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Two Simultaneous Leaks Isolation in a Pipeline by Transient Responses

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Abstract— This paper presents a procedure to isolate two simultaneous leaks in a pipeline using the parameter estimation framework with a reduction in the research interval of the four unknown parameters when only pressure and flow rate are measured at the extremes of the line. Considering the steady state parameter z_{eq} which characterizes the subset of two indistinguishable leaks, a family of dynamic models with constraints is obtained. This family, called $\mathcal{F}$, allows to reformulate in a simple way the leaks identification task in which diverse off-line optimization algorithms can be used to solve it. The formulation of the optimization task with constraints is the main contribution of this work. The advantages of the family $\mathcal{F}$ in term of the search interval reduction are discussed using the parameters of a water pilot pipeline of $135m$ long in which the $\mathcal{L}_2$ norm of the upstream flow error is minimized using real data.

I. INTRODUCTION

The automatic detection and location of multi-leak in the fluid transport systems with pipelines is a challenge for the safe process and supervision engineering. Several schemes of leak location based in the mathematical model of the fluid have been developed. Shields [1] and Kowalczuk [2] designed residual generators using a finite dimension model and assuming fix space discretization in the set of partial differential equations (PDE) which describes the fluid behavior. However, these methods are not robust with respect to uncertainty in the position and can be only applied to locate leaks in limited cases. The accuracy in the position obtained with these methods can be increase using the procedure given in [3]; this latest is robust and efficient, again only if one leak exists. Verde [4] formulated the location problem assuming sequential leaks and scanning a search interval in the pipeline where the leak position is estimated with an adaptive law. This procedure is successful if a new leak appears after the other has been located. Moreover, diverse methods which assumed only one leak have been tested; they delivered the same wrong value z_{eq}, when they are tested with the same data for two simultaneous leaks. The reasons of this fact are [5]:

- The impossibility to isolate two leaks with any residual evaluation algorithm based on steady state conditions since a non unique solution exists; and
- The parameterization by the virtual leak located in the position z_{eq} which depends on the orifice size and position of the two leaks.

On the other hand, in [6] a leak detection method is presented which is based on transient responses and the authors use the Fourier Transform of the pressure signals to get the leak information. Specifically, the position of one leak can be detected, but the approach assumes that the pipeline is working around an operation point. The sensitivity study for the pipeline model with two leaks given in [7] concluded that: the transient of the fluid is rich enough to isolate most of the two leaks cases, and if the model neglected fast dynamics, the locations are un precise.

Therefore, we propose a dynamic model with unknown order and tackle the two leaks isolation task using the framework of parameter identification. In particular, the parameter z_{eq} together with physical constraints of the pipeline are used to determine a family of nonlinear models of high order $\mathcal{F}$ with finite cardinality and three unknown parameters. This family characterizes the indistinguishable leaks with steady state data and describes at the same time, suitable transients for identification purpose. Thus, static and dynamic complementary conditions are used to reduce the search intervals for the unknown parameters simplifying the isolation task. Strictly speaking, diverse available off-line optimization methods, can be used to determine the unknown parameter of the family $\mathcal{F}$. Here, the toolbook Optimization of MATLAB software [8] is used.

The paper is organized as follows. In Section II starting from the PDE and assuming two leaks in the pipeline, static relations which must satisfy the physical variables of the system in leaks conditions are given. The key parameter z_{eq} in these relations characterizes the subset of indistinguishable leaks in the steady state. In Section III one proposes rules to select the space discretization which determines the minimal finite dimension of the dynamic model, and one introduces the family of models $\mathcal{F}$ with only three unknown parameters in faulty conditions. Section IV summarizes the isolation algorithm and in Section V a study case using simulated data of a water pilot pipeline is discussed and shows the advantage of the novel model. Finally, the conclusions are given in the last section.

II. PIPELINE MODEL

Consider the one-dimensional nonlinear model of a fluid in a pipeline with distributed parameters given by

$$\frac{\partial Q(z,t)}{\partial t} + gA\frac{\partial H(z,t)}{\partial z} + \mu Q(z,t)\left|Q(z,t)\right| = 0 \quad (1)$$

$$b^2\frac{\partial Q(z,t)}{\partial z} + gA\frac{\partial H(z,t)}{\partial t} = 0 \quad (2)$$

which is obtained using momentum and energy equations and assuming incompressible fluid [9], with $H(z,t)$ the pressure head (m), $Q(z,t)$ the flow rate (m^3/s), z the length coordinate (m), t the time coordinate (s), g the gravity (m/s^2), A the section cross-area (m^2), D the pipeline diameter (m), b the speed of sound (m/s) and $\mu = \frac{f}{2DA}$ where f is the friction coefficient.

Consider the existence of two leaks arbitrarily located at point $p_1 = z_1$ and $p_2 = z_1 + z_2$ of a pipeline with length L. The outflow associated to each leak

$$Q_f(p_i,t) = \lambda_i\sqrt{H(p_i,t)} \quad \text{for } i = 1,2 \quad (3)$$

with $\lambda_i > 0$ produces a discontinuity in the eqs. (1) and (2) and new boundary conditions in the system appear. If the up and down stream flows and the pressures at the extremes of the line are measured, diverse combination of boundaries can be used. Defining

$$\begin{bmatrix} y_{1s} & y_{2s} \end{bmatrix} := \lim_{t\to\infty}\begin{bmatrix} Q(t,0) & Q(t,L) \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} u_{1s} & u_{2s} \end{bmatrix} := \lim_{t\to\infty}\begin{bmatrix} H(t,0) & H(t,L) \end{bmatrix} \quad (5)$$

one obtains from (1) and (2) a steady state constraint for two simultaneous leaks given by

$$y_{1s} - y_{2s} = \lambda_1\sqrt{u_{1s} - \frac{\mu z_1}{a_1}y_{1s}^2} + \lambda_2\sqrt{u_{2s} + \frac{\mu(L-z_2)}{a_1}y_{2s}^2} \quad (6)$$

with $a_1 = gA$. Moreover, Verde et al [5] shown that the parameter

$$z_{eq} := \frac{a_1(u_{1s} - u_{2s})}{\mu(y_{1s}^2 - y_{2s}^2)} - \frac{Ly_{2s}^2}{(y_{1s}^2 - y_{2s}^2)} \quad (7)$$

with $y_{1s} \neq y_{2s}$, characterizes all the two leaks which can not be isolated from the pipeline data in steady state. This value can be estimated straightforward and it has been validated with data of a water pilot pipeline [7]. Since, eq. (7) is obtained assuming only two leaks, it cannot be used for the general multi-leak isolation task.

Fig. 1 shows the pressure profile in steady state with two leaks located at z_1 and $z_1 + z_2$ respectively. One can see that the profile of the pressure along the first and last section is equivalent to the profile of one leak located at point z_{eq} and the following conditions between positions satisfy

$$z_1 < z_{eq}, \qquad z_2 > z_{eq} - z_1 \quad (8)$$

In the case of one leak, its position is obtained from $z_{eq} = z_1$, and z_2 could take any value satisfying the second inequality of (8).

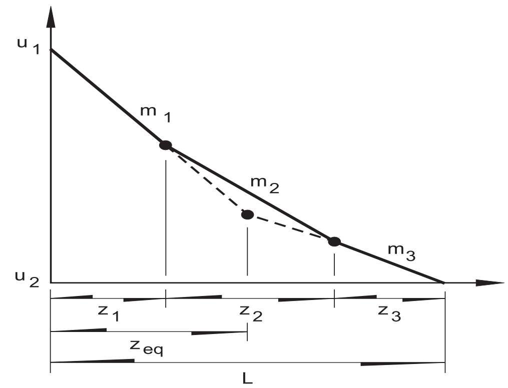

Fig. 1. Pressure drops with 2 leaks at points z_1 and $(z_1 + z_2)$

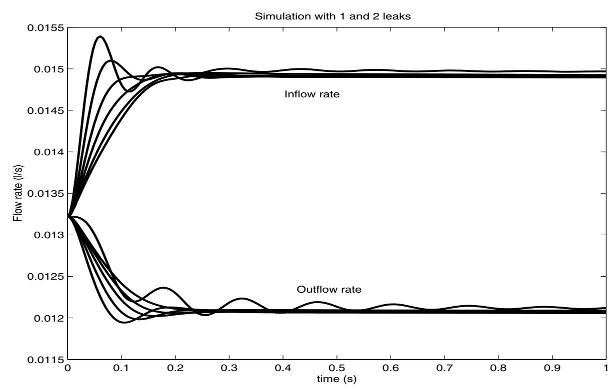

Fig. 2. Transient response simulation with a leak located at $50m$ and with five pairs of leaks for $z_{eq} = 50m$

Moreover from (7) and Fig. 1 one obtains

$$(y_{1s}^2 - y_{2s}^2)(z_{eq} - z_1) = z_2(Q_{eqs}^2 - y_{2s}^2) \quad (9)$$

with $Q_{eqs} = \lim_{t\to\infty} Q_{eq}(t, z_{eq})$, the steady flow rate at z_{eq}.

Since, there are three equations (6), (7) and (9), and four unknown parameters λ_1, λ_2, z_1 and z_2, an infinite number of quadruplets $\wp_j(z_{eq}) = (z_{1_j}, \lambda_{1_j}, z_{2_j}, \lambda_{2_j})$ for a given z_{eq} meets the condition

$$\lim_{t\to\infty}(y\wp_j(z_{eq}) - y\wp_k(z_{eq})) = 0 \quad j \neq k \quad (10)$$

with $y\wp_{\tilde{n}}(z_{eq})$ the output vector for the set $\wp_{\tilde{n}}(z_{eq})$.

In the case that (9) satisfies with $z_1 = z_{eq}$, only one leak exists and diverse methods can be used to identify it. On the contrary, the transient response of the fluid in leak conditions must be used to identify the parameters associated to the leaks. Therefore, z_{eq} characterizes the subset of unidentificable pairs of two leaks in term of the measurable variables in steady state (inlet-flow, outlet-flow and total pressure drop).

As example, Fig. 2 shows the flows responses for a leak located at $50m$ and five cases of two leaks with $z_{eq} = 50m$.

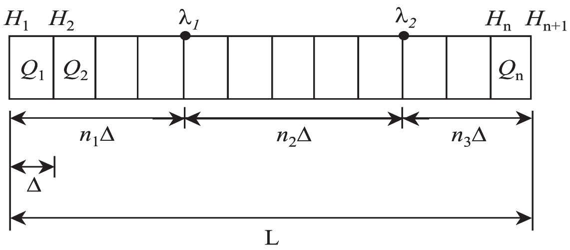

Fig. 3. Variables with uniform sections in the pipeline

III. DYNAMIC MODEL WITH LEAKS IN THE PIPE

Assuming that the leaks positions coincide with the space discretization and choosing the output and input vectors by

$$y = \begin{bmatrix} Q(t,0) & Q(t,L) \end{bmatrix}^T \quad u = \begin{bmatrix} H(t,0) & H(t,L) \end{bmatrix}^T,$$

one proposes to discretize the differentials (1) and (2) with respect to z by a first order approximation to get a lumped parameters model, in which the dimension is not a priori fixed, obtaining

$$\begin{bmatrix} \dot{Q}_1 \\ \dot{H}_2 \\ \dot{Q}_2 \\ \vdots \\ \dot{H}_{n_1} \\ \vdots \\ \dot{H}_{n_{1,2}} \\ \vdots \\ \dot{Q}_{n-1} \\ \dot{H}_n \\ \dot{Q}_n \end{bmatrix} = \begin{bmatrix} -\mu Q_1|Q_1| - \frac{a_1}{\Delta}(H_2 - u_1) \\ \frac{a_2}{\Delta}(Q_1 - Q_2) \\ -\mu Q_2|Q_2| + \frac{a_1}{\Delta}(H_2 - H_3) \\ \vdots \\ \frac{a_2}{\Delta}(Q_{n_1-1} - Q_{n_1} - \sqrt{H_{n_1}}\lambda_1) \\ \vdots \\ \frac{a_2}{\Delta}(Q_{n_{1,2}-1} - Q_{n_{1,2}} - \sqrt{H_{n_{1,2}}}\lambda_2) \\ \vdots \\ -\mu Q_{n-1}|Q_{n-1}| + \frac{a1}{\Delta}(H_{n-1} - H_n) \\ \frac{a_2}{\Delta}(Q_{n-1} - Q_n) \\ -\mu Q_n|Q_n| + \frac{a1}{\Delta}(H_n - u_2) \end{bmatrix} \tag{11}$$

with $n_{1,2} = n_1 + n_2$, $a_2 = \frac{b^2}{gA}$, the flow Q_i and pressure H_i are the discretize variables given in Fig. 3. Note that the section size Δ of the space discretization must satisfy

$$(n_1 + n_2 + n_3)\Delta = n\Delta = L \tag{12}$$

with n the number of the sections. One can see from Fig. 3, that the leaks separation and Δ determines the accuracy of the approximated dynamic model (11) and its order.

A. Constraints for the Space Discretization

It is known that a discretization of a PDE means a compromise between transient accuracy and computational issues [10]. For the leaks isolation, one suggests to make a sensitivity analysis of the dynamic response of (11) with respect to the discrete space Δ, to determine the minimal order n of the model which follows the transient response of the pipe and satisfies at the same time the steady state constraint (7). This idea to establish a compromise between an accuracy dynamic model and a low number of unknown parameters using the known parameter z_{eq} has not been used before to solve the leak isolation task.

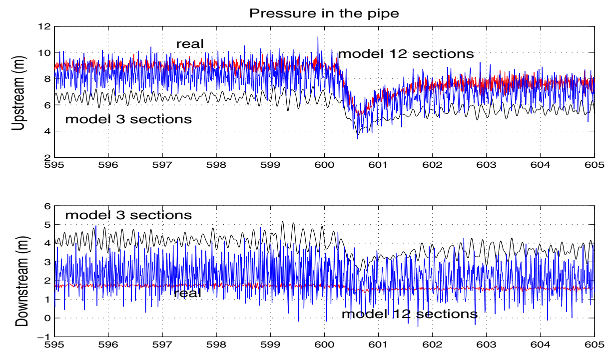

Fig. 4. Transients of the pressure for three values of Δ

For the considered water pipeline of long $L = 132.5m$, Fig. 4 shows the responses of the pressure transient with respect to real and simulated data with sections $\Delta_1 = 44m$ and $\Delta_2 = 11.04m$. Since, a section smaller than Δ_2 in eq. (11) do not improved strongly the transient pressure, an upper boundary for the discretized space $\Delta_\top$ given by

$$\Delta_\top = \Delta_2 \approx 0.08L \tag{13}$$

is suggested.

On the other hand, because z_{eq} has to satisfy (8) and n_1 and n_2 have to satisfy

$$z_1 = n_1\Delta, \qquad z_2 = n_2\Delta; \tag{14}$$

if z_{eq} is located near the extremes of the pipe, the constraints (8) and (14) could generate a limit for the section Δ smaller than $\Delta_\top$. To be sure that at least two sections exist from the extremes of the line to z_{eq}, a new upper limit for Δ must be selected, such that, both (8) and (14) with $n_1 = 1$ and $n_2 = 1$, satisfy.

Then, if $\Delta_\top < z_{eq} < L - \Delta_\top$, one suggests to use $\Delta = \Delta_\top$ in (11), since this value yields a good transient response and the static constraints satisfy. On the contrary, if $L - \Delta_\top < z_{eq}$ or $z_{eq} < \Delta_\top$ a reduction in the section size is required. These rules are summarized as follows:

- R1: If $\Delta_\top < z_{eq} < L - \Delta_\top$, then $\Delta = \Delta_\top$.
- R2: If $z_{eq} < \Delta_\top$, then $\Delta = \frac{z_{eq}}{2}$.
- R3: If $L - z_{eq} < \Delta_\top$, then $\Delta = \frac{L - z_{eq}}{2}$.

B. Search Intervals Reduction

Taking into account that the unknown integers n_1, n_2 and n_3 are related, one can determine their limits, such that (8) and (14) are satisfied. Mixing both constraints, the interval is reduced to

$$n_1\Delta < z_{eq} < (n_1+n_2)\Delta \tag{15}$$

and the extremes of n_2 are

$$n_{2min} > \frac{z_{eq}}{\Delta} - n_1 \qquad n_{2max} = n - n_1 - 1 \tag{16}$$

On the other hand, since the integers $n_1\Delta$ and $n_2\Delta$ must as well satisfy equation (9), a less conservative reduction is achieved if one considers only as candidate pairs the set $\mathcal{S}_{n1,n2} = (n_1, n_2)$ that satisfies conditions (15) and (16) and generates simultaneously an error

$$e_{n1,n2} := (y_{1s}^2 - y_{2s}^2)(z_{eq} - n_1\Delta) - n_2\Delta(Q_{eq}^2 - y_{2s}^2) \tag{17}$$

less than a threshold T_e.

To enclose the search interval for the parameter λ_2, using (6) after the evaluation of z_{eq} and the selection of Δ, one gets

$$\lambda_2 \in I_{\lambda_2} = [\lambda_{2_{min}}, \lambda_{2_{max}}] \tag{18}$$

with

$$\lambda_{2min} = \frac{(Q_{eqs} - y_{2s})}{\sqrt{u_{2s} + \frac{\mu(L-z_{eq})}{a_1} y_{2s}^2}} \tag{19}$$

$$\lambda_{2max} = \frac{(Q_{eqs} - y_{2s})}{\sqrt{u_{2s} + \frac{\mu\Delta}{a_1} y_{2s}^2}} \tag{20}$$

C. Generation of the Family of Models $\mathcal{F}$

The selection of the value Δ according to rules (R1, R2, R3) allows to get from (6) and (9) the relations

$$n_1\Delta = z_{eq} - k_1 n_2 \Delta \tag{21}$$

$$\lambda_1 = \frac{k_2}{\sqrt{p(n_1)}} \tag{22}$$

with known constants

$$k_1 = \frac{Q_{eqs}^2 - y_{2s}^2}{y_{1s}^2 - y_{2s}^2}, \qquad k_2 = y_{1s} - Q_{eqs}$$

and function

$$p(n_1) = u_{1e} - \frac{\mu n_1 \Delta}{a_1} y_{1s}^2.$$

Then, assuming that the pipe is in steady state when leaks appear, equations (21) and (22) can be substituted in the dynamic model (11), to get the family of models $\mathcal{F}$

$$\begin{aligned}
\dot{\tilde{Q}}_1 &= -\mu\tilde{Q}_1|\tilde{Q}_1| + \frac{a_1}{\Delta}(u_1 - \tilde{H}_2)\\
\dot{\tilde{H}}_2 &= \frac{a_2}{\Delta}\left(\tilde{Q}_1 - \tilde{Q}_2\right)\\
\dot{\tilde{Q}}_2 &= -\mu\tilde{Q}_2|\tilde{Q}_2| + \frac{a_1}{\Delta}(\tilde{H}_2 - \tilde{H}_3)\\
&\vdots\\
\dot{\tilde{H}}_{n_1} &= \frac{a_2}{\Delta}\left(\tilde{Q}_{n_1-1} - \tilde{Q}_{n_1} - \overbrace{\frac{k_2}{\sqrt{p(n_1)}}\sqrt{\tilde{H}_{n_1}}}^{t1}\right)\\
&\vdots\\
\dot{\tilde{H}}_{n_{1,2}} &= \frac{a_2}{\Delta}\left(\tilde{Q}_{n_{1,2}-1} - \tilde{Q}_{n_{1,2}} - \overbrace{\lambda_2\sqrt{\tilde{H}_{n_{1,2}}}}^{t2}\right)\\
&\vdots\\
\dot{\tilde{Q}}_{n-1} &= -\mu\tilde{Q}_{n-1}|\tilde{Q}_{n-1}| + \frac{a_1}{\Delta}(\tilde{H}_{n-1} - \tilde{H}_n)\\
\dot{\tilde{H}}_n &= \frac{a_2}{\Delta}\left(\tilde{Q}_{n-1} - \tilde{Q}_n\right)\\
\dot{\tilde{Q}}_n &= -\mu\tilde{Q}_n|\tilde{Q}_n| + \frac{a_1}{\Delta}(\tilde{H}_n - u_2)
\end{aligned} \tag{23}$$

This family is general, has a satisfactory transient behavior, and depends only on three unknown parameters (n_1, n_2, λ_2). Moreover, the pair (n_1, n_2) must be a member of $\mathcal{S}_{n1,n2}$ and the parameter λ_2 is inside the interval I_{λ_2} for a given z_{eq}. Note that, for every candidate pair (n_1, n_2) the terms associated to the leaks (t_1, t_2) must be introduced in the specific pair of state variables $(\tilde{H}_{n_1}, \tilde{H}_{n_{1,2}})$. Moreover, if $k_2 = 0$ from (22) one concludes that $\lambda_1 = 0$ and (23) is still valid for the one leak case.

IV. ISOLATION ALGORITHM

To estimate the parameters (n_1, n_2, λ_2), for the family (23) with their constraints diverse pattern recognition processes in a recursive framework can be applied [11]. Since the parameters can be identified only using data during the transient response of the output, one could re-use the data set if the settling time of the system response is shorter than the converge time of the estimator.

Taking into account the constraints for the pairs (n_1, n_2) and the search interval for λ_2, the two leaks location problem in a pipeline can be formulated as the minimization of

$$E(n_1^*, n_2^*, \lambda_2^*) = \min_{n_1,n_2,\lambda_2} \int_{t=0}^{T} \left(Q_1(t) - \tilde{Q}_1(n_1, n_2, \lambda_2)\right)^2 dt \tag{24}$$

subject to $(n_1, n_2) \in \mathcal{S}_{n1,n2}$ and $\lambda_2 \in I_{\lambda_2}$

where Q_1 is the upstream flow data, $\tilde{Q}_1(n_1, n_2, \lambda_2)$ is the first component of the output of family $\mathcal{F}$ and T is the settling time of the leaks response of the data. Since the cardinality of the set $\mathcal{S}_{n1,n2}$ is finite, the functional E is practically optimized with respect to the parameter λ_2 for a finite set of members of $\mathcal{F}$. Several optimization tools can be used straightforward for this purpose. Here, the optimization and simulink toolbooks of MATLAB software has been used [8].

A. Algorithm

The set of above introduced equations can be summarized as a leaks isolation algorithm in which the first steps allow to reduce the possible members of the family $\mathcal{F}$ where the leaks can be located. The second part of the algorithm strictly speaking consists to determine the parameter λ_2 such that the $\mathcal{L}_2$ norm of the error between data and output model is minimizing considering the feasible members of $\mathcal{F}$.

- Step 1. If a leak condition is detected, estimate the parameter z_{eq} by (7).
- Step 2. If flow Q_{eqs} at position z_{eq} is equal to y_{2s}, $\exists$ only one leak at position z_{eq}. On the contrary, define a family of dynamic model $\mathcal{F}$ to locate the two leaks according the following steps.
- Step 3. Select the parameter Δ following rules (R1,R2,R3).
- Step 4. Evaluate the error (17) considering the pairs (n_1, n_2) which satisfy (15) and (16).
- Step 5. Determine the search interval I_{λ_2} using (19) and (20).
- Step 6. Generate the set $\mathcal{S}_{n1,n2}$, get in order the members according to the values of (17) and set up $i = 1$.
- Step 7. Minimize (24) with respect to λ_2 for the pair i of $\mathcal{S}$.
- Step 8. If the minimization with all members of $\mathcal{S}$ is achieved, go to the next step. On the contrary, increase the index i and go to step 7.
- Step 9. The parameter λ_2^* and the pair i which yield the smallest value of (24) are selected as the second leak outflow and the leaks positions respectively and λ_1 is obtained from (22).

V. SIMULATION RESULTS

Consider the parameters of the water pilot pipeline of length $L = 132.56(m)$, given in [7]. Assume that two simultaneous leaks with $\lambda_1 = 6.2283 \times 10^{-5}$ and $\lambda_2 = 1.0534 \times 10^{-4}$ are located at 44.18 and $88.37m$ of the line. Considering the steady state value of the measurable variables, one can evaluate (7) obtaining the key value $z_{eq} = 71.65m$.

Taking into account the long of the pipeline and the value of z_{eq}, one can select $\Delta = 11.04m$ by rule R1. This value divides the line in 12 uniform sections.

On the other hand, considering that

$$n_1 \leq \frac{z_{eq}}{\Delta} = 6.43$$

and (16), the subset of family $\mathcal{F}$ or candidate members is characterized by the pairs (n_1, n_2) given in Table I. It means, the family of models is reduced to 30 possible combinations of pairs (n_1, n_2) where the leaks parameters must be searched.

TABLE I
Set of pairs (n_1, n_2) of the subset of $\mathcal{F}$

n_1	n_2
6	1, 2, 3, 4, 5
5	2, 3, 4, 5, 6
4	3, 4, 5, 6, 7
3	4, 5, 6, 7, 8
2	5, 6, 7, 8, 9
1	6, 7, 8, 9, 10

Moreover, since n_1 and n_2 must satisfy (17), another reduction in the cardinality of the subset of $\mathcal{F}$ can be done. Table II shown the value of the error (17) for the 30 members. One can see that only the last two cases yielded an error of order 10^{-6} and can be considered as candidate models to locate the leaks. The cases correspond to $n_1 = 4$ and $n_2 = 4$ and $n_1 = 1$ and $n_2 = 9$. So, only two models will be tested in (24) during the minimization with respect to the parameter λ_2.

TABLE II
Classification of the subset of $\mathcal{F}$

n_1	n_2	$e_{n1,n2}$	n_1	n_2	$e_{n1,n2}$
6	5	-3.4031×10^{-4}	3	7	-1.0005×10^{-4}
5	6	-2.8726×10^{-4}	6	2	-9.7004×10^{-5}
6	4	-2.5921×10^{-4}	4	3	9.0203×10^{-5}
1	6	2.4936×10^{-4}	1	8	8.7154×10^{-5}
4	7	-2.3421×10^{-4}	1	10	-7.5052×10^{-5}
5	5	-2.0616×10^{-4}	4	5	-7.2003×10^{-5}
2	5	1.9631×10^{-4}	3	5	6.2152×10^{-5}
3	8	-1.8116×10^{-4}	2	8	-4.7001×10^{-5}
6	3	-1.7811×10^{-4}	5	3	-4.3952×10^{-5}
1	7	1.6826×10^{-4}	5	2	3.7151×10^{-5}
4	6	-1.5311×10^{-4}	2	7	3.4102×10^{-5}
3	4	1.4325×10^{-4}	3	6	-1.8950×10^{-5}
2	9	-1.2810×10^{-4}	6	1	-1.5902×10^{-5}
5	4	-1.2505×10^{-4}	4	4	9.1001×10^{-6}
2	6	1.1520×10^{-4}	1	9	6.0511×10^{-6}

Finally, evaluating (19) and (20) the following search interval for λ_2 is obtained

$$\lambda_2 \in \left[1.2512 \times 10^{-4}, 1.0002 \times 10^{-4}\right]$$

Minimizing the criterion (24) for the two models, the values of Table III are obtained. In this table the parameter λ_1 is estimated by (22). The third column λ_{2i} corresponds to the initial value of the parameter λ_2 used in the optimization. One can be seen that for the pair (4,4) the estimation converges to the same value of $\hat{\lambda}_2$ for different initial conditions in the optimization procedure. On the contrary, for the model with $n_1 = 1$ and $n_2 = 9$ the optimization approach generates different values of $\hat{\lambda}_2$ depending on the initial condition λ_{2i}.

TABLE III

Parameters Estimation for the two models considered in the optimization with different initial conditions

n_1	n_2	λ_{2i}	$\hat{\lambda}_2$	$\hat{\lambda}_1$
4	4	1.2512×10^{-4}	1.0983×10^{-4}	6.0214×10^{-5}
4	4	1.0002×10^{-4}	1.0983×10^{-4}	6.0214×10^{-5}
1	9	1.2512×10^{-4}	1.2512×10^{-4}	5.5592×10^{-5}
1	9	1.0002×10^{-4}	1.2219×10^{-4}	5.5592×10^{-5}

In Fig. 5 the upstream flow errors between the plant and both members of the family $\mathcal{F}$ are shown. e_1 corresponds to the error between the plant and the model with the pair $(1, 9)$ and e_2 corresponds to the pair$(4, 4)$. One can say that the error e_2 associated with the true leaks positions is closer to zero than e_1. This result validate the substitution of (21) and (22) in the model (23).

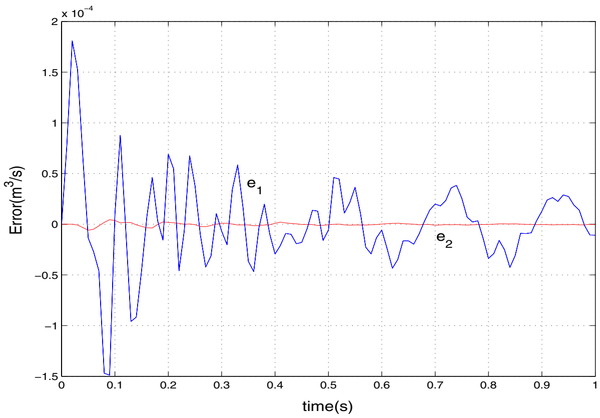

Fig. 5. Evolution of the error between the pipeline and the candidate models

The estimation errors for the leaks outflows with the pair $(4, 4)$ is given by

$$\lambda_1 - \hat{\lambda}_1 = 0.2669 \times 10^{-5}, \quad \lambda_2 - \hat{\lambda}_2 = 0.449 \times 10^{-5}$$

and they are tolerated, since the acuracy in the leak position is more important than the outflow. Note that, the procedure given here cannot be implemented on-line, since the parameter z_{eq} can be only obtained after the transient effects of the leaks in the response are neglected.

VI. CONCLUSIONS

This paper presented an off-line minimization procedure based on a combination of unsteady and steady state constraints to identify the four parameters associated to the existence of two leaks in a pipeline. To describe the fluid, a finite dimension nonlinear family of models $\mathcal{F}$ assuming flow and pressure measurements at the extremes of the pipeline is obtained in term of a key parameter z_{eq}. It is shown that steady state conditions of the fluid with leaks, can be complemented with a finite dimension family $\mathcal{F}$ of models, to reduce the search intervals of the two leaks isolation problem. The search intervals reduction for the location task has not been presented before and is the main contribution of this paper. For the optimization task diverse software toolbook can be used. Simulation results are discussed and show that parameters are satisfactory estimated. Finally, it is to remark that the generalization of the multi-leak task is an open problem and for each new leak two parameters must be identified in the dynamic model.

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References

[1] D. Shields, S. Ashton, and S. Daley, "Design of nonlinear observers for detecting faults in hydraulic sub-sea pipelines," Control Engineering Practice, vol. 9, pp. 297–311, 2001.

[2] J. Korbicz, J. M. Koscielny, Z. Kowalczuk, and W. Cholewa, Fault Diagnosis. Germany: Springer, 2004.

[3] C. Verde, "Minimal order nonlinear observer for leak diagnosis in pipelines," Journal od Dynamics Systems Mesurements and Control, ASME, vol. 126, pp. 467–472, September 2004.

[4] C. Verde, "Accomodation of multi-leaks positions in a pipeline," Control Engineering Practice, vol. 13, pp. 1071–1078, 2005.

[5] C. Verde, G. Bornard, and S. Gentil, "Isolability of multi-leaks in a pipeline," in Proceedings 4th MATHMOD, Vienna, ARGESIM Verlag, ISBN-3-901608-24-9, February 2003.

[6] M. Ferrante and B. Brunone, "Pipe system diagnosis and leak detection by unsteady-stae tests: 1. harmonic analysis," Advances in Water resources, vol. 26, pp. 95–105, 2003.

[7] N. Visairo, "Detección y localización de fugas en un ducto," Ph Thesis CENIDET, Mexico, 2004.

[8] MATLAB, Optimization. Natick, Massachuesetts: MathWorks, Inc., 2005.

[9] M. C. Chaudry, Applied hydraulic transients. New York, USA: Von Nostrand Reinhold Co, 1979.

[10] A. B. de Almeida and E. Koelle, Fluid transients in pipe networks. U.S.A.: Computational Mechanics Publications Co-publisher Applied Science, 1992.

[11] B. Boashash, Time Frequency Signal Analysis and Processing. Elsevier, 2003.

[12] D. Luenberger, Linear and Nonlinear Programming. USA: Addison Wesley, 1984.