# Architecture and Scheduling Algorithm for a Multi-Channel Wireless Infrastructure Network with Diversity Management Layer

Yong Huang, Weibo Gong and Dev Gupta

Abstract-802.11a protocol provides 12 non-overlapping frequency bands, but the common wireless LAN usually only uses one channel. In this paper we propose a "super" Wi-Fi network architecture, which uses the multiple non-overlapping channels simultaneously to improve the total network bandwidth with minimum changes to the existing devices. To be compliable to the existing Wi-Fi network hardware and 802.11 protocols, a software based diversity management layer (DML) is introduced above the MAC layer to control and support the multi-channel. To minimize the total transmission time, a central scheduler is needed to decide how to allocate the stations' requests into several simultaneous channels. Since the scheduling time is limited, we develop a Lagrangian relaxation based scheduling algorithm for this scheduling problem. The simulation results show that the algorithm is faster than Cplex and provides good sub-optimal solutions. We also propose a backup-resource method to further reduce the scheduling time when the number of stations in the system increases.

## I. INTRODUCTION

In recent years, the wireless LANs have been widely deployed in enterprises, public areas and home environment. Because of its implementation convenience and low maintenance cost, wireless LAN is becoming a very popular lastmile connection. On the other hand, wireless LANs are still suffering from lower throughput comparing to wired LANs.

One of the factors for the lower throughput is the limited wireless spectrum. In order to improve the wireless LAN throughput, IEEE 802.11a and IEEE 802.11b standards define 12 and 3 non-overlapping channels, respectively. With these non-overlapping channels, more users can be allocated in the same spot without interfering to each other. But in most current wireless LANs, usually only one channel is used. As the prices of 802.11 devices have decreased a lot during recent years, it is attractive to find methods for utilizing the existing multiple channels based on current 802.11 devices.

Raniwala *et al.* [1] proposed an 802.11-based multi-hop wireless ad hoc network architecture. The multi-channel capacity is used by a wireless mesh network, which is a midnetwork between the end-users and the backbone network. From the implementation point of view, it's a complicated architecture and more hardware devices are needed in comparison to a standard 802.11 network. Also from the enduser's point of view, end-users still can only communicate with a traffic aggregation device using one channel.

In this paper we propose a software-based architecture, which we refer to as a super Wi-Fi network. Super Wi-Fi net-

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work is designed for an enterprise and can support hundreds of stations. It simultaneously uses up to 12 available nonoverlapping channels with the IEEE 802.11a hardware devices to improve the overall network bandwidth. To be compliable to the existing Wi-Fi network hardware and 802.11 protocols, a pure software-based diversity management layer (DML) is proposed, which locates above standard 802.11 MAC layer to control and support the multiple channels.

Instead of using the pure CSMA/CA mechanism, a super Wi-Fi network uses contention periods to transmit request and request-grant messages and uses contention-free periods to transmit data. In this paper, a Lagrangian relaxation based Pricing Algorithm (LPA) is introduced to optimize the total data transmission time. In order to reduce the scheduling time, a part of the channel capacity is reserved as the backup resource to accommodate occasional and minor violations of the channel resource constraint. Simulation studies are presented.

The rest of this paper is organized as follows. Section II introduces the system architecture and diversity management layer. In section III, a scheduling problem is defined with the goal to optimize the total transmission time subject to Wireless Access Point (WAP) capacity constraints. Section IV gives a Lagrangian relaxation based pricing algorithm to solve the scheduling problem. In section V, we propose a novel backup-resource method to further reduce the scheduling time. Section VI concludes the paper.

#### **II. SYSTEM ARCHITECTURE**



Fig. 1. Super Wi-Fi Network

Super Wi-Fi network uses current standard 802.11 wireless hardware. Each station just uses one Network Interface Card (NIC) and only the software driver needs to be upgraded. The system consists of several standard WAPs, which are working on different non-overlapping channels, respectively. Fig 1 depicts the network architecture. The AP1, AP2, ..., AP12 are 12 standard 802.11a access points. The number of the access points might vary according to different size of enterprises and different number of stations. These access points are geographically distributed around the enterprise cells and connected to the Internet/WAN via a switch. The stations in the enterprise zone have the capability to communicate with any WAP, but the channel conditions vary. A separate coordinator, the central scheduler PC, is also connected to the switch. Each station should establish a Virtual Private Network (VPN) connection with the scheduler when it joins the network during a contention period. A VPN channel is the control channel between the central scheduler and the station. The control messages transmitted through the VPN channel are:

- Channel Condition Message (CCM): the channel conditions are described as the effective throughput from WAPs to the stations. They are measured at each station and reported to the central scheduler only when some channel conditions are changed;
- Access Request Message (ARM): the message from a station to request for transmitting certain bytes of data. The data size is decided by the application and the QoS requirement;
- Access Grant Message (AGM): the message from the central scheduler to inform a station which channel should be used and when to transmit the requested data;
- Indication of Incoming Data Message (IIDM): the message from the central scheduler to inform a station to listen on a specified channel for incoming data at indicated time.

Before any data transmission, a station should always send ARM to the scheduler via the pre-established VPN channel, including the size, QoS parameters, etc. After the scheduler receives all the requests, it will execute a central scheduling algorithm to decide how to allocate the requests. In order to solve the scheduling problem in limited time, sub-optimal solution is acceptable. When the system is highly loaded, it is usually hard to obtain a feasible solution very fast. In that case, some requests will be dropped according to the QoS priorities. In section V, we give a guideline to use a fraction of the channel capacity as a backup-resource to reduce the scheduling time.

When the scheduling results are available, the scheduler will send the grant information as AGMs back via VPN channels. When an AGM or IIDM is received, the station will switch to the indicated channel to transmit or receive data for the assigned time period. When the data transmission is over, each station switches back to its VPN channel, unless it receives explicit message to change the WAP associated with its VPN channel. In a super Wi-Fi network, the VPN channel is the only control channel and is also implemented on the wireless medium. Each VPN channel can be described as a  $(Station_i, WAP_j)$  pair and the VPNs with the same access point share a contention based VPN period. Fig 2 shows the VPN period and the data period.



Fig. 2. VPN Period and Data Period

During a VPN period, the stations submit their requests, ARMs, using the CSMA/CA mechanism. The VPN interval is a parameter indicating the time between two VPN periods and has a fixed value.

The scheduler will use the next VPN period to broadcast the scheduling results. 802.11 MAC provides a virtual carrier-sensing mechanism, network allocation vector (NAV), to predict the future traffic on the medium. Each station's DML should preset the network allocation vector (NAV) at the beginning of each VPN period such that the stations will not take control of the medium. When the scheduling result is received, the station's DML will update the NAV and access the wireless medium at the scheduled time.

## A. Diversity Management Layer

The DML is a new MAC sub-layer between the upper layers and the standard 802.11 MAC layer. An important guideline for the design of super Wi-Fi network is to be compliant to 802.11 standards. So DML should support and manage the multi-channel in the manner that the multichannel is transparent to the standard 802.11 MAC. Fig 3 presents the DML in the protocol stack.



Fig. 3. DML in protocol stack

The basic services of the DML include:

- Manage the VPN channel;
- Communicate with the central scheduler;
- Periodically measure the channel condition.

The VPN channel is the only control channel between the DML of each station and the central scheduler. The DML should setup the VPN channel when the station joins the network. The station can change the VPN channel if the

channel condition of the current VPN is very bad. The DML will close the VPN channel when the station leaves.

The DML communicates with the central scheduler to determine which channel to use for each data transmission. It will form a request frame and tell the physical layer to switch channel upon receiving the channel access assignment. The request frame has the information of packet size, QoS parameter and measured channel conditions or preferred AP set. The central scheduler will aggregate all the requests and run a centralized channel allocation algorithm. The scheduling results or channel access assignment will then be broadcasted to all stations' DMLs.

Another task of the DML is to periodically measure the conditions of the multiple channels. The channel conditions have crucial impact on the throughput and are helpful for the central scheduler to determine the channel allocation. The CCM can be either submitted periodically or piggyback with requests or data frames.

# **III. PROBLEM FORMULATION**

In wireless networks, the time to switch from one channel to another channel should be considered. For a typical 802.11a wireless card, the switching time is 150-200 microsecond. Frequent channel switching will make the scheduling algorithm complicated and also cost more energy overhead. In this paper we assume that a request from one station can only be transmitted via one channel during a scheduling period.

## A. Notations

We use the following notations throughout this paper. We label the n stations as  $S_1, S_2, ..., S_n$  and the m wireless APs as  $W_1, W_2, ..., W_m$  ( $m \le 12$ ). Since these wireless APs work on different channel frequencies, we use  $W_j$  to indicate channel j too. We also need the following notations:

- m: number of WAPs;
- n: number of stations;
- $q_{i,j}$ : effective data rates in bit/second (bps) unit for  $S_i$  to transmit data via channel  $W_j$ .  $q_{i,j}$  depends on the channel conditions, such as the distance between station and WAP and signal to noise ratio  $SNR_{i,j}$ . We assume that the  $q_{i,j}$  can be measured and aware to the central scheduler;
- $b_i$ : total bandwidth request of station  $S_i$  in bit unit;
- $x_{i,j}$ : 0-1 decision variable.  $x_{i,j} = 1$  dedicates the request of station  $S_i$  is transmitted via  $W_j$  and  $x_{i,j} = 0$ , otherwise;
- $t_{i,j}$ : transmission time used by  $S_i$  via  $W_j$ ,  $t_{i,j} = b_i \cdot x_{i,j}/q_{i,j}$ ;
- $T_j$ : the length of available contention free period of channel  $W_j$ .

# B. The Scheduling Problem

For each scheduling cycle, the objective is to minimize the total transmission time for the whole system. So the scheduling problem can be formulated as an optimization problem:

$$Minimize \sum_{i=1}^{n} \sum_{j=1}^{m} t_{i,j} = \sum_{i=1}^{n} \sum_{j=1}^{m} \frac{b_i \cdot x_{i,j}}{q_{i,j}}$$
(1)

subject to the WAP bandwidth resource limit:

$$\sum_{i=1}^{n} t_{i,j} = \sum_{i=1}^{n} \frac{b_i \cdot x_{i,j}}{q_{i,j}} \le T_j, \text{ for all j,}$$
(2)

and the total request constraint of each station:

$$\sum_{j=1}^{m} x_{i,j} \le 1, x_{i,j} = 0 \text{ or } 1.$$
(3)

This is a generalized assignment problem in integer programming [2]. There are several well-known solutions. However, the scheduling interval, namely the interval between two VPN periods, is too small for those methods to converge and the optimal result cannot give any explicit indication for the next scheduling cycle. We propose a Lagrangian relaxation based iterative algorithm to solve this problem.

## **IV. SCHEDULING ALGORITHM**

#### A. Algorithm

Lagrangian relaxation based algorithms have been successfully used to solve some integer programming or mix-integer programming problems, such as the job shop scheduling [3] and power system scheduling [4][5]. The basic idea of Lagrangian relaxation method is to relax the WAP bandwidth resource constraints (2) by introducing the Lagrangian multipliers  $\lambda = [\lambda_1, ..., \lambda_m]^T$ .  $\lambda_j$  is associated to the bandwidth constraint of channel  $W_j$ . Using the objective function (1) and constraints (2), the Lagrangian is formulated as:

$$L = \sum_{i=1}^{n} \sum_{j=1}^{m} t_{i,j} + \sum_{j=1}^{m} \lambda_j (\sum_{i=1}^{n} t_{i,j} - T_j)$$
(4)

By observing the structure of the Lagrangian function (4) and using the duality theorem [6], we transfer the original optimization problem to a two-level maximum-minimum problem. Given multipliers  $\lambda$ , the low-level minimum optimization consists of the following individual sub problems:

(P-i), i = 1, 2, ..., n  

$$\min L_i(\lambda) = \sum_{j=1}^m t_{i,j} + \sum_{j=1}^m \lambda_j (t_{i,j} - \frac{T_j}{n})$$

$$= \sum_{j=1}^m \left( (1 + \lambda_j) \frac{b_i}{q_{i,j}} \cdot x_{i,j} - \lambda_j \frac{T_j}{n} \right) \quad (5)$$

subject to constraints (3).

Intuitively we can view  $\lambda_j$  as a virtual price for using channel  $W_j$ . The pricing information can directly show how popular each WAP was and give the guideline for system upgrade. Each station wants to minimize its own transmission time but it also has to pay the price for using the bandwidth. It is straightforward to solve the sub problem.

$$J_i^* = \arg\min_j \left( (1+\lambda_j) \frac{b_i}{q_{i,j}} \cdot x_{i,j} - \lambda_j \frac{T_j}{n} \right) \tag{6}$$

Let  $x_{i,J_i^*} = 1$  and other  $x_{i,j} = 0$  if  $j \neq J_i^*$ . The time to solve each sub problem is o(m).

Given  $\lambda$ , let  $L_i^*(\lambda)$  denotes the optimal value for the sub problem (P-i). Then the upper level dual problem is:

$$\psi^* = \max_{\lambda(\lambda_j \ge 0)} \psi(\lambda) \text{ with } \psi(\lambda) = \sum_{i=1}^n L_i^*(\lambda)$$
 (7)

Since the discrete integer decision variables are involved at the low level minimum problems, the dual function (7) may not be differentiable at certain points. Therefore, a subgradient algorithm is used to update the multipliers  $\lambda$ .

$$\lambda_j(k+1) = \max[0, \lambda_j(k) + \alpha(k)g_j(k)], \text{ for j=1,...,m}$$
 (8)

where k is the high-level iteration number and  $\alpha$  is the step size.  $g_j(k)$  is the subgradient of  $\psi(\lambda)$  with respect to  $\lambda_j(k)$ and defined as:

$$g_j(k) = \sum_{i=1}^n (\frac{b_i}{q_i, j} \cdot x_{i,j}(k)) - T_j$$
(9)

where  $\{x_{i,j}(k)\}\$  are the solutions of the sub problems given  $\lambda(k)$ .

The adaptive step size at iteration k,  $\alpha(k)$ , is given by:

$$\alpha(k) = \gamma \frac{\psi^* - \psi(k)}{\sum_{j=1}^m g_j(k)^2}$$
(10)

where  $\gamma$  is a positive scaling value between 0 and 2.  $\psi^*$  is the optimal value of  $\psi$  with  $\psi(k) \leq \psi^*$ . Proof of the convergence of updating procedure is given in [7]. The basic idea is: if  $\lambda^*$  is one in the optimal set, since g(k) is the subgradient of  $\psi(\lambda)$ , the following relation holds:

$$\psi^* - \psi(k) \le g(k) \cdot (\lambda^* - \lambda(k)) \tag{11}$$

If  $\lambda(k)$  is not optimal, then the direction of g(k) makes an acute angle from  $\lambda(k)$  to  $\lambda^*$ . Consequently,  $\lambda(k + 1)$  will move closer to  $\lambda^*$  than  $\lambda(k)$ . Thus, the updating process will make the  $\lambda(k)$  converge to the optimal set even if the objective function is not monotonic.

The high-level updating procedure will stop when the change of  $\lambda$  is small enough. But the dual solution is generally infeasible, i.e. the constraints of (2) are not satisfied. We use a heuristic method to generate a feasible solution. The key idea is to change the decision variable  $x_{i,j}$ , where j is one of the congested WAPs, while this change will minimize the increase of objective function.

We now present the summary of the algorithm:

Step 0: [Initialize]

Good initialization of the Lagrangian multipliers  $\lambda$  can significantly reduce the number of high-level iterations. Since the requests of stations usually change smoothly, in our algorithm we inherit the converged  $\lambda$  of previous scheduling cycle as  $\lambda^0$ . Step 1: [Solve low-level sub problems] Given  $\lambda(k)$  ( $k \ge 0$ ), for i = 1, ..., n, find

$$J_i^* = \arg\min_j \left( (1+\lambda_j) \frac{b_i}{q_{i,j}} \cdot x_{i,j} - \lambda_j \frac{T_j}{n} \right)$$

set  $x_{i,J_i^*} = 1$  and other  $x_{i,j} = 0$  if  $j \neq J_i^*$ 

Step 2: [Update high-level multipliers]

- (a) Calculate the objective value of  $\psi(k)$ ;
- (b) Calculate the step size  $\alpha(k)$  and the subgradient accord-
- ing to equations (10) and (9), respectively;
- (c) Update  $\lambda$  according to equation (8). Step 3: [Check Convergence]

Check the stopping criteria, if

$$\|\lambda(k+1) - \lambda(k)\| < \epsilon$$

where  $\epsilon$  is a small positive number, or k is larger than a preset number, stop. Also, considering the scheduling time is limited, the algorithm will stop if all constraints of (2) are satisfied, i.e. a feasible solution is obtained. Else, go to Step 1.

#### Step 4: [Generate feasible solution]

Let  $J_o = \{j | W_j \text{ is overbooked}\}$  describes the set of overbooked WAPs, i.e. the assigned request time to the WAP is larger than the allocatable time length. Then the heuristic method to generate a feasible solution is: for all  $j \in J_o$ , find the  $j^*$   $(j^* \notin J_o)$ , such that the difference of objective function values is minimum when  $x_{i,j} = 1$  is changed to  $x_{i,j^*} = 1$  while other decision variables are fixed. If  $j^*$  doesn't exist, then feasible solution cannot be obtained.

After adjustment, if  $W_j$  is not congested, remove j from  $J_o$ . If  $J_o$  is empty, report feasible solution. Else, go to step 4.

# B. Simulation Study

We simulate a super Wi-Fi network with four access points and different number of stations. The request length of each station follows an exponential distribution with expectation of 1/m. 90% of the stations are lightly loaded users with expectation of 1024 bits. 5% stations are normally loaded with 1/m = 2048 bits and the rest 5% are heavily loaded with 1/m = 4096 bits. The effective date rates  $q_{i,j}$  are uniformly distributed among 1Mbp, 2Mbps, 5Mbps and 11Mbps. The scheduling cycle is set to 10ms and  $T_i = 10$  for all j.

We compared our pricing algorithm with the Cplex 9 when the number of stations varies from 80 to 160. Our Lagrangian relaxation based Pricing Algorithm (LPA) is implemented in C++. The simulations were executed on a 2.66GHz Linux Workstation with 1G RAM.

In practice, the optimal value  $\psi^*$  is generally unknown before the dual function is solved. Instead, its estimation is used [8]. In our simulation, we use an adaptive step sizing method [5]. In every high-level iteration, we fix the norm of  $\lambda(k+1)-\lambda(k)$  to be a positive number and use this parameter to control the actual step size. The adaptive step sizing means we decrease this number whenever the violation of constraint increases. Otherwise, we increase this parameter. Fig 4 shows the dual function value vs. the high-level iteration number.



Fig. 4. Dual Function Value vs. Dual Iterations



Fig. 5. Relative Difference of Objective Values



Fig. 6. Scheduling Time: LPA vs. Cplex

Fig 5 compares the relative difference of objective function values between Cplex and LPA. The relative difference is defined as  $(\psi_{LPA} - \psi_{Cplex})/\psi_{Cplex}$ . Although LPA can only obtain a sub-optimal solution, the difference between the objective values is very small.

Fig 6 presents the evolution of scheduling time. It is obvious that LPA is faster than Cplex. The scheduling time is less than 10ms even when the number of stations increases to more than 150. Fig 7 illustrates the high-level iteration



Fig. 7. High-level Iteration Numbers vs. Number of Stations

numbers of LPA and Cplex. Because the iteration number of LPA is much smaller than that of Cplex, LPA saves much more scheduling time.

#### V. BACKUP-RESOURCE ANALYSIS

Although our scheduling algorithm is faster than the Cplex algorithm, when the system load increases, the scheduling time still grows fast. In this section, we propose a new backup-resource method to significantly reduce the scheduling time, especially when the system load increases.

Backup-resource means during each scheduling process, a fraction of the channel resource is used for the backup purpose. For example, we use  $(1 - \alpha)T_j$  as the available  $j^{th}$  channel resource instead of  $T_j$  to run the scheduling algorithm.  $\alpha$  is the factor of the resource to be reserved for backup. If the WAP resource constraint (2) is violated, the backup resource will be used to accommodate what causes the violation. With the help of the backup resource, in general the scheduling time will be significantly reduced. On the other hand, since the high-level iteration is reduced, the solution may not be very close to the optimal one.

We need to address several issues here. First, how does the backup-resource method affect the request loss probability? Second, how much should be the backup resource, i.e. what's the appropriate value of  $\alpha$ ? Third, how much can the backup-resource improve the scheduling time?

To answer the first two questions, we use a simple M/M/m/n/n pure loss model to analyze the change of the loss probability against different  $\alpha$  values. We assume that each WAP has m servers with constant capacity and no buffer. So if a request arrives and all the servers are occupied, the request will be lost. There are totally n stations using the WAP with the same arrival rate  $\lambda$  and each station cannot have two requests served at the same time. The service time of each request follows exponential distribution with expectation  $1/\mu$ . With backup-resource the service rate will reduce to  $(1 - \alpha)\mu$ . Then the request loss probability,  $P_{loss}$ , is[9]:

$$P_{loss} = \frac{C_n^m (\frac{\lambda}{1-\alpha)\mu})^m}{\sum_{i=0}^m C_n^i (\frac{\lambda}{(1-\alpha)\mu})^i}$$
(12)



Fig. 8.  $P_{loss}$  vs.  $\alpha$ 

Lets define  $\rho = \frac{n\lambda}{m\mu}$  to describe different load conditions. Fig 8 shows how the  $P_{loss}$  changes against  $\alpha$  for different  $\rho$ . When  $\alpha$  is less than 0.15, the  $P_{loss}$  increases little for small and medium  $\rho$ . It suggests that reserving part of the capacity as backup-resource won't affect the  $P_{loss}$  much when system is not highly loaded. Fig 9 and Fig 10 show the simulation results of the improvement of the scheduling time and high-level iteration number for different  $\alpha$ , respectively. For a small system or light loaded system, backup-resource method doesn't improve much because the scheduling is already fast. It helps especially when the number of stations is large. If  $\alpha$  is too small, the backup-resource might not be able to accommodate the violation part. When  $\alpha = 0.05$  the scheduling time got no significant improvement.



Fig. 9. Improvement of scheduling time

# VI. CONCLUSION

In this paper, a software based super Wi-Fi network architecture is proposed to use the multiple non-overlapping wireless channels simultaneously. A diversity management layer above the standard IEEE 802.11 MAC layer is introduced to support the multi-channel features. Time is divided into contention period and contention-free period. The contention period is for stations to submit requests and receive grand of request message, while the contention-free period is for



Fig. 10. Improvement of Iteration Number

data transmission. A station can transmit data with several Wireless Access Points (WAPs). Which WAP to use should be decided by a central scheduler. The central scheduler has the full knowledge of the channel conditions measured at each station and the goal of the scheduling problem is to minimize the total transmission time of all subscribed stations subject to the bandwidth constraints at each WAP. A Lagrangian relaxation based pricing algorithm is proposed to solve the scheduling problem. Simulation study shows the pricing algorithm is much faster than Cplex 9. The decided "pricing" information can be used as the initial pricing value for next scheduling period and improve the converging speed. We also discussed a novel idea to reduce the scheduling time, the backup-resource method, to further reduce the scheduling time.

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