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Robust Decentralized MPC Algorithm for a class of Dynamically Interconnected Constrained Systems

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Abstract— **Model predictive control (MPC) is one of the most promising and significant control approaches because of its ability to handle control problems for constrained systems. In this paper, we propose a robust decentralized MPC algorithm for dynamically interconnected large-scale constrained uncertain systems with disturbances. Each decentralized MPC controller determines the control sequence, satisfying input and state constraints in the presence of uncertainties and disturbances, based only on the nominal model of its own subsystem and information broadcasted from other controllers. This controller uses the information on the reachable state bound of its own subsystem and dynamically interconnected subsystems which is predicted one-step before. This algorithm guarantees robust stability in the presence of model uncertainties and disturbances.**

I. Introduction

Model predictive control (MPC) is one of the most promising and significant control approaches because of its ability to handle control problems for the systems having input, state and output constraints. In general, it determines the control sequence by solving on-line a constrained finite horizon centralized optimization problem at each time instance and subsequently only the first control in this sequence is implemented [14][16][18]. This type of MPC scheme has been vigorously studied for many years, with many research results concerning not only constraint satisfaction but also feasibility, stability and robustness [2][3][11][13][15][19]. In this line of research, a single MPC controller determines all the control inputs of system based on the information about overall system. However, when we consider that large-scale dynamically interconnected systems with constraints, the complexity of on-line optimal control problem of such systems could cause practical difficulties; i.e., it requires a significant computation burden. Therefore, for such systems, ordinary MPC techniques based on centralized optimization problem may not be so useful.

One way to cope with computation issues is to use decentralized control schemes. In the field of MPC, the development of decentralized control schemes combined with various MPC approaches attracts a lot of attention recently. The standard features of decentralized MPC are as follows; there are distinct MPC controllers, one for each subsystem. To each controller, the optimal control problem is assigned only for its subsystem. Then, it is solved to determine its own control sequence based on information about itself and interconnected subsystems. In the MPC literature, several decentralized control approaches using MPC techniques for dynamically interconnected systems have been proposed [1][6][7][17]. However, there exists a drawback such that the constraints of input, states and output of each subsystem are not considered. In general, it is considerably difficult to guarantee stability and feasibility of decentralized MPC algorithms for constrained subsystems, especially when uncertainties and disturbances exist [8]. Therefore, the main research issue in this field is to develop a robust decentralized MPC algorithm for constrained subsystems that is globally feasible and stabilize the overall interconnected subsystems.

The purpose of this paper is to develop a robust decentralized MPC algorithm for a class of dynamically interconnected constrained systems, illustrated in Fig. 1, with model uncertainties and disturbances. It is formulated based on the framework in [5], [9] and [10] where the comparison models are introduced to predict the future reachable bound of system states in the presence of uncertainties and disturbances.

We first describe each subsystem as linear time-invariant discrete-time local model with uncertainties, disturbances and also the terms which denote the influences from other interconnected subsystems. Next, we describe a framework for designing decentralized MPC scheme to control each local subsystem. In order to formulate it, we introduce the comparison models for each subsystem to predict the future reachable bounds of its state and state prediction error. Then, in a similar manner in [5], [9] and [10], the given constraints are modified based on these bounds to guarantee the robust stability and moreover the original robust MPC problem can be transformed into a nominal one without uncertain terms using these comparison models.

The key feature is that each decentralized MPC controller can determine the control sequence, satisfying given input/state constraints in the presence of uncertainties and disturbances, based only on the nominal model of its own local subsystem and information on reachable state bounds of other subsystems. Therefore, the proposed decentralized MPC controller not only solve the optimal control problem of its own subsystem, but also predict the reachable future bound of it local state variables and communicate it to the other controllers to modify the constraints in their control schemes at each time instant.

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Notation and Preliminaries: The symbol x_l^i denotes the lth element of vector x^i. The notation $\|x\|$ and $\|x\|_p$ denote the Euclidean and p-norms of a vector x; when x is a

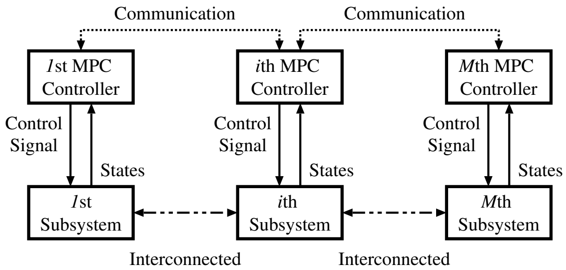

Fig. 1. Composition of decentralized MPC controllers and subsystems

scalar $|x|$ denotes its absolute value. We denote the largest singular value of a matrix M as Let $\overline{\sigma}(M)$ denotes the largest singular value of a matrix M. The maximum and minimum eigenvalues of a matrix M are denoted by $\overline{\lambda}(M)$ and $\underline{\lambda}(M)$, respectively. The notation $M \succ 0$ means that M is a symmetric positive definite; $M^{\frac{1}{2}}$ denotes the unique positive definite square root of $M \succ 0$.

II. Problem Statement

A. System Description

In this paper, we consider the problem of controlling M dynamically interconnected subsystems. Each subsystem, denoted by superscript $i \in \mathbb{Z}$, $\mathbb{Z} = \{1, 2, \cdots, M\}$, is described by linear time-invariant discretized dynamics as follows:

$$x^i(t+1) = A^{ii}x^i(t) + B^i u^i(t) + B_d^i d^i(t) + \sum_{j=1, j\neq i}^{M} A^{ij} x^j(t) \quad (1)$$

where $x^i \in \mathbb{R}^n$, $u^i \in \mathbb{R}^m$, $d^i(t) \in \mathbb{R}^p$ are the state, the control signal and the disturbance of the ith subsystem, respectively, and $A^{ii} \in \mathbb{R}^{n\times n}$, $A^{ij} \in \mathbb{R}^{n\times n}$, $B^i \in \mathbb{R}^{n\times m}$, $B_d^i \in \mathbb{R}^{n\times p}$. Let $x_0^i := x^i(0)$ denotes a given initial state of the ith subsystem. Moreover, we assume that the disturbance $d^i(t) \in \mathbb{R}^p$ consists of a state-dependent uncertainty $d_1^i \in \mathbb{R}^{p_1}$ and a bounded disturbance $d_2^i \in \mathbb{R}^{p_2}$ similarly to [5]. These satisfy

$$d^i(t) := [(d_1^i(t))^T,\ (d_2^i(t))^T]^T \in D^i(x^i(t)),\ \forall t \geq 0,$$
$$D^i(x^i(t)) := D_1^i(x^i(t)) \times D_2^i$$

where

$$D_1^i(x^i(t)) := \{d_1^i(t) \in \mathbb{R}^{p_1} : \|d_1^i(t)\| \leq \|x^i(t)\|\},$$
$$D_2^i := \{d_2^i \in \mathbb{R}^{p_2} : \|d_2^i\| \leq 1\}$$

for $\forall i \in \mathbb{Z}$. Then, the matrices B_d^i in (1) can be defined as $B_d^i := [B_{d_1}^i, B_{d_2}^i]$ where $B_{d_1}^i \in \mathbb{R}^{n\times p_1}$ and $B_{d_2}^i \in \mathbb{R}^{n\times p_2}$ which correspond to $d_1^i(t)$ and d_2^i, respectively.

B. Constraints

It is assumed that each subsystem is subjected to the following constraints both on the fully measurable state x^i and the control input u^i of the ith subsystem, which must be fulfilled at all time instant $t \geq 0$.

$$\begin{aligned} &x^i(t) \in X^i,\ X^i := \{x^i(t) \in \mathbb{R}^n : |x_l^i(t)| \leq \psi_l^i, \forall l\}, \\ &u^i(t) \in U^i,\ U^i := \{u^i(t) \in \mathbb{R}^m : |u_l^i(t)| \leq \eta_l^i, \forall l\} \end{aligned} \quad (2)$$

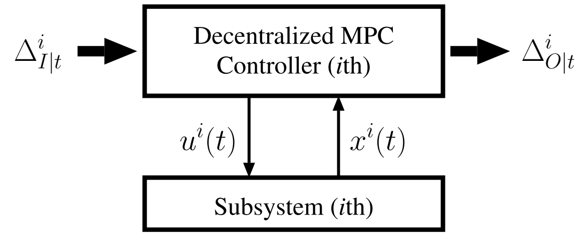

Fig. 2. Information flow at time instant t

where $\psi_l^i > 0$ and $\eta_l^i > 0$ denote state and input constraint levels of each subsystem, respectively, and $\forall i \in \mathbb{Z}$. We then define the terminal constraint set X_f^i for ith subsystem as

$$X_f^i := \{x^i(t) \in \mathbb{R}^n : V(x^i) \leq 1\},\ V(x^i) := \sqrt{(x^i)^T P^i x^i}$$

for $P^i \succ 0$ which satisfies the following assumption similar to those used in [5], [9] and [10].

Assumption 1 A given feedback gain $K^i \in \mathbb{R}^{m\times n}$ and $P^i \succ 0$ for ith subsystem where $i \in \mathbb{Z}$ satisfy

$$1 \leq \sqrt{\underline{\lambda}(P^i)} \min_l \psi_l^i,\ \|K^i\| \leq \sqrt{\underline{\lambda}(P^i)} \min_l \eta_l^i \quad (3)$$

$$\alpha^i < 1,\ 0 \leq \frac{\beta^i}{1-\alpha^i} < 1, \quad (4)$$

where

$$\alpha^i := a_1^i + a_2^i, \quad \beta^i := a_3^i + \sum_{j=1, j\neq i}^{M} \frac{a_4^{ij}}{\sqrt{\underline{\lambda}(P^j)}},$$
$$Q^i := P^i - (F^{ii})^T P^i F^{ii}, \quad F^{ii} := A^{ii} + B^i K^i$$

and

$$a_1^i := \sqrt{1 - \frac{\underline{\lambda}(Q^i)}{\overline{\lambda}(P^i)}}, \quad a_2^i := \frac{\overline{\sigma}((P^i)^{\frac{1}{2}} B_{d_1}^i)}{\sqrt{\underline{\lambda}(P^i)}},$$
$$a_3^i := \overline{\sigma}((P^i)^{\frac{1}{2}} B_{d_2}^i), \quad a_4^{ij} := \overline{\sigma}((P^i)^{\frac{1}{2}} A^{ij}).$$

Remark 1: In the above assumption, (3) and (4) guarantee $u^i(t) = K^i x^i(t) \in U^i$ is hold for all $x^i \in X_f^i$. Then, by this feedback control, X_f^i is a robustly invariant set for (1) (Refer to [4], [5], [9], [10]).

III. Robust Decentralized MPC Algorithm

The goal of this section is to develop a robust decentralized MPC algorithm which guarantees the ultimate boundedness of the closed-loop system of each subsystem while satisfying control and state constraints (2) for any disturbances. Figure 2 illustrates the information flow into and from the ith MPC controller at time instant t in the proposed robust decentralized MPC method. $\Delta_{I|t}^i$ denotes the information broadcasted by all other MPC controllers and $\Delta_{O|t}^i$ denotes the information broadcasted by the ith MPC controller to all other MPC controllers at time instant t. The information exchanged between ith controller and all other MPC controllers is as follows:

$$\Delta^i_{I|t} := \{w^j_{\cdot|t-1},\ \nu^j_{\cdot|t-1},\ \hat{x}^j_{\cdot|t-1}\},\ \forall j \in \mathbb{Z}\backslash j \neq i,$$
$$\Delta^i_{O|t} := \{w^i_{\cdot|t-1},\ \nu^i_{\cdot|t-1},\ \hat{x}^i_{\cdot|t-1}\} \tag{5}$$

where $w^i_{\cdot|t}$ and $\nu^i_{\cdot|t}$ denote the upper bounds predicted at time instant t; $w^i_{\cdot|t}$ is on the future state trajectory of the ith subsystem and $\nu^i_{\cdot|t}$ is on the future state prediction error, which are described in detail in the follows. $\hat{x}^i_{\cdot|t}$ denotes the predicted future value of $x^i(\cdot)$. Note that the exchanged ones are the information predicted at one-step before. It is also important that the ith MPC controller needs to receive information only from other MPC controllers that directly influence the dynamics of the ith subsystem. At time instant t, the ith MPC controller determines the control input $u^i(t)$ which satisfies the given constraints (2) and predicts the future bounds $w^i_{\cdot|t}$ and $\nu^i_{\cdot|t}$ its subsystem over the prediction horizon N based on the information $\Delta^i_{I|t}$ obtained through communication and the measured state. Then, it transmits the information $\Delta^i_{O|t}$ to all other controllers of the dynamically interconnected subsystems.

Next, we define the finite horizon constrained optimal control problem for the proposed robust decentralized MPC method for the ith subsystem. In this method, the following control which consists of a state feedback control with a gain $K^i \in \mathbb{R}^{m\times n}$ and an open-loop control $\tilde{u}^i \in \mathbb{R}^m$ is adopted to regulate the state of the ith subsystem subsystem[3][5][9][10][12].

$$u^i(t) = K^i x^i(t) + \tilde{u}^i(t). \tag{6}$$

Consider the ith subsystem (1) at time instant t and let $\hat{x}^i_{t|t}$ be the measured state $x^i(t)$ of this subsystem. Then, the optimization problem of the ith MPC controller to determine the optimal open-loop control $\tilde{u}^i(t)$ of (6) which guarantees the constraint fulfillment is described as follows:

Optimization problem for decentralized MPC

$$\min_{\tilde{u}^i} J_d(x^i(t), \tilde{u}^i_{t+k|t}) := \sum_{k=0}^{N-1} (\tilde{u}^i_{t+k|t})^T R^i \tilde{u}^i_{t+k|t} \tag{7}$$

where $R^i \succ 0$ is a given diagonal matrix and subject to

(a) *Prediction model*: The future state $x^i(t+k)$ of (1) is predicted based on the control input (6) and the following model

$$\hat{x}^i_{t+k+1|t} = F^{ii}\hat{x}^i_{t+k|t} + B^i\tilde{u}^i_{t+k|t} + \sum_{j=1,j\neq i}^{M} A^{ij}\hat{x}^j_{t+k|t-1} \tag{8}$$

where $\hat{x}^i_{t|t} := x^i(t)$ and $k = 0, 1, \cdots, N-1$.

(b) *Comparison models*: Since the future state is predicted based on (8), the prediction error will inevitably occur. Thus, it is necessary to evaluate the prediction error of $\hat{x}^i_{\cdot|t}$ due to the system uncertainties, the disturbances and the effects of dynamically interconnected subsystems and then modify the given constraint (2) based on it. Note that the disturbance depends on the future state $x^i(t+k)$ of the ith real subsystem and furthermore the future state $x^j(t+k)$ of other subsystems, which are dynamically interconnected to the ith subsystem, as shown in (1). To that aim, we introduce the following two scalar systems into the optimization problem:

$$w^i_{t+k+1|t} = \alpha^i w^i_{t+k|t} + a^i_3 + \sum_{l=1}^{m} b^i_l|\tilde{u}^i_{l,t+k|t}| + \sum_{j=1,j\neq i}^{M} c^{ij} w^j_{t+k|t-1} \tag{9}$$

$$\nu^i_{t+k+1|t} = a^i_1\nu^i_{t+k|t} + a^i_2 w^i_{t+k|t} + a^i_3 + \sum_{j=1,j\neq i}^{M} c_{ij}\nu^j_{t+k|t} \tag{10}$$

where $b^i_l := \overline{\sigma}((P^i)^{\frac{1}{2}}B^i_l)$, $c^{ij} := a^{ij}_4/\sqrt{\underline{\lambda}(P^j)}$ and B^i_l denotes the lth column of B^i. The initial states of (9) and (10) are $w^i_{t|t} = V(x^i(t))$ and $\nu^i_{t|t} = 0$, respectively. These are derived in a similar way to the method described in [5], [9], [10]. To ensure the robust stability and the constraint fulfillment, the open-loop control $\tilde{u}^i_{\cdot|t}$ should be determined so that the following constraint is satisfied.

$$w^i_{t+k+1|t} \in W^i(t+k+1, w^i_{\cdot|t}, w^i_{\cdot|t-1})$$
$$W^i := \{w^i \in \mathbb{R} : w^i_{t+k+1|t} \leq w^i_{t+k+1|t-1}\} \tag{11}$$

(c) *Modification of constraints*: As mentioned above, there exists a prediction error between $x^i(t+k)$ and $\hat{x}^i_{t+k|t}$. Therefore, the constraints on the predicted state $\hat{x}^i_{\cdot|t}$ and the predicted control input $\hat{u}^i_{\cdot|t}$ should be modified based on (2), (9) and (10) in order to guarantee the robust stability and the fulfillment of the given constraints (2) as follows:

$$\hat{X}^i(t+k+1, w^i_{\cdot|t}, \nu^j_{\cdot|t-1}) :=$$
$$\{\hat{x}^i \in \mathbb{R}^n : |\hat{x}^i_{l,t+k+1|t}| \leq \psi^i_l - \psi^i_l(t+k+1, w^i_{\cdot|t}, \nu^j_{\cdot|t-1})\}$$
$$\hat{U}^i(t+k, w^i_{\cdot|t}, \nu^j_{\cdot|t-1}) :=$$
$$\{\hat{u}^i \in \mathbb{R}^m : |\hat{u}^i_{l,t+k|t}| \leq \eta^i_l - \eta^i_l(t+k, w^i_{\cdot|t}, \nu^j_{\cdot|t-1})\} \tag{12}$$

where

$$\psi^i_l(t+k+1, w^i_{\cdot|t}, \nu^j_{\cdot|t-1}) := \sum_{s=0}^{k} \|\zeta^i_{1,l}(s)\| \frac{w^i_{t+k+1-s|t}}{\sqrt{\underline{\lambda}(P^i)}}$$
$$+ \sum_{s=0}^{k} \|\zeta^i_{2,l}(s)\| + \sum_{s=0}^{k}\sum_{j=1,j\neq i}^{M} \|\zeta^{ij}_{3,l}(s)\| \frac{\nu^j_{t+k+1-s|t-1}}{\sqrt{\underline{\lambda}(P^j)}}$$

$$\eta^i_l(t+k, w^i_{\cdot|t}, \nu^j_{\cdot|t-1}) := \sum_{s=0}^{k} \|\xi^i_{1,l}(s)\| \frac{w^i_{t+k-s|t}}{\sqrt{\underline{\lambda}(P^i)}}$$
$$+ \sum_{s=0}^{k} \|\xi^i_{2,l}(s)\| + \sum_{s=0}^{k}\sum_{j=1,j\neq i}^{M} \|\xi^{ij}_{3,l}(s)\| \frac{\nu^j_{t+k-s|t-1}}{\sqrt{\underline{\lambda}(P^j)}}$$

$$\zeta^i_\ell(s) := (F^{ii})^s B^i_{d_\ell}, \quad \zeta^{ij}_3(s) := (F^{ii})^s A^{ij},$$

$$\xi^i_\ell(s) := \begin{cases} K^i(F^{ii})^{s-1}B^i_{d_\ell} & for\ s \neq 0 \\ 0 & for\ s = 0 \end{cases},$$

$$\xi_3^{ij}(s) := \begin{cases} K^i(F^{ii})^{s-1}A^{ij} & for\ s \neq 0 \\ 0 & for\ s = 0 \end{cases},$$

and $\zeta_{\ell,l}(s)$ and $\xi_{\ell,l}(s)$ denote the lth rows of $\zeta_\ell(s)$ and $\xi_\ell(s)$, respectively. Therefore, the optimal open-loop control in (7) should be determined so that

$$\begin{aligned} &\hat{x}^i_{t+k+1|t} \in \hat{X}^i(t+k+1, w^i_{\cdot|t}, \nu^j_{\cdot|t-1}), \\ &K^i\hat{x}^i_{t+k|t} + \tilde{u}^i_{t+k|t} \in \hat{U}^i(t+k, w^i_{\cdot|t}, \nu^j_{\cdot|t-1}) \end{aligned} \tag{13}$$

is hold.

(d) *Constraint on* $w^i_{\cdot|t}$: Similarly to [5], [9], [10], the following constraints on $w^i_{\cdot|t}$ in (9) and (10) are introduced in order to guarantee the feasibility at each time instant of the optimization problem.

$$w^i_{t+k|t} \leq \omega^i \ where\ \omega^i \geq \max\{V(x^i_0), 1\}, \tag{14}$$

$$w^i_{t+N|t} \leq 1 \tag{15}$$

where $k = 0, 1, \cdots, N-1$. Note that ω^i should be bounded to ensure the the feasibility at all time instants in a similar way to [5], [9], [10], but it remains as a future research work. Also note that (15) guarantees $x^i(k+N) \in X^i_f$ which can be easily verified in the following Lemma 1.

In the above optimization, the finite horizon constrained optimal control problem (7)-(15) without disturbance terms is solved using only the measured state x^i of each subsystem at the current time t and the information obtained through the communication with dynamically interconnected subsystems. Several comments on the proposed optimization problem are in order:

• The states $w^i_{\cdot|t}$ and $\nu^i_{\cdot|t}$ in (9) and (10) enable us to obtain the future upper bounds both on $V(x^i)$ and $V(z^i)$ where $z^i := x^i - \hat{x}^i$, which is described in the following lemma.

Lemma 1 For any $\tilde{u}^i_{t+k|t}$ satisfying (11), the states $w^i_{\cdot|t}$ and $\nu^i_{\cdot|t}$ in (9), (10) and the state $x^i(\cdot)$ of the ith subsystem in (1) satisfy

$$V(x^i(t+k)) \leq w^i_{t+k|t} \ and\ V(z^i(t+k)) \leq \nu^i_{t+k|t} \tag{16}$$

where $\forall i \in \mathbb{Z}$ and $k = 0, 1, \cdots, N$.

• The constraint sets $\hat{X}^i$ and $\hat{U}^i$ in (12) modified based on (9) and (10) and the constraint set W^i on $w^i_{\cdot|t}$ in (11) satisfy the following property.

Theorem 1 For a given $x^i(t) \in \mathbb{R}^n$, any open-loop trajectory $\tilde{u}^i_{t+k|t}$, $k = 0, 1, \cdots, N-1$ which satisfies (8)-(13) also satisfies the constraints for the real subsystem in (1) such as

$$\begin{aligned} &x^i(t+k+1) \in X^i, \\ &K^i x^i(t+k) + \tilde{u}^i_{t+k|t} \in U^i \end{aligned} \tag{17}$$

for all possible $d^i(t+k) \in D^i(x^i(t+k))$.

• At the first time instant $t = 0$, we determine the control inputs $\tilde{u}^i(0)$, $\forall i \in \mathbb{Z}$, for all subsystems by solving the optimization problem of the centralized MPC method proposed in [5], [9], [10]. In this case, M dynamically interconnected subsystems can be described as follows:

$$\boldsymbol{x}(t+1) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{B}_d\boldsymbol{d}(t) \tag{18}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^{(n\times M)}$, $\boldsymbol{u}(t) \in \mathbb{R}^{(m\times M)}$ and $\boldsymbol{d}(t) \in \mathbb{R}^{(p\times M)}$ defined as

$$\begin{aligned} \boldsymbol{x}(t) &:= \begin{bmatrix} (x^1(t))^T & (x^2(t))^T & \cdots & (x^M(t))^T \end{bmatrix}^T \\ \boldsymbol{u}(t) &:= \begin{bmatrix} (u^1(t))^T & (u^2(t))^T & \cdots & (u^M(t))^T \end{bmatrix}^T \\ \boldsymbol{d}(t) &:= \begin{bmatrix} (d^1(t))^T & (d^2(t))^T & \cdots & (d^M(t))^T \end{bmatrix}^T \end{aligned}$$

and

$$\boldsymbol{A} := \begin{bmatrix} A^{11} & A^{12} & \cdots & A^{1M} \\ A^{21} & A^{22} & \cdots & A^{2M} \\ \vdots & \vdots & \ddots & \vdots \\ A^{M1} & A^{M2} & \cdots & A^{MM} \end{bmatrix} \in \mathbb{R}^{(n\times M)\times(n\times M)}$$

$$\boldsymbol{B} := \begin{bmatrix} B^1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & B^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & B^M \end{bmatrix} \in \mathbb{R}^{(n\times M)\times(m\times M)}$$

$$\boldsymbol{B}_d := \begin{bmatrix} B^1_d & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & B^2_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & B^M_d \end{bmatrix} \in \mathbb{R}^{(n\times M)\times(p\times M)} \tag{19}$$

For the systems described as (18) and the control defined such as

$$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{x}(t) + \tilde{\boldsymbol{u}}(t) \tag{20}$$

where

$$\begin{aligned} \tilde{\boldsymbol{u}}(t) &:= \begin{bmatrix} (\tilde{u}^1(t))^T & (\tilde{u}^2(t))^T & \cdots & (\tilde{u}^M(t))^T \end{bmatrix}^T \\ \boldsymbol{K} &:= \begin{bmatrix} K^1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & K^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & K^M \end{bmatrix} \in \mathbb{R}^{(m\times M)\times(n\times M)} \end{aligned} \tag{21}$$

based on (6), we can derive easily various constraints similar to (9), (12), (13), (14) and (15) (Refer to [5], [9], [10]). Then, by solving the following finite horizon constrained optimal control problem

$$\min_{\tilde{\boldsymbol{u}}} J_c(\boldsymbol{x}(t), \tilde{\boldsymbol{u}}_{t+k|t}) := \sum_{k=0}^{N-1} \tilde{\boldsymbol{u}}^T_{t+k|t}\boldsymbol{R}\tilde{\boldsymbol{u}}_{t+k|t} \tag{22}$$

where $\boldsymbol{R} := diag(R^1, R^2, \cdots, R^M)$, we can determine the optimal open-loop control $\tilde{\boldsymbol{u}}(0)$. For $t \geq 1$, we apply the proposed decentralized MPC method to determine the control input for each subsystem. Note that if the solution of the above centralized optimization problem (22) cannot be found, it means that the problem is infeasible and thus the proposed decentralized optimization problem also cannot be solved. The above MPC controller (22) developed based on the centralized manner needs the knowledge of the overall subsystems and computes all the control sequences at the same time. Thus, the practical implementation of such MPC technique in large-scale dynamically interconnected systems requires a significant computation burden. In order to avoid such problem, we have developed the above decentralized MPC scheme.

• The comparison model (9) is a nonlinear equation of $\tilde{u}^i_{t+k|t}$. Similarly to [5], [9], [10], we modify it by replacing $\tilde{u}^i_{t+k|t} \in \mathbb{R}^m$ with $\chi^i_{t+k|t} \in \mathbb{R}^m$ as

$$w^i_{t+k+1|t} = \alpha^i w^i_{t+k|t} + a^i_3 + \sum_{l=1}^{m} b^i_l \chi^i_{l,t+k|t} + \sum_{j=1, j\neq i}^{M} c^{ij} w^j_{t+k|t} \tag{23}$$

with an additional constraint such as

$$|\tilde{u}^i_{t+k|t}| \leq \chi^i_{t+k|t}, \ k = 0, 1, \cdots, N-1. \tag{24}$$

Then, the cost function $J_d(x^i(t), \tilde{u}^i_{\cdot|t})$ can be rewritten as

$$J_d(x^i(t), \chi^i_{\cdot|t}) := \sum_{k=0}^{N-1} (\chi^i_{t+k|t})^T R^i \chi^i_{t+k|t}. \tag{25}$$

Therefore, the modified problems have only the linear constraints and can be solved by a standard quadratic programming (QP) method with free variables $\tilde{u}^i_{\cdot|t}$ and $\chi^i_{\cdot|t}$.

• When we implement this algorithm, it is not necessary for each MPC controller to communicate with all other MPC controllers. The ith MPC controller needs the information to receive from and send to jth MPC controller only if A^{ij} in (1) is nonzero. If the system is loosely interconnected, the amount of communication will not be large.

IV. Stability of Decentralized MPC Algorithm

In this section, it is presented that the proposed decentralized MPC algorithm guarantees the robust stability of the overall subsystems in (1). We first assume that the following assumption on the feasibility for the optimization in (7) is satisfied.

Assumption 2 Assume the optimization problem in (7) is feasible at the current time instant t for ω^i which satisfies Assumption 1. Then, at the next time instant $t+1$,

$$\tilde{u}^i_{t+k|t+1} = \begin{cases} \tilde{u}^{i*}_{t+k|t}, & k = 1, 2, \cdots, N-1 \\ 0, & k = N \end{cases} \tag{26}$$

could be a feasible (but not necessarily optimal) control sequence, where $\tilde{u}^{i*}_{t+k|t}$ denotes the optimal control sequence determined at time instant t.

The above assumption means that if a feasible solution to the initial optimization problem in (7) is found, then all subsequent optimization problem will be feasible and the given constraints (2) will be satisfied at every time instants for any disturbance $d^i(t+k) \in D^i(x^i(t+k))$. Based on the above assumption, we get the following Theorem 2 describing the property of stability of the proposed robust decentralized MPC method.

Theorem 2 Assume the optimization problem for the centralized MPC method in (18) is feasible at $t = 0$. Then, this optimal control sequence $\tilde{u}^{i*}_{k|t=0}$ could be one of the feasible control sequences at the next time instant $t = 1$. Furthermore, the proposed MPC method has the following properties.

(i) The optimization in (7) is feasible at time $t \geq 1$.
(ii) There exists t^i_c satisfying

$$\|x^i(t)\| \leq \frac{a^i_3 + \sum_{j=i, j\neq i}^{M} c^{ij} w^j_{t_c|t_c-1}}{\sqrt{\underline{\lambda}(P^i)}\,(1-\alpha^i)}, \ \forall t \geq t_c \tag{27}$$

Proof: We show (i) by induction. The optimization problem is feasible at $t = 0$ by assumption. Assume now it is feasible at each $t = \delta$ ($\delta = 1, 2, \cdots, k$). Then, since Assumption 2 means that the control in (26) is feasible at $t = \delta + 1$, (i) is proved.

In order to prove (ii), we next show that the optimal cost $J_d(x^i(t), \tilde{u}^{i*}_{\cdot|t})$ is nonincreasing. At the time instant $t+1$, the feasible solution in (26) satisfies

$$J_d(x^i(t+1), \tilde{u}^i_{t+k|t+1}) \leq J_d(x^i(t), \tilde{u}^{i*}_{t+k|t}) \tag{28}$$

since

$$\begin{aligned} &J_d(x^i(t+1), \tilde{u}^i_{t+k|t+1}) - J_d(x^i(t), \tilde{u}^{i*}_{t+k|t}) \\ &= \sum_{k=1}^{N} (\tilde{u}^i_{t+k|t+1})^T R^i \tilde{u}^i_{t+k|t+1} - \sum_{k=0}^{N-1} (\tilde{u}^{i*}_{t+k|t})^T R^i \tilde{u}^{i*}_{t+k|t} \\ &= -(\tilde{u}^{i*}_{t|t})^T R^i \tilde{u}^{i*}_{t|t} \leq 0. \end{aligned} \tag{29}$$

From the optimality of $J_d(x^i(t+1), \tilde{u}^{i*}_{t+k|t+1})$, it is also satisfied that

$$J_d(x^i(t+1), \tilde{u}^{i*}_{t+k|t+1}) \leq J_d(x^i(t+1), \tilde{u}^i_{t+k|t+1}) \tag{30}$$

Thus, from (28) and (30), the optimal cost is nonincreasing, i.e.,

$$J_d(x^i(t+1), \tilde{u}^{i*}_{t+k|t+1}) \leq J_d(x^i(t), \tilde{u}^{i*}_{t+k|t}). \tag{31}$$

is hold. Since the optimal cost is nonincreasing and bounded by 0 from below, it is satisfied that $J_d(x^i(t), \tilde{u}^{i*}_{t+k|t}) \to c$ as $t \to \infty$ for a constant $c \geq 0$. This implies that, for each $\varepsilon^i > 0$, there exists $t^i_1 > 0$ such that

$$0 \leq J_d(x^i(t), \tilde{u}^{i*}_{t+k|t}) - J_d(x^i(t+1), \tilde{u}^{i*}_{t+k|t+1}) < \varepsilon^i \tag{32}$$

where $\forall t \geq t_1^i$. But, from (28), (29) and (30), we have

$$\begin{aligned}
&(\tilde{u}_{t|t}^{i*})^T R^i \tilde{u}_{t|t}^{i*} \\
&= J_d(x^i(t), \tilde{u}_{t+k|t}^{i*}) - J_d(x^i(t+1), \tilde{u}_{t+k|t+1}^{i}) \\
&\leq J_d(x^i(t), \tilde{u}_{t+k|t}^{i*}) - J_d(x^i(t+1), \tilde{u}_{t+k|t+1}^{i*}).
\end{aligned} \tag{33}$$

Thus, (32) and (33) imply

$$\tilde{u}_{t|t}^{i*} \rightarrow 0 \ as \ t \rightarrow \infty. \tag{34}$$

From (34), we can choose t_1^i such that

$$\|\tilde{u}_{t|t}^{i*}\|_\infty \leq \varepsilon_1^i, \ \forall t \geq t_1^i. \tag{35}$$

From (9), (16) and (35), it follows that

$$\begin{aligned}
&V(x^i(t+1)) \\
&\leq \alpha^i V(x^i(t)) + a_3^i + \sum_{l=1}^{m} b_l^i |\tilde{u}_l^{i*}(t)| + \sum_{j=i,j\neq i}^{M} c^{ij} w_{t|t-1}^j \\
&\leq \alpha^i V(x^i(t)) + a_3^i + \gamma_1^i + \gamma_2^i(t_1^i)
\end{aligned} \tag{36}$$

where

$$\gamma_1^i := \varepsilon_1^i \sum_{l=1}^{m} b_l^i, \quad \gamma_2^i(t_1^i) := \sum_{j=i,j\neq i}^{M} c^{ij} w_{t_1^i|t_1^i-1}^j \tag{37}$$

and $\forall t \geq t_1^i$. Note that Assumption 1 assures that $w_{\cdot|t}^i$ is nonincreasing. Therefore,

$$\begin{aligned}
&V(x^i(t_1+k)) \\
&\leq (\alpha^i)^k V(x^i(t_1^i)) + \sum_{s=0}^{k-1} (\alpha^i)^k (a_3^i + \gamma_1^i + \gamma_2^i(t_1^i)) \\
&= (\alpha^i)^k V(x^i(t_1^i)) + \frac{(1-(\alpha^i)^k)(a_3^i + \gamma_1^i + \gamma_2^i(t_1^i))}{1-\alpha^i}
\end{aligned} \tag{38}$$

and the right-hand side of (38) converges to

$$\frac{a_3^i + \gamma_1^i + \gamma_2^i(t_1^i)}{1-\alpha^i} \ as \ k \rightarrow \infty. \tag{39}$$

Since γ_1^i can be chosen arbitrarily, there exists a finite time t_c^i $(\geq t_1^i)$ which satisfies

$$V(x^i(t)) < \frac{a_3^i + \gamma_1^i + \gamma_2^i(t_c^i)}{1-\alpha^i}, \ \forall t \geq t_c^i \tag{40}$$

Thus, (27) follows form (40). □

V. Conclusion

In this paper, we have proposed a robust decentralized MCP algorithm for a class of dynamically interconnected large-scale systems having both input and state constraints and model uncertainties and disturbances. This method is developed basically based on the comparison models which enables us to predict the future reachable bounds of states of each subsystem and difference between the real and predicted states. The developed MPC controller determines the input sequence and predicts the future reachable state bound based on the information exchanged between only dynamically interconnected subsystems. The determined control input satisfies the given constraints in the presence of uncertainties and disturbances, and also the robust stability of overall subsystems is guaranteed. As a future research works, we will make investigation into the decentralized MPC method which guarantees the feasibility at all time instants. Also we will show the effectiveness of the proposed algorithm by numerical examples.

References

[1] L. Acar, "Boundaries of the receding horizon control for interconnected systems," *Journal of Optimization and Theory and Application*, vol. 84, no. 2, 1995.

[2] T. A. Badgwell, "Robust model predictive control of stable linear systems," *International Journal of Control*, vol. 68, no. 4, pp. 797–818, 1997.

[3] A. Bemporad, "Reducing conservativeness in predictive control of constrained systems with disturbances," in *Proc. 37th IEEE Conf. Decision and Control*, Florida, USA, 1998, pp. 1384–1391.

[4] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.

[5] H. Fukushima and R. R. Bitmead, "Robust constrined predictive control using comparison model," *Automatica*, vol. 41, no. 1, pp. 97–106, 2005.

[6] D. Jia and B. H. Krogh, "Distributed model predictive control," in *Proc. American Control Conf.*, Arlington, Virginia, USA, 2001, pp. 2767–2772.

[7] ——, "Min-max feedback model predictive control for distributed control with communication," in *Proc. American Control Conf.*, Anchorage, Alaska, USA, 2002, pp. 4507–4512.

[8] F. B. K. Tamas and G. J. Balas, "Hierarchical design of decentralized receding horizon controller for decoupled systems," in *Proc. 43rd IEEE Conf. Decision and Control*, Atlantis Paradise Island, Bahamas, 2004, pp. 1592–1597.

[9] T. H. Kim, H. Fukushima, and T. Sugie, "Robust adaptive model predictive control based on comparison model," in *Proc. 43rd IEEE Conf. Decision and Control*, Atlantis Paradise Island, Bahamas, 2004, pp. 2041–2046.

[10] T. H. Kim and T. Sugie, "Discrete-time adaptive model predictive control based on comparison model," in *The 16th IFAC World Congress*, Prague, Czech Republic, 2005.

[11] M. V. Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities," *Automatica*, vol. 32, no. 10, pp. 1361–1379, 1996.

[12] Y. I. Lee and B. Kouvaritakis, "Constrained receding horizon predictive control for systems with disturbances," *International Journal of Control*, vol. 72, no. 11, pp. 1027–1032, 1999.

[13] ——, "Robust receding horizon predictive control for systems with uncertain dynamics and input saturation," *Automatica*, vol. 36, no. 10, pp. 1497–1504, 2000.

[14] D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert, "Constrained model predictive control: stability and optimality," *Automatica*, vol. 36, no. 6, pp. 789–814, 2000.

[15] H. Michalska and D. Q. Mayne, "Robust receding horizon control of constrained nonlinear systems," *IEEE Trans. Auto. Control*, vol. 38, no. 11, pp. 1623–1632, 1993.

[16] M. Morari and J. H. Lee, "Model predictive control: past, present and future," *Computers and Chemical Engineering*, vol. 23, pp. 667–682, 1999.

[17] M. Motee and B. Sayyar-Rodsari, "Optimal partitioning in distributed model predictive control," in *Proc. American Control Conf.*, Denver, Colorado, USA, 2003, pp. 5300–5305.

[18] J. B. Rawlings, "Tutorial overview of model predictive control," *IEEE Control System Magazine*, vol. 20, no. 3, pp. 38–52, 2000.

[19] P. O. M. Scokaert and D. Q. Mayne, "Min-max feedback model predictive control for constrined linear systems," *IEEE Trans. Auto. Control*, vol. 43, no. 8, pp. 1136–1142, 1998.