by $\boldsymbol{x}_i$. We use the norm for the vector $\boldsymbol{x}$ as follows:

$$\|\boldsymbol{x}\| \triangleq \sqrt{\langle \boldsymbol{x},\ \boldsymbol{x} \rangle}.$$

## II. SYSTEM DESCRIPTION AND PROBLEM SETTING

Consider the continuous-time SISO system described by

$$\sum_{i=0}^{n_f} \alpha_i \frac{\mathrm{d}^i y(t)}{\mathrm{d}t^i} = u(t), \qquad t \in [0, \quad T] \tag{1}$$

where $u(t) \in L_2[0,\ T]$ and $y(t) \in L_2[0,\ T]$ are the input and the output, respectively, and $\alpha_i \in \mathbb{R}$ $(i = 0, 1, \cdots, n_f)$ are coefficient parameters. We assume the following:

- The initial state is at the origin (i.e., $y(0) = 0$, $\dot{y}(0) = 0$, $\cdots$ and so on).
- The system order $n_f$ is known.

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- The parameters $\alpha_i$ $(i = 0, 1, \cdots, n_f)$ are unknown.
- We can measure $\tilde{y}(t)$ , the output contaminated with noise, at the sampling time, i.e.,

$$\tilde{y}(t) = y(t) + w(t), \qquad t = 0, T_s, 2T_s, \cdots, NT_s \quad (2)$$

  where $T_s$ is the sampling period satisfying $T/T_s = N$ $(\in \mathbb{Z})$ and $\{w(iT_s)\}$ $(i = 0, 1, 2, \cdots, N)$ are zero mean white noises.
- We can repeat the experiments at the same initial condition on the time interval $[0,\ T]$.
- The reference signal $r \in L_2[0,\ T]$ is given in advance, which is smooth and its initial state is at the origin.

For the system, we consider the following two problems:

**A) ILC problem:** Find the input update law $\{\phi,\ \xi\}$

$$\nu^{k+1}(t) = \phi(\nu^k(t), r(t), \tilde{y}^k(t)) \quad (3)$$
$$u^k(t) = \xi(\nu^k(t)) \quad (4)$$

such that the tracking error $e(t)$ satisfies

$$e^k(t) \triangleq r(t) - y^k(t) \to 0, \quad (k \to \infty)$$

Here, the superscript $k$ denotes the trial number and $\nu(t)$ is an appropriate intermediate state variable for learning.

**Remark 1:** In the conventional ILC case, we use $\nu^k(t) = u^k(t)$, $\phi(\nu^k(t), r(t), \tilde{y}^k(t)) = \nu^k(t) + H\dot{e}^k(t)$ and $\xi(\nu^k(t)) = \nu^k(t)$.

**B) ID problem:** Based on I/O data $\{u(iT_s), \tilde{y}(iT_s)\}$ $(i = 0, 1, 2, \cdots, N)$, determine the parameters $\alpha_i$ $(i = 0, 1, \cdots, n_f)$.

## III. Noise tolerant ILC based on sampled data

In this section, we propose a noise tolerant ILC method based on sampled I/O data.

### A. Preliminaries

Let $V_d(t)$ and $\boldsymbol{\alpha}^*$ be defined by

$$\begin{aligned} V_d(t) &\triangleq \left[v_{d0}(t), v_{d1}(t), \ldots, v_{dn_f}(t)\right] \\ &\triangleq \left[r(t), \frac{\mathrm{d}r(t)}{\mathrm{d}t}, \ldots, \frac{\mathrm{d}^{n_f} r(t)}{\mathrm{d}t^{n_f}}\right] \\ \boldsymbol{\alpha}^* &\triangleq [\alpha_0, \alpha_1, \ldots, \alpha_{n_f}]^T. \end{aligned}$$

Note that $\boldsymbol{\alpha}^*$ represents the true parameter value. Then, (1) implies that the optimal input $u^*(t)$ defined by

$$u^*(t) = V_d(t)\boldsymbol{\alpha}^*$$

achieves perfect tracking, i.e., $y(t) = r(t)$. Therefore, it is enough for us to seek $u^*(t)$ in the finite dimensional parameter space ($\mathbb{R}^{n_f+1}$ ) rather than the infinite dimensional one (i.e., $L_2[0,\ \ T]$) in ILC. So we update the input of the form

$$u^k(t) = V_d(t)\boldsymbol{\alpha}^k. \quad (5)$$

This is one of the most important observation made by Hamamoto et al. [8]. We will try to find the update algorithm for $\boldsymbol{\alpha}^k$.

Now, we use the following symbols. Since data are available only on sampled time, we define

$$\boldsymbol{u}^k \triangleq [u^k(0), u^k(T_s), \cdots, u^k(NT_s)]^T \in \mathbb{R}^{N+1}$$
$$\tilde{\boldsymbol{y}}^k \triangleq [\tilde{y}^k(0), \tilde{y}^k(T_s), \cdots, \tilde{y}^k(NT_s)]^T \in \mathbb{R}^{N+1}$$

In the same way, $\boldsymbol{u}^* \in \mathbb{R}^{N+1}$, $\boldsymbol{y}^k \in \mathbb{R}^{N+1}$, $\boldsymbol{r} \in \mathbb{R}^{N+1}$, $\boldsymbol{e}^k \in \mathbb{R}^{N+1}$, and $\boldsymbol{w}^k \in \mathbb{R}^{N+1}$ are defined. Also, the error measurement $\tilde{\boldsymbol{e}}^k \in \mathbb{R}^{N+1}$ are defined from

$$\tilde{e}(t) \triangleq r(t) - \tilde{y}(t)\ (= e(t) - w(t)).$$

Let $\tilde{V}_d \in \mathbb{R}^{(N+1)\times(n_f+1)}$ be defined by

$$\tilde{V}_d \triangleq \begin{bmatrix} v_{d0}(0) & v_{d1}(0) & \ldots & v_{dn_f}(0) \\ v_{d0}(T_s) & v_{d1}(T_s) & \ldots & v_{dn_f}(Ts) \\ \vdots & \vdots & \ldots & \vdots \\ v_{d0}(NT_s) & v_{d1}(NT_s) & \ldots & v_{dn_f}(NT_s) \end{bmatrix},$$

which yields $\boldsymbol{u}^* = \tilde{V}_d\boldsymbol{\alpha}^*$. We assume

$$\mathrm{rank}(\tilde{V}_d^T\tilde{V}_d) = n_f + 1 \quad (6)$$

Further, let QR decomposition of $\tilde{V}_d$ be

$$\tilde{V}_d = UR, \qquad U^TU = I_{n_f+1}. \quad (7)$$

where $U \triangleq \left[\boldsymbol{f}_0, \boldsymbol{f}_1, \ldots, \boldsymbol{f}_{n_f}\right] \in \mathbb{R}^{(N+1)\times(n_f+1)}$ and $R \in \mathbb{R}^{(n_f+1)\times(n_f+1)}$ is a nonsingular upper triangular matrix. Define $\boldsymbol{a}^k \in \mathbb{R}^{n_f+1}$ by

$$\boldsymbol{a}^k \triangleq R\boldsymbol{\alpha}^k, \quad (8)$$

then $\tilde{V}_d\boldsymbol{\alpha} = U\boldsymbol{a}^k$ holds. So from (5), we obtain

$$\boldsymbol{u}^k = U\boldsymbol{a}^k. \quad (9)$$

Conversely, $\boldsymbol{a}^k$ is uniquely determined by

$$\boldsymbol{a}^k = U^T\boldsymbol{u}^k$$

Similarly, we define

$$\boldsymbol{a}^* \triangleq U^T\boldsymbol{u}^*, \quad \boldsymbol{b}^k \triangleq U^T\boldsymbol{y}^k, \quad \boldsymbol{b}^* \triangleq U^T\boldsymbol{r}^*,$$
$$\tilde{\boldsymbol{\varepsilon}}^k \triangleq U^T\tilde{\boldsymbol{e}}^k, \quad \boldsymbol{\varepsilon}^k \triangleq U^T\boldsymbol{e}^k, \quad \boldsymbol{\eta}^k \triangleq U^T\boldsymbol{w}^k.$$

Note that $\boldsymbol{a}^k$ and $\boldsymbol{b}^k$ are vector representation in the $n_f+1$ dimensional space $\mathrm{Im}\tilde{V}_d$ of input $\boldsymbol{u}^k$ and output $\boldsymbol{y}^k$, respectively, with respect to the bases $\{\boldsymbol{f}_0, \cdots, \boldsymbol{f}_{n_f}\}$.

### B. Learning Law

Now we propose the following learning law:

**[Proposed ILC]**

$$\boldsymbol{a}^{k+1} = H_1\boldsymbol{a}^k + H_2\boldsymbol{\xi}^k \quad (10)$$
$$\boldsymbol{\xi}^{k+1} = \boldsymbol{\xi}^k + \tilde{\boldsymbol{\varepsilon}}^{k+1} \quad (11)$$

for $k = 0, 1, 2, \cdots$ with the initial condition

$$\boldsymbol{a}^0 = 0, \qquad \boldsymbol{\xi}^0 = 0, \quad (12)$$

where $H_1 \in \mathbb{R}^{(n_f+1)\times(n_f+1)}$ and $H_2 \in \mathbb{R}^{(n_f+1)\times(n_f+1)}$ are gain matrices to be determined. Note that the plant (1) is represented by

$$\boldsymbol{b}^k = L_f\boldsymbol{a}^k \quad (13)$$

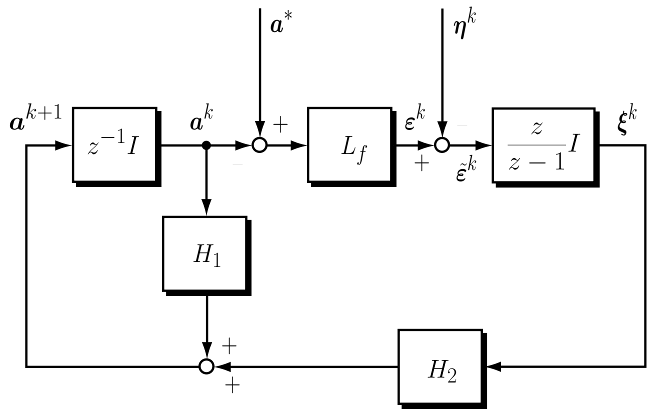


Fig. 1. Proposed learning law using integral operator

in this parameter space, where $L_f \in \mathbb{R}^{(n_f+1)\times(n_f+1)}$ is a constant matrix uniquely determined from the plant. We will discuss how to compute $L_f$ later. The proposed learning law is depicted in Fig. 1, where $z$ denotes the forward shift operator with respect to the trial number $k$ ( e.g., $\boldsymbol{a}^{k+1} = z\boldsymbol{a}^k$). Eq. (11) represents the integrator (or accumulator, more precisely), which is described by $z/(z-1)$.

Next, we consider the convergence condition of the proposed law. Noting that

$$\tilde{\varepsilon}^{k+1} = \boldsymbol{b}^* - \boldsymbol{b}^{k+1} - \boldsymbol{\eta}^{k+1}, \tag{14}$$

from (10), (11), (13), we have

$$\begin{bmatrix}\boldsymbol{a}^{k+1}\\ \boldsymbol{\xi}^{k+1}\end{bmatrix} = A_H \begin{bmatrix}\boldsymbol{a}^{k}\\ \boldsymbol{\xi}^{k}\end{bmatrix} + \begin{bmatrix}0 & 0\\ L_f & -I\end{bmatrix}\begin{bmatrix}\boldsymbol{a}^{*}\\ \boldsymbol{\eta}^{k+1}\end{bmatrix} \tag{15}$$

$$A_H \triangleq \begin{bmatrix}H_1 & H_2\\ -L_fH_1 & I-L_fH_2\end{bmatrix}$$

If $A_H$ is not a stable matrix (i.e., some eigenvalues are not inside the unit disc), the parameters does not converge. Therefore, suppose that $A_H$ is stable. In addition, temporally, consider the noise-free case (i.e., $\boldsymbol{\eta}^k = 0$). Then, as $k \to \infty$, we obtain

$$\begin{bmatrix}\boldsymbol{a}^{\infty}\\ \boldsymbol{\xi}^{\infty}\end{bmatrix} = (I - A_H)^{-1}\begin{bmatrix}0\\ L_f\end{bmatrix}\boldsymbol{a}^*$$

$$(I - A_H)\begin{bmatrix}\boldsymbol{a}^{\infty}\\ \boldsymbol{\xi}^{\infty}\end{bmatrix} = \begin{bmatrix}0\\ L_f\end{bmatrix}\boldsymbol{a}^*.$$

This implies that

$$\boldsymbol{a}^{\infty} = \boldsymbol{a}^*$$
$$H_1\boldsymbol{a}^{\infty} + H_2\boldsymbol{\xi}^{\infty} = \boldsymbol{a}^*.$$

Therefore, the parameter converges to the desired value, i.e.,

$$\boldsymbol{a}^k \to \boldsymbol{a}^*, \qquad (k \to \infty) \tag{16}$$

holds. So we have $\boldsymbol{u}^k \to \boldsymbol{u}^*$, which implies that tracking error $\boldsymbol{e}^k$ converges to 0. The above arguments is summarized as follows.

**Theorem 1:** For the plant (1), suppose we adopt the learning law (9), (10), (11). Then

$$\boldsymbol{e}^k \to 0, \qquad (k \to \infty)$$

is satisfied, if there is no measurement noise and the following condition holds.

$$\begin{bmatrix}H_1 & H_2\\ -L_fH_1 & I-L_fH_2\end{bmatrix} \text{is stable} \tag{17}$$

Note that the convergence dose not depend on each matrix value ($H_1$, $H_2$ or $L_f$) as long as $A_H$ is stable. This is one of the reasons why we adopt the integrator in the learning law.

As for noise effect reduction, we can state the following: Since $\boldsymbol{w}^k$ (noise) and each column of $U$ (determined by the reference signal) are not correlated, it is expected that each entry of $\boldsymbol{\eta}^k = U^T\boldsymbol{w}^k$ is small. In addition, with the aid of (11), (12) and (14), we have

$$\boldsymbol{\xi}^k = \sum_{i=0}^{k-1}(\boldsymbol{b}^* - \boldsymbol{b}^i) - \sum_{i=0}^{k}\boldsymbol{\eta}^i$$

So, from (10), the noise effect on $\boldsymbol{a}^{k+1}$ seems to be small. This is another reason why we adopt the integrator in the learning law. Since its effectiveness must be verified quantitatively through numerical simulation, we will perform simulation in the noisy circumstance later.

### C. Choice of learning gains

In what follows, we will show how to choose the learning gain. To this end, first we need to estimate $L_f$. Since, the plant (1) is SISO LTI, when we inject the input sequence $\boldsymbol{u}^k$, the corresponding output $\boldsymbol{y}^k$ is given by

$$\boldsymbol{y}^k = P\boldsymbol{u}^k \tag{18}$$

irrespective of $k$, where $P \in \mathbb{R}^{(N+1)\times(N+1)}$ is specified by

$$P = \begin{bmatrix}p_0 & 0 & 0 & \dots & 0\\ p_1 & p_0 & 0 & \dots & 0\\ p_2 & p_1 & p_0 & \dots & 0\\ \vdots & \vdots & \vdots & \dots & \vdots\\ p_N & p_{N-1} & p_{N-2} & \dots & p_0\end{bmatrix}.$$

The first column of $P$ is the output $\boldsymbol{y}$ when we inject the input $\boldsymbol{u} = [1, 0, 0, \cdots, 0]$. Since $\boldsymbol{u}^k = U\boldsymbol{a}^k$ and $\boldsymbol{b}^k = U^T\boldsymbol{y}^k$ hold, (18) implies that $\boldsymbol{b}^k = U^TPU\boldsymbol{a}^k$ is satisfied for arbitrary $\boldsymbol{a}^k$. That is, matrix $L_f$ is computed from $U$ and $P$ as follows:

$$L_f = U^TPU. \tag{19}$$

Therefore, we can obtain an estimate of $P$, say $\hat{P}$, by getting the impulse response through experiment. Then, we have an estimate of $L_f$ by computing $\hat{L}_f = U^T\hat{P}U$. Though the impulse response may be contaminated by measurement noise, its effect is reduced considerably by this matrix production operation.

As for the choice of $H_1$ and $H_2$, it is easy to find such gains that stabilize $A_H$. Since this is essentially the state feedback stabilization problem, we can adopt, for example, pole assignment technique. In this case, the pole location must be chosen from the trade-off between the convergence speed and noise tolerance.

### *D. Numerical example*

We will evaluate the effectiveness of the proposed method through simulation in this subsection.

Consider the third order plant whose transfer function is given by

$$P(s) = \frac{1}{s^3 + 10s^2 + 30s + 8}. \tag{20}$$

The time interval of each trial is $T = 10$[s], and the sampling period is 10[ms]. Namely, the number of data used for one trial is 1001. Suppose the reference signal $r(t)$ be the step response of the system whose transfer function is given by

$$P_r(s) = \frac{2^4}{(s+2)^4}. \tag{21}$$

The measurement noise of the output is the white one with zero mean and variance $\sigma^2$. The variance $\sigma^2$ of the measurement noise is chosen so that the noise to signal ratio (NSR) will be 10[%]. The NSR is defined as

$$\mathrm{NSR} \triangleq \frac{\|\boldsymbol{w}^k\|}{\|\boldsymbol{r}\|}.$$

What we have to choose in the proposed ILC is the learning gains $H_1$ and $H_2$. Employing the pole assignment technique, we choose them in such a way that all eigenvalues of $A_H$ are located at 0.9. Then, we perform hundred iteration of trials by using the proposed learning control law.

Fig. 2 shows the measured output $\tilde{y}(t)$ contaminated by noise at the 3rd, 25th, and 100th trials of the learning control. The reference signal is also shown by thick line. Though the output measurement is near zero all the time at 3rd trial, it looks that the tracking performance is good (at the 100th trial). In order to see the real performance, the true output $y(t)$ of each case is plotted in Fig. 3. It turns out that the true output of 100th trial is almost the same as the reference. There is no difference between them in this scale of Fig. 3. Fig. 4 shows the relation between the true tracking error $\|\boldsymbol{e}^k\|$ (rather than measured error) and the iteration number of trails. We can see that almost perfect tracking is achieved after 60th trail. It is surprising that the proposed learning control achieves such a nice performance in this noisy circumstances. These figures show that the proposed method is really noise tolerant.

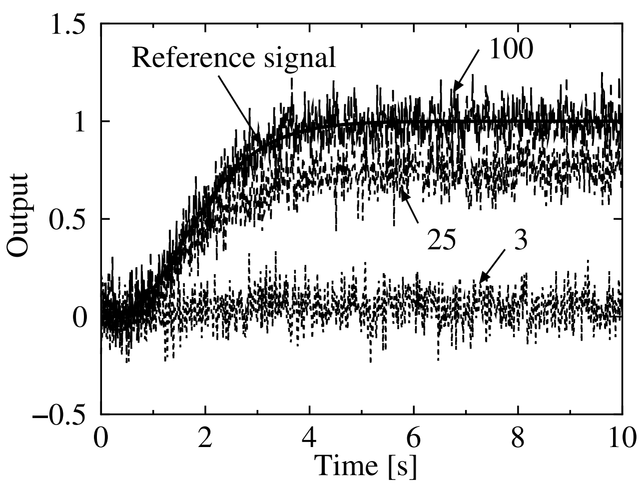


Fig. 2. Measured output behavior of the proposed ILC

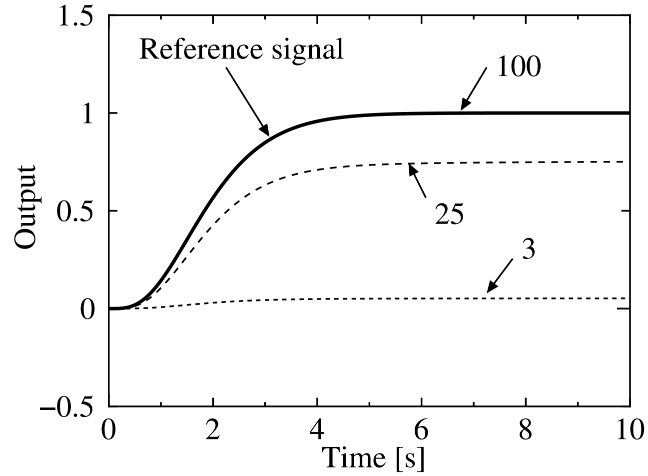


Fig. 3. True output behavior

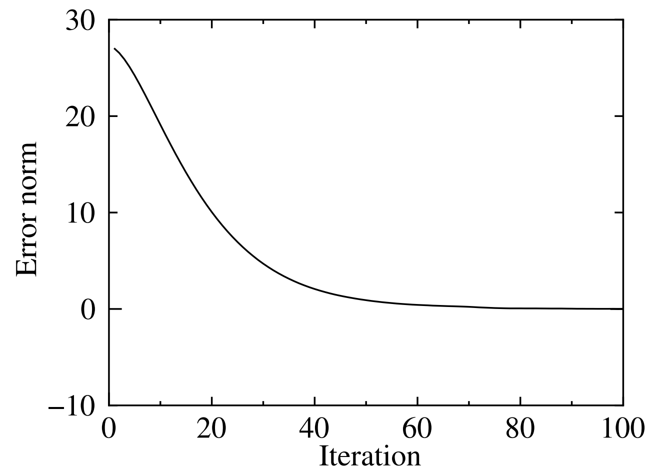


Fig. 4. Tracking error $\|\boldsymbol{e}^k\|$ vs iteration number

## IV. Application to identification of continuous-time systems

### *A. The identification algorithm*

When the learning control yields the desired input $\boldsymbol{u}^*$, we already have $\boldsymbol{a}^*$ as shown in (16). Therefore, from (8), we can obtain the system parameter by $\boldsymbol{\alpha}^* = R^{-1}\boldsymbol{a}^*$.

From the view point of identification, it is important how to (choose) the reference $r(t)$. At the current stage, we do not have any systematic methods of the choice. However, from various simulation experience, one candidate of $r(t)$ is

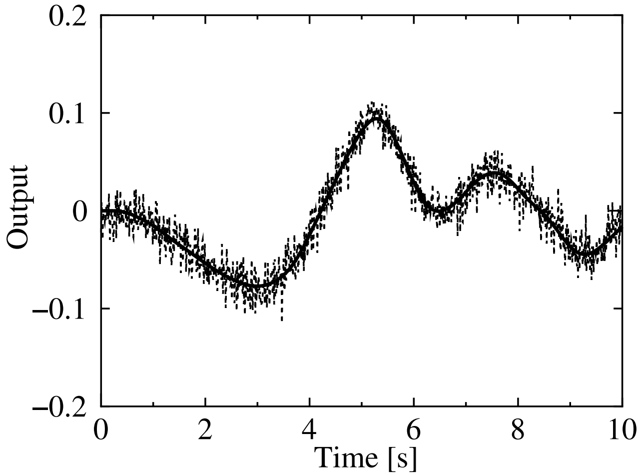


Fig. 5. $r(t)$ and $\tilde{y}(t)$ at 100th trial (NSR:30[%])

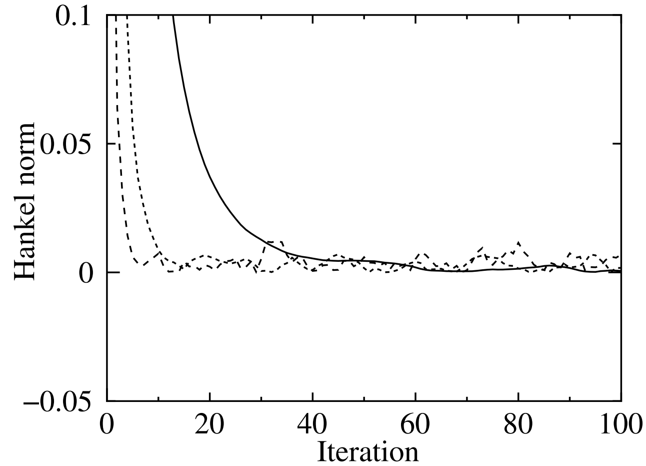


Fig. 7. Hankel norm $\|P(s) - \hat{P}(s)\|_H$ in each trial

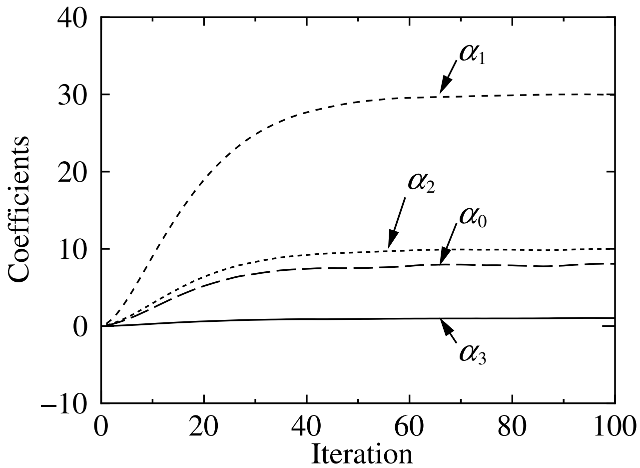


Fig. 6. Identifi ed coeffi cients $\hat{\alpha}^k$ in each trial

a white noise filtered by higher order linear systems. We will show this point through simulation in the following.

*B. Numerical evaluation*

We consider the same system (20) as the plant to be identified. The time horizon and the sampling time $T_s$ are also the same, i.e., $T = 10$[s] and $T_s = 10$[ms]. Also, we use the same learning gain as before (i.e., its pole location is at $p = 0.9$). The reference signal $r(t)$ is chosen to be the output of the system (21) when a white noise is injected, which is shown by the thick line in Fig. 5. In this identification, we set the NSR of the measured output to be 30[%].

The behavior of the measured output $\tilde{y}(t)$ at the 100th trial is also shown by dotted line in Fig. 5. This figure tells us that the output tracks the reference very well. So we can expect that an accurate model of the plant is obtained. In fact, this is the case. Fig.6 shows the estimated coefficients at each trial $k$ $(= 1, 2, \cdots, 100)$ in the proposed ILC. From this figure, we see that the denominator coefficients of the plant (i.e., $\alpha_0 = 8$, $\alpha_1 = 30$, $\alpha_2 = 10$, $\alpha_3 = 1$) are almost accurately estimated, though the convergence may be slow.

Furthermore, it is confirmed that, the identification accuracy is improved in each trial in terms of the Hankel norm of the error system $\|P(s) - \hat{P}(s)\|_H$ from Fig.7, where the solid line shows its value at each trial. The figure also shows the cases where the eigenvalues of $A_H$ are located at $p = 0.7$ and $p = 0.5$, respectively. When we use the ILC with pole location at $p = 0.5$, the error norm converges to the neighborhood of zero less than ten iteration as shown by the broken line, while its behavior is a little bit more sensitive to noise. The case with poles at $p = 0.7$ is shown by dotted line, which exhibits the convergence speed is slower but more insensitive to noise compared to the case of $p = 0.5$. Therefore, as stated in Section III, the pole location must be chosen by taking account of the trade-off between the convergence speed and noise tolerance.

*C. Comparison with conventional methods*

Finally, we try to give a comparison with some of the existing identification methods. Since the conditions of identification experiments, such as input signals to be used, are different, it seems to be difficult to perform a fair comparison. However, a comparison with the following identification methods may give us some idea about the effectiveness of the proposed method.

- The indirect method by the subspace identification method (N4SID)
- The direct method by the Instrumental-Vector State-Variable Filter approach (SVF)[7]
- The direct method by the Instrumental-Vector Fourier Modulating Function (FMF)[7]

The plant is the same as before. However, when we perform the above identification methods, the number of sampled I/O data is chosen to be 10000 which is ten times of the data in one ILC trial. The input signal is white noise and the identification experiments are conducted for various NSR of the measured output ($0 \sim 50$) [%]. Furthermore, the cut-off frequency of the SVF element is set as 8 [rad/s], and the

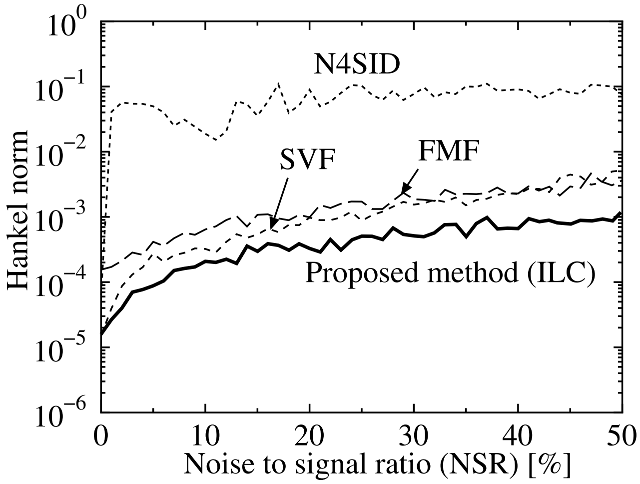


Fig. 8. The relation between Hankel norm and NSR in each method

frequency band of the system to be identified in FMF is set as 5 [rad/s]. This seems to be near optimal choice as far as we tried in terms of the reduction of variance of the identified coefficients.

The relation between NSR and the Hankel norm of the error system in various methods are shown in Fig. 8. In the figure, the solid line shows the case when we use the proposed ILC method at the 100th trial. The dotted line, the short dashed line and the dashed line show N4SID, SVF and FMF, respectively. Since estimation accuracy depends on noise of each identification experiment, the value of Hankel norms shown in the figure is the average of 20 sets of experiments. From Fig.8, we see that the proposed ILC based identification method is robust against noise and may be useful as an alternative identification method.

## V. Conclusion

In this paper we have proposed a new iterative learning control (ILC) method which is robust against measurement noise. The method utilizes the ILC with a finite-dimensional prescribed I/O space, and reduces the noise effect by employing the past trial data effectively. A concrete design procedure based on I/O sampled-data is given. We have also demonstrated that the proposed ILC method can be used for identification of a class of linear continuous-time systems. This provides an alternative identification methods from an entirely different viewpoint from the conventional one(s). One advantage is that the parameters of the continuous-time system can be identified without the use of time-derivative of I/O data. In addition, we are able to confirm the identification accuracy by looking at the tracking performance during the experiments. Numerical examples are given to demonstrate its effectiveness.

On of the interesting future research topics is to extend this method to a broader class of systems.

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