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Virtual Leader Approach to Coordinated Control of Multiple Mobile Agents with Asymmetric Interactions

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***Abstract*— This paper considers multiple mobile agents moving in the space with point mass dynamics. We introduce a set of coordination control laws that enable the group to generate the desired stable flocking motion. The control laws are a combination of attractive/repulsive and alignment forces, and the control law acting on each agent relies on the state information of its flockmates and the external reference signal. By using the control laws, all agent velocities asymptotically approach the desired velocity, collisions are avoided between the agents, and the final tight formation minimizes all agent global potentials. Moreover, we show that the velocity of the center of mass either is equal to the desired velocity or exponentially converges to it. Furthermore, when the velocity damping is taken into account, we can properly modify the control laws to generate the same stable flocking motion. Finally, for the case that not all agents know the desired common velocity, we show that the desired flocking motion can still be guaranteed. Numerical simulations are worked out to illustrate our theoretical results.**

I. PROBLEM FORMULATION

We consider a group of N agents moving in an n-dimensional Euclidean space, each has point mass dynamics described by

$$\begin{aligned} \dot{x}^i &= v^i, \\ m_i \dot{v}^i &= u^i, \quad i = 1, \cdots, N, \end{aligned} \tag{1}$$

where $x^i \in \mathbf{R}^n$ is the position vector of agent i, $v^i \in \mathbf{R}^n$ is its velocity vector, $m_i > 0$ is its mass, and $u^i \in \mathbf{R}^n$ is the (force) control input acting on agent i.

Our objective is to make the entire group move at a desired velocity and maintain constant distances between the agents. We first consider the ideal case, that is, we ignore the velocity damping. In this case, in order to achieve our control objective, we try to regulate each agent velocity to the desired velocity, reduce the velocity differences between agents, and at the same time, regulate their distances such that their global potentials become minimum. Hence, we choose the control law u^i for agent i to be

$$u^i = \alpha^i + \beta^i + \gamma^i, \tag{2}$$

where α^i is used to regulate the potentials among agents, β^i is used to regulate the velocity of agent i to the weighted average of its flockmates, and γ^i is used to regulate the momentum of agent i to the desired final momentum (all to be designed later). α^i is derived from the social potential fields which is described by artificial social potential function V^i, a function of the relative distances between agent i and its flockmates. Collision-free and cohesion in the group can be guaranteed by this term. β^i reflects the alignment or velocity matching with neighbors among agents. γ^i is designed to regulate the momentum among agents based on the external signal (the desired velocity). By using such a of momentum regulation, we can obtain the explicit convergence rate of the center of mass (CoM) of the system.

Remark 1: The design of α^i and β^i indicates that, during the course of motion, agent i is influenced only by its "neighbors", whereas γ^i reflects the influence of the external signal on the agent motion.

Certainly, in some cases, the velocity damping can not be ignored. For example, objects moving in viscous environment and mobile objects with high speeds such as supersonic aerial vehicles, are subjected to the influence of velocity damping. Then, in this case, the model in (1) should be in the following form

$$\begin{aligned} \dot{x}^i &= v^i, \\ m_i \dot{v}^i &= u^i - k_i v^i, \end{aligned} \tag{3}$$

where $k_i > 0$ is the "velocity damping gain", $-k_i v^i$ is the velocity damping term, and u^i is the control input for agent i. Here we assume that the damping force is in proportion to the magnitude of velocity and the damping gains k_i, $i = 1, \cdots, N$ are not equal to each other. In order to achieve our control objective, we need to compensate for the velocity damping. Hence, we modify the control law u^i to be

$$u^i = \alpha^i + \beta^i + \gamma^i + k_i v^i. \tag{4}$$

II. MAIN RESULTS

In this section, we investigate the stability properties of multiple mobile agents with point mass dynamics described in (1). We will present explicit control input in (2) for the terms α^i, β^i, and γ^i. We will employ matrix analysis and algebraic graph theory as basic tools for our discussion. Some concepts and results can be found in [11]–[12].

Due to the complexity of the agent interactions, we will define two kinds of structure topologies to describe the information flows between the agents. Throughout this paper, we assume that each agent is equipped with two onboard sensors: the position sensor which is used to sense the position information of the flockmates and the velocity sensor which is used to sense the velocity information of its neighbors, and assume that all sensors can sense

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instantaneously. In what follows, we will use an undirected graph $\mathcal{G}$ to describe the position sensor information flow and use a weighted directed graph $\mathcal{D}$ to describe the velocity sensor information flow. Following [2], we make the following definitions.

Definition 1: (Position neighboring graph) The position neighboring graph, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, is an undirected graph consisting of a set of vertices, $\mathcal{V} = \{n_1, \cdots, n_N\}$, indexed by the agents in the group, and a set of edges, $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V} \mid n_j \sim n_i\}$, containing unordered pairs of vertices that represent the position neighboring relations.

Definition 2: (Velocity neighboring graph) The velocity neighboring graph, $\mathcal{D} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, is a weighted directed graph consisting of a set of vertices, $\mathcal{V} = \{n_1, \cdots, n_N\}$, indexed by the agents in the group, and a set of arcs, $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V} \mid n_j \sim n_i\}$, containing ordered pairs of vertices that represent the velocity neighboring relations. $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N \times N}$ is the weight matrix which consists of the interaction coefficients between the agents.

Note that, in the velocity neighboring graph $\mathcal{D}$, an arc (n_i, n_j) represents an unidirectional velocity information exchange link from n_i to n_j, which means that agent i can obtain the velocity information of agent j.

In this paper, we consider a group of mobile agents with fixed topology. In order to make the final potential of each agent be global minimum and at the same time, ensure collision-free in the group, we assume that the position neighboring graph is complete. Let $\mathcal{I} = \{1, 2, \cdots, N\}$. For the velocity neighboring graph, we denote the set $\mathcal{N}_i = \{j \mid a_{ij} > 0\} \subseteq \mathcal{I} \backslash \{i\}$ which contains all neighbors of agent i, where a_{ij} is the weight of arc (n_i, n_j).

Definition 3: [2] (Potential function) Potential V^{ij} is a differentiable, nonnegative, radially unbounded function of the distance $\|x^{ij}\|$ between agents i and j, such that $V^{ij}(\|x^{ij}\|) \to \infty$ as $\|x^{ij}\| \to 0$, and V^{ij} attains its unique minimum when agents i and j are located at a desired distance, where $x^{ij} = x^i - x^j$ denotes the relative position vector between agents i and j.

Functions V^{ij}, $i, j = 1, \cdots, N$ are the artificial social potential functions that govern the interindividual interactions. One example of such potential functions is $V^{ij}(\|x^{ij}\|) = a \ln \|x^{ij}\|^2 + \frac{b}{\|x^{ij}\|^2}$, where a and b are some positive constants. It is easy to see that V^{ij} attains its unique minimum $a(1 + \ln(b/a))$ when $\|x^{ij}\| = \sqrt{b/a}$.

Then, the total potential of agent i can be expressed as

$$V^i = \sum_{j=1, j \neq i}^{N} V^{ij}(\|x^{ij}\|).$$

Note that, in this section, we assume that all agents can detect the external signal, that is, they all know the desired final velocity. In the case that not all agents know the mission signal, we will discuss the flocking control problem in a separate section. In what follows, we only present the detailed analysis for the ideal case, since for the nonideal case, we only need to add the terms k_i $(i = 1, \cdots, N)$ to cancel the velocity damping.

In the ideal case, we take the control law u^i to be

$$u^i = -\sum_{j=1, j \neq i}^{N} \nabla_{x^i} V^{ij} - \sum_{j \in \mathcal{N}_i} w_{ij}(v^i - v^j) - m_i(v^i - v^0), \quad (5)$$

where $v^0 \in \mathbf{R}^n$ is the desired common velocity and is a constant vector, $w_{ij} \geq 0$, and $w_{ii} = 0$, $i, j = 1, \cdots, N$ represent the interaction coefficients. $w_{ij} > 0$ if agent i can obtain the velocity information of agent j, and is 0 otherwise. We denote $W = [w_{ij}]$ as the interaction coefficient matrix (coupling matrix) on agent velocity associated with the velocity neighboring graph $\mathcal{D}$. Thus, when $\mathcal{D}$ is connected, $W + W^T$ is irreducible. The control law in (5) implies that we adopt the local velocity regulation and the global potential regulation to achieve our control objective. Such a regulation is due to the complexity of the interactions between agents (or particles) in nature.

Weight Balance Condition [8]: consider the weight matrix $W = [w_{ij}] \in \mathbf{R}^{N \times N}$, for all $i = 1, \cdots, N$, we assume that $\sum_{j=1}^{N} w_{ij} = \sum_{j=1}^{N} w_{ji}$.

Proposition 1: [3] Let $\mathcal{D}$ be a weighted directed graph such that the weight balance condition is satisfied. Then $\mathcal{D}$ is strongly connected if and only if it is weakly connected.

Throughout this paper, we assume that the coupling matrix satisfies the weight balance condition. Hence, if $\mathcal{D}$ is weakly connected, then it must be strongly connected.

A. Stability Analysis

Theorem 1: By taking the control law in (5), all agent velocities in the group described in (1) asymptotically approach the desired common velocity, collision avoidance is ensured between the agents, and the group final configuration minimizes all agent global potentials.

This theorem becomes apparently true after Theorem 2 is proved, so we proceed to present Theorem 2.

We define the error vectors: $e_p^i = x^i - v^0 t$, and $e_v^i = v^i - v^0$, where t is time variable and v^0 is the desired common velocity. Then e_v^i represents the velocity difference vector between the actual velocity and the desired velocity of agent i. It is easy to see that $\dot{e}_p^i = e_v^i$, and $\dot{e}_v^i = \dot{v}^i$. Hence, the error dynamics is given by

$$\begin{aligned} \dot{e}_p^i &= e_v^i, \\ \dot{e}_v^i &= \frac{1}{m_i} u^i, \quad i = 1, \cdots, N. \end{aligned} \quad (6)$$

By the definition of V^{ij}, it follows that $V^{ij}(\|x^{ij}\|) = V^{ij}(\|e_p^{ij}\|) := \widetilde{V}^{ij}$, where $e_p^{ij} = e_p^i - e_p^j$, and hence $\widetilde{V}^i = V^i$ and $\nabla_{e_p^i} \widetilde{V}^{ij} = \nabla_{x^i} V^{ij}$. Thus, the control input for agent i in the error system has the following form

$$u^i = -\sum_{j=1, j \neq i}^{N} \nabla_{e_p^i} \widetilde{V}^{ij} - \sum_{j \in \mathcal{N}_i} w_{ij}(e_v^i - e_v^j) - m_i e_v^i. \quad (7)$$

Consider the following positive semi-definite function

$$J = \frac{1}{2} \sum_{i=1}^{N} \left(\widetilde{V}^i + m_i e_v^{iT} e_v^i \right). \quad (8)$$

It is easy to see that J is the sum of the total artificial potential energy and the total kinetic energy of all agents in the error system. Define the level set of J in the space of agent velocities and relative distances in the error system $\Omega = \{(e_v^i, e_p^{ij}) \mid J \le c,\ c > 0\}$. In what follows, we will prove that the set Ω is compact. In fact, the set $\{e_v^i, e_p^{ij}\}$ with $J \le c$ is closed by continuity. Moreover, boundedness can be proved by the connectivity of the position neighboring graph. More specifically, from $J \le c$, we have $\widetilde{V}^{ij} \le c$. On the other hand, since the potential function V^{ij} is radially unbounded, $\widetilde{V}^{ij}$ is also radially unbounded, and there is a positive constant d such that $\|e_p^{ij}\| \le d$ for all $i, j = 1, \cdots, N$. By similar analysis, we have $e_v^{iT} e_v^i \le 2c/m_i$, thus $\|e_v^i\| \le \sqrt{2c/m_i}$.

By the symmetry of $\widetilde{V}^{ij}$ with respect to e_p^{ij} and by $e_p^{ij} = -e_p^{ji}$, it follows that $\frac{\partial \widetilde{V}^{ij}}{\partial e_p^{ij}} = \frac{\partial \widetilde{V}^{ij}}{\partial e_p^{i}} = -\frac{\partial \widetilde{V}^{ij}}{\partial e_p^{j}}$, and therefore

$$\frac{d}{dt}\sum_{i=1}^{N} \frac{1}{2}\widetilde{V}^i = \sum_{i=1}^{N} \nabla_{e_p^i}\widetilde{V}^i \cdot e_v^i.$$

Theorem 2: By taking the control law in (7), all agent velocities in the system described in (6) asymptotically approach zero, collision avoidance is ensured between the agents, and the group final configuration minimizes all agent global potentials.

Proof: Choosing the positive semi-definite function J defined as in (8) and calculating the time derivative of J along the solution of the error system (6), we have

$$\dot{J} = -\frac{1}{2}e_v^T\left((L + L^T) \otimes I_n\right)e_v - e_v^T(M \otimes I_n)e_v, \quad (9)$$

where $e_v = (e_v^{1T}, \cdots, e_v^{NT})^T$ is the stack vector of all agent velocity vectors in the error system; $L = [l_{ij}]$ with

$$l_{ij} = \begin{cases} -w_{ij}, & i \neq j, \\ \sum_{k=1,k\neq i}^{N} w_{ik}, & i = j, \end{cases}$$

is the Laplacian matrix of the weighted velocity neighboring graph if we set the corresponding edge weight of the graph to be w_{ij}; $M = \mathrm{diag}(m_1, \cdots, m_N)$; I_n is the identity matrix of order n and $\otimes$ stands for the Kronecker product.

By the definition of matrix L and the weight balance condition, it is easy to see that $L + L^T$ is symmetric, each row sum is equal to 0, the diagonal entries are positive, and all the other entries are nonpositive. By matrix theory [12], all eigenvalues of $L + L^T$ are nonnegative. Hence, matrix $L + L^T$ is positive semi-definite. Furthermore, it is easy to see that matrix M is positive definite. Thus $\dot{J} \le 0$, and $\dot{J} = 0$ implies that $e_v^1 = e_v^2 = \cdots = e_v^N$ and they all must equal zero. This occurs only when $v^1 = \cdots = v^N = v^0$. Thus $\dot{e}_v^i = \dot{v}^i = \mathbf{0}$ for all $i = 1, \cdots, N$. Based on LaSalle's invariance principle [13], the system trajectories converge to the largest positively invariant subset of the set defined by $E = \{e_v | \dot{J} = 0\}$. In E, the agent velocity dynamics in the error system is

$$\dot{e}_v^i = -\frac{1}{m_i}\sum_{j=1,j\neq i}^{N} \nabla_{e_p^i}\widetilde{V}^{ij} = -\frac{1}{m_i}\nabla_{e_p^i}\widetilde{V}^i. \quad (10)$$

Thus, in steady state, all agent velocities in the error system no longer change and equal zero, and moreover, from (10), the potential $\widetilde{V}^i$ of each agent i is globally minimized. Collision-free can be ensured between the agents since otherwise it will result in $\widetilde{V}^i \to \infty$. ■

From the proof of Theorem 2, it follows that, in steady state, all agent actual velocities no longer change and are equal to the desired velocity.

Remark 2: In the velocity neighboring graph, if the nonzero interaction coefficients all equal 1, then the weight balance condition implies that for each vertex, the number of arcs starting at it is equal to the number of arcs ending on it. The graphs satisfying such properties have been defined as the balanced graphs [9].

B. The Motion of the CoM

In what follows, we will analyze the motion of the CoM of system (1).

The position vector of the CoM in system (1) is defined as $x^* = (\sum_{i=1}^N m_i x^i)/(\sum_{i=1}^N m_i)$. Thus the velocity vector of the CoM is $v^* = (\sum_{i=1}^N m_i v^i)/(\sum_{i=1}^N m_i)$. By using control law (5), and by the symmetry of function V^{ij} with respect to x^{ij} and the weight balance condition, we get

$$\dot{v}^* = -v^* + v^0. \quad (11)$$

Suppose the initial time $t_0 = 0$, and $v^*(0) = v_0^*$. By solving (11), we obtain $v^* = v^0 + (v_0^* - v^0)e^{-t}$. Thus, it follows that, if $v_0^* = v^0$, then the velocity of the CoM is invariant and equals v^0 for all the time; if $v_0^* \neq v^0$, then the velocity of the CoM exponentially converges to the desired velocity v^0 with a time constant of 1 s. Therefore, from the analysis above, we have the following theorem.

Theorem 3: By taking the control law in (5), if the initial velocity of the CoM is equal to the desired velocity, then it is invariant for all the time; otherwise it will exponentially converge to the desired velocity with a time constant of 1s.

Remark 3: Note that, by the calculation above, we can see that, when the coupling matrix satisfies the weight balance condition, the velocity variation of the CoM does not rely on the neighboring relations or the magnitudes of the interaction coefficients. It is obvious that, when there is no external signal acting on the group and the motion of each agent is only based on the state information of its flockmates, under the weight balance condition, the velocity of the CoM is invariant.

C. Convergence Rate Analysis

From (9), it is easy to see that the interaction coefficients can influence the decaying rate of the total energy J, hence, it can also influence the convergence rate of the system. In what follows, we will present some qualitative analysis.

Let us again consider the dynamics of the error system. From the analysis in Theorem 2, we know that $\dot{J} \le 0$, and $\dot{J} = 0$ occurs only when $e_v^1 = \cdots = e_v^N = \mathbf{0}$, that is, only when all agents have reached the desired velocity. In other words, if there exists one agent whose velocity is different from the desired velocity, then the energy function J is strictly monotone decreasing with time. Of course, before the group forms the final tight configuration, there might be the case that all agent velocities have reached the desired value, but due to the regulation of the potentials among agents, it instantly changes into the case that not all agents have the desired velocity. Hence, the decaying rate of energy is equivalent to the convergence rate of the system. From (9), it follows that $\dot{J} = -\frac{1}{2}e_v^T\left(\left(L + L^T + 2M\right) \otimes I_n\right) e_v$. By matrix theory [12], we have $\dot{J} \leq -\frac{1}{2}\lambda_{\min}^* e_v^T e_v \leq -m_{\min} e_v^T e_v$, where $\lambda_{\min}^* > 0$ denotes the smallest real eigenvalue of matrix $L + L^T + 2M$, and $m_{\min} := \min_{i\in\mathcal{I}}\{m_i\}$. In the case that the velocity neighboring graph $\mathcal{D}$ is connected and not all agents have reached the common velocity, we have $\dot{J} \leq -\frac{1}{2}(\lambda_2 + 2m_{\min})e_v^T e_v$, where $\lambda_2 > 0$ denotes the second smallest real eigenvalue of matrix $L + L^T$. However, if the velocity neighboring graph is not connected, without loss of generality, we assume that $L + L^T = \mathrm{diag}\left(\left(L_1 + L_1^T\right), \cdots, \left(L_r + L_r^T\right)\right)$, and $M = \mathrm{diag}(M_1, \cdots, M_r)$ is the corresponding agent mass matrix, where $1 < r \leq N$ represents the number of the connected velocity neighboring graphs, $L_i + L_i^T$ is the Laplacian matrix associated with the ith connected component $\mathcal{D}_i$. When not all agents in the same connected subgroups have reached the common velocity, we have $\dot{J} \leq -\frac{1}{2}\min_{i\in\mathcal{I}^*}\left\{\lambda_2^i \widehat{e}_v^{iT}\widehat{e}_v^i\right\} - m_{\min} e_v^T e_v$, where λ_2^i denotes the second smallest real eigenvalue of matrix $L_i + L_i^T$, $\widehat{e}_v^i$ is the stack vector of all agent velocity vectors in the connected component $\mathcal{D}_i$, and $\mathcal{I}^* = \{1, \cdots, r\}$. Therefore, we have the following conclusion: The convergence rate of the system relies on the interaction coefficients as well as agent masses, and it is always not faster than the convergence rate of the CoM. Furthermore, if the initial velocity of the CoM is not equal to the desired velocity, then the fastest convergence rate of the system does not exceed the exponential convergence rate with a time constant of 1 s.

Remark 4: Note that, when the group has achieved the final steady state, the control input above equals zero.

III. DISCUSSIONS ON VARIOUS CONTROL LAWS

In the sections above, we introduced a set of control laws that enable the group to generate the desired stable flocking motion. However, it should be clear that control law (5) is not the unique control law to produce the desired motion for the group. In this section, we provide some other useful control laws. For simplicity, we only present the control laws for the group moving in the ideal case, since in the nonideal case, we only need to add the terms $k_i v^i$ $(i = 1, \cdots, N)$ to cancel the velocity damping. The analysis and proof are quite similar to the proof of Theorem 2, so we only present the control laws and the corresponding Lyapunov function.

Suppose that α^i and β^i rely on agent i's mass, and take the control law acting on agent i to be

$$u^i = -\sum_{j=1,j\neq i}^{N} m_i \nabla_{x^i} V^{ij} - \sum_{j\in\mathcal{N}_i} m_i w_{ij}(v^i - v^j) - m_i(v^i - v^0). \quad (12)$$

We still consider the error system (6) and choose the following Lyapunov function $J = \frac{1}{2}\sum_{i=1}^{N}(\widetilde{V}^i + e_v^{iT}e_v^i)$. The rest analysis is similar to Theorem 2, thus is omitted.

Definition 4: Define the center of the system of agents as $\overline{x} = (\sum_{i=1}^{N} x^i)/N$. The average velocity of all agents is defined as $\overline{v} = (\sum_{i=1}^{N} v^i)/N$.

It is obvious that the velocity of the system center is just the average velocity of all agents.

Using the control law in (12), we have $\dot{\overline{v}} = -\overline{v} + v^0$. Suppose the initial time $t_0 = 0$, and $\overline{v}(0) = \overline{v}_0$. We get $\overline{v} = v^0 + (\overline{v}_0 - v^0)e^{-t}$. It is obvious that, if $\overline{v}_0 = v^0$, then the velocity of the system center is equal to the desired velocity v^0 for all the time, and if $\overline{v}_0 \neq v^0$, then the velocity of the system center exponentially converges to the desired velocity with a time constant of 1 s.

IV. EXTENSIONS AND DISCUSSIONS

In this section, we investigate the case that not all agents know the desired velocity. We assume that the velocity neighboring graph is weakly connected and there exists at least one agent who knows the desired velocity. In the case that there is no external signal acting on the group, the collective dynamic behaviors of the agent group have been analyzed in [3].

Without loss of generality, suppose that agent i $(i = 1, \cdots, N_1)$ $(1 \leq N_1 < N)$ can detect the external reference signal, and agent j $(j = N_1 + 1, \cdots, N)$ cannot detect the reference signal. The control law acting on each agent i is taken to be

$$u^i = -\sum_{j=1,j\neq i}^{N} \nabla_{x^i} V^{ij} - \sum_{j\in\mathcal{N}_i} w_{ij}(v^i - v^j) - h_i m_i(v^i - v^0) \quad (13)$$

for all $i = 1, \cdots, N$, where $h_i = 1$ for all $i = 1, \cdots, N_1$ and $h_i = 0$ for all $i = N_1 + 1, \cdots, N$.

We still consider the error system (6). Using control law (13) and taking Lyapunov function (8), we have $\dot{J} = -\frac{1}{2}e_v^T\left(\left(L + L^T\right) \otimes I_n\right) e_v - e_v^T(\widehat{M} \otimes I_n)e_v$, where $\widehat{M} = \mathrm{diag}(h_1 m_1, \cdots, h_N m_N)$. From the proof of Theorem 2, we obtain that matrix $L + L^T$ is positive semi-definite. By the connectivity of graph $\mathcal{D}$, it follows that $L + L^T$ is irreducible and the eigenvector associated with the single zero eigenvalue is $\mathbf{1} = [1, \cdots, 1]^T \in \mathbf{R}^N$. From the proof of Theorem 2 in [10], we obtain that $e_v^T\left(\left(L + L^T\right) \otimes I_n\right) e_v = 0$ if and only if $e_v^1 = \cdots = e_v^N$. On the other hand, by the definition of h_i and m_i, it follows that matrix $\widehat{M}$ is positive semi-definite, and $e_v^T(\widehat{M} \otimes I_n)e_v = 0$ if and only if $e_v^i = \mathbf{0}$ for all $i = 1, \cdots, N_1$. Hence, $\dot{J} \leq 0$, and $\dot{J} = 0$ implies that $e_v^1 = \cdots = e_v^N = \mathbf{0}$. Following similar analysis as in the previous sections, we can conclude that the desired stable flocking motion can be achieved. Due to space limitation, we omit the detailed proof.

Remark 5: If there exists only one agent in the group who can detect the external reference signal, the group can still generate the desired stable flocking motion. This is of practical interest in control of multi-agent systems.

V. SIMULATIONS

In this section, we will present some numerical simulations for the system described by (1) in order to illustrate the theoretic results obtained in the previous sections.

These simulations are performed with ten agents moving in the plane whose initial positions, velocities and the velocity neighboring relations are set randomly, but they satisfy: 1) all initial positions are set within a circle of radius $R = 15$ m centered at the origin, 2) all initial velocities are set with arbitrary directions and magnitudes within the range of [0, 10] m/s, and 3) the velocity neighboring graph is connected. All agents have different masses and they are set randomly in the range of (0, 1] kg. Note that, because the position neighboring graph is complete, we will not describe it. In the following figures, we only present the velocity neighboring relations.

The following simulations are all performed with the same group having the same initial state. However, different control laws are taken in the form of (13) with the explicit potential function $V^{ij} = \frac{1}{2}\ln\|x^{ij}\|^2 + \frac{5}{2\|x^{ij}\|^2}$, $i, j = 1, \cdots, 10$. The interaction coefficient matrix W is generated randomly such that $\sum_{j=1}^{10} w_{ij} = \sum_{j=1}^{10} w_{ji}$, $w_{ii} = 0$, and the nonzero w_{ij} satisfy $0 < w_{ij} < 1$ for all $i, j = 1, \cdots, 10$. We run all simulations for 200 seconds.

Fig. 1 presents the group initial state and the interaction topology. Figs. 2 and 3 describe the group state in the case that the motion of the group is not influenced by any external signal. When we send a signal to the group and try to make all agents move at a desired velocity $v^0 = [0.1, -0.1]^T$, Figs. 4 and 5 show the results in our simulation with the control laws taken in the form of (5), whereas Figs. 6 and 7 show the simulation results with the assumption that there are only two agents, labelled by circles, who know the desired velocity. It can be seen from them that the desired stable flocking motion can be achieved. Fig. 8 shows the motion trajectories of the CoM and Fig. 9 (d) depicts the corresponding curves of the errors between the velocities of the CoM and the desired velocity, where the star represents the initial position of the CoM, and (a), (b), and (c) represent the corresponding states of the CoM in three simulations, respectively. Figs. 9 (a)–(c) are the velocity error plots to describe the errors between the agent actual velocities and the desired velocity in three simulations, and the dashed lines in Fig. 9 (c) depict the velocity error curves of the agents who can detect the external reference signal. Fig. 9 explicitly demonstrates that, when there is no external signal acting on the group, all agent velocities asymptotically approach a common velocity, and the velocity of the CoM is invariant in time and is equal to the final common velocity, otherwise, the velocities of all agents and the CoM asymptotically approach the desired velocity. Moreover, from Fig. 9, we can see an interesting phenomenon that increasing the numbers of agents receiving external signal do not necessarily imply a faster convergence rate of the group to the desired velocity. This is consistent with some real situations in nature.

Figs 2, 4, and 6 show that for a given initial condition, a group of agents may exhibit different transient behavior, depending on different choice of individuals for feeding in external signal. This suggests the possibility for adjusting the transient process by appropriately selecting the subgroup of agents on which to apply the external signal. From Figs 3, 5 and 7, we can see that the final tight configuration of the group is not unique, but in each case, the final potential of each agent is globally minimum according to the theoretical results.

Numerical simulations also indicate that the desired stable flocking can be achieved by using control law (13).

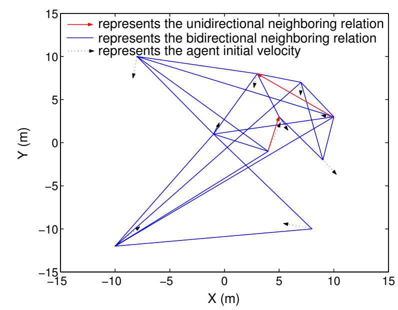

Fig. 1. The group initial state (t=0 s)

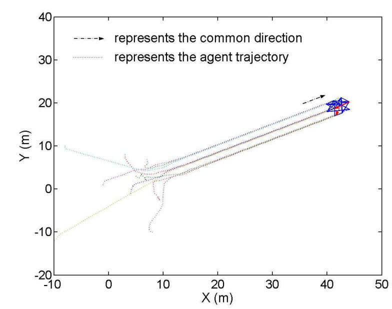

Fig. 2. Final configuration and trajectories (t=200 s)

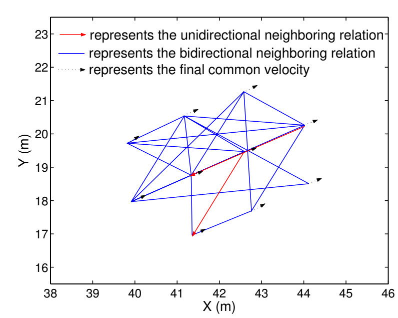

Fig. 3. Final configuration and velocities (t=200 s)

REFERENCES

[1] C. W. Reynolds, Flocks, herds, and schools: A distributed behavioral model, *Computer Graphics (ACM SIGGRAPH '87 Conference Proceedings)*, vol. 21, no. 4, 1987, pp. 25–34.

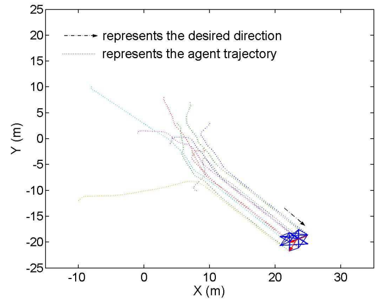

Fig. 4. Final configuration and trajectories (t=200 s)

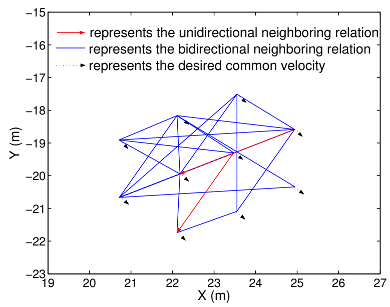

Fig. 5. Final configuration and velocities (t=200 s)

[2] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Stable flocking of mobile agents, Part I: Fixed topology; Part II: Dynamic topology", in *Proc. IEEE Conference on Decision and Control*, Maui, Hawaii USA, vol. 2, December 2003, pp. 2010–2015; 2016–2021.

[3] H. Shi, L. Wang, and T. Chu, "Coordination of multiple dynamic agents with asymmetric interactions", in *Proc. IEEE International Symposium on Intelligent Control*, Limassol, Cyprus, June 2005, pp. 1423–1428.

[4] N. E. Leonard and E. Fiorelli, "Virtual leaders, artificial potentials and coordinated control of groups", in *Proc. IEEE Conference on Decision and Control*, Orlando, Florida USA, vol. 3, December 2001, pp. 2968–2973.

[5] S. Mu, T. Chu, and L. Wang, Coordinated collective motion in a motile particle group with a leader, *Physica A*, vol. 351, June 2005, pp. 211–226.

[6] L. Wang, H. Shi, and T. Chu, "Flocking control of groups of mobile autonomous agents via local feedback", in *Proc. IEEE International Symposium on Intelligent Control*, Limassol, Cyprus, June 2005, pp. 441–446.

[7] H. Shi, L. Wang, T. Chu, and M. Xu, "Flocking control of multiple interactive dynamical agents with switching topology via local feedback", *Lecture Notes in Artificial Intelligence*, Springer-Verlag, vol. 3630, 2005, pp. 604–613.

[8] L. Wang, H. Shi, T. Chu, W. Zhang, and L. Zhang, "Aggregation of forging swarms", *Lecture Notes in Artificial Intelligence*, Springer-Verlag, vol. 3339, 2004, pp. 766–777.

[9] R. Olfati-Saber and R. M. Murray, Consensus problems in networks of agents with switching topology and time-delays, *IEEE Transactions on Automatic Control*, vol. 49, no. 9, 2004, pp. 1520–1533.

[10] T. Chu, L. Wang, and T. Chen, "Self-organized motion in a class of anisotropic swarms: Convergence vs oscillation", in *Proc. American Control Conference*, Portland, OR, USA, June 2005, pp. 3474–3479.

[11] C. Godsil and G. Royle, *Algebraic Graph Theory*, New York: Springer-Verlag, 2001.

[12] R. A. Horn and C. R. Johnson, *Matrix Analysis*, New York: Cambridge Univ. Press, 1985.

[13] H. K. Khalil, *Nonlinear Systems*, Upper Saddle River, NJ: Prentice–Hall, 1996.

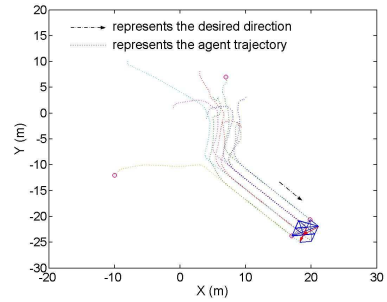

Fig. 6. Final configuration and trajectories (t=200 s)

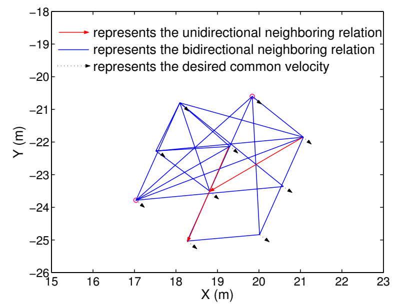

Fig. 7. Final configuration and velocities (t=200 s)

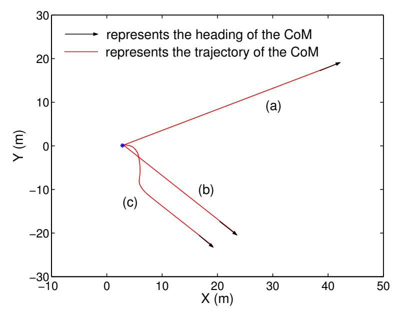

Fig. 8. The trajectories and headings of the CoM (t=200 s)

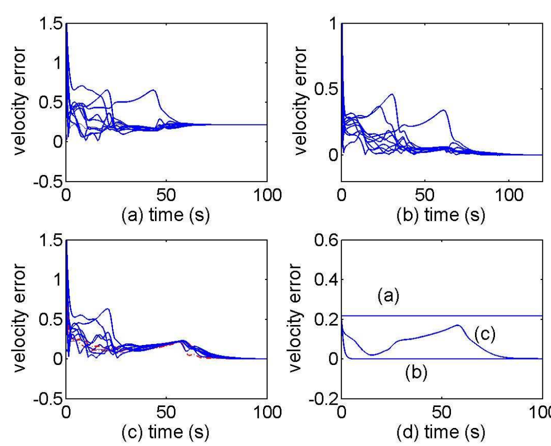

Fig. 9. The velocity error plots