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Tracking Control for Robot Manipulators with Kinematic and Dynamic Uncertainty

D. Braganza, W. E. Dixon, D. M. Dawson, B. Xian

Abstract— **The control objective in many robot manipulator applications is to command the end-effector motion to achieve a desired response. To achieve this objective a mapping is required to relate the joint/link control inputs to the desired Cartesian position and orientation. If there are uncertainties or singularities in the mapping, then degraded performance or unpredictable responses by the manipulator are possible. To address these issues, an adaptive tracking controller is developed in this paper for robot manipulators with uncertainty in the kinematic and dynamic models. The controller is developed based on the unit quaternion representation so that singularities associated with three parameter representations are avoided.**

I. INTRODUCTION

The control objective in many robot manipulator applications is to command the end-effector motion to achieve a desired response. The control inputs are applied to the manipulator joints, and the desired position and orientation is typically encoded in terms of a Cartesian coordinate frame attached to the robot end-effector with respect to the base frame (i.e., the so-called task-space variables). Hence, a mapping (i.e., the solution of the inverse kinematics) is required to convert the desired task-space trajectory into a form that can be utilized by the joint space controller. If there are uncertainties or singularities in the mapping, then this can result in degraded performance or unpredictable responses by the manipulator. Several parameterizations exist to describe orientation angles in the task-space to joint-space mapping, including three-parameter representations (e.g., Euler angles, Rodrigues parameters) and the four-parameter representation given by the unit quaternion. Three-parameter representations always exhibit singular orientations (i.e., the orientation Jacobian matrix in the kinematic equation is singular for some orientations), while the unit quaternion represents the end-effector orientation without singularities. By utilizing the singularity free unit quaternion, the emphasis in this paper is to develop a tracking controller that compensates for uncertainty throughout the kinematic and dynamic models. Some previous task-space control formulations based on the unit quaternion can be found in [1], [2], [15], [19] and [21]. A quaternion-based resolved acceleration controller was presented in [2], and quaternion-based resolved rate and resolved acceleration task-space controllers were proposed in [21]. Output feedback task-space controllers using quaternion feedback were presented in [15] for the regulation problem and in [1] for the tracking problem. Model-based and adaptive asymptotic full-state feedback controllers and an output feedback controller based on a model-based observer were developed in [19] based on the quaternion parameterization.

A common assumption in most of the previous robot controllers (including all of the aforementioned quaternion-based task-space control formulations) is that the robot kinematics and manipulator Jacobian are assumed to be perfectly known. From a review of literature, few controllers have been developed that target uncertainty in the manipulator forward kinematics and Jacobian. For example, Cheah et al. [3]-[8] developed several approximate Jacobian feedback controllers that exploit a static, best-guess estimate of the manipulator Jacobian to achieve task-space regulation objectives despite parametric uncertainty in the manipulator Jacobian. In [20], Yazarel and Cheah develop a task-space adaptive controller for set point control of robots with uncertainties in the gravity regressor matrix and kinematics. In [10], Dixon developed an adaptive regulation controller for robot manipulators with uncertainty in the kinematic and dynamic models. The result in [10] also accounted for actuator saturation since the maximum commanded torque could be a priori determined due to the use of saturated feedback terms in the controller.

All of the aforementioned controllers that account for kinematic uncertainty are based on the three-parameter Euler angle representation. Moreover, all of the previous results only target the set point regulation problem. The only result that targets the more general tracking control problem for manipulators with uncertain kinematics is given in [9]. The result in [9] is also based on the Euler angle representation and the controller requires the measurement of the task space velocity. Hence motivated by previous work, an adaptive tracking controller is developed in this paper for robot manipulators with uncertainty in the kinematic and dynamic models. The controller is developed based on the unit quaternion representation so that singularities associated with three parameter representations are avoided. In addition, the developed controller does not require the measurement of the task space velocity. The stability of the controller is proven through a Lyapunov-based stability analysis.

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D. Braganza and D. M. Dawson are with the Electrical and Computer Engineering Dept., Clemson University, Clemson, SC 29634, USA.

W. E. Dixon is with the Mechanical and Aerospace Engineering Dept., University of Florida, Gainesville, FL 32611, USA. wdixon@ufl.edu

B. Xian is with the Dept. of Mechanical Engineering and Material Sciences, Duke University, Durham, NC 27705, USA.

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II. ROBOT DYNAMIC AND KINEMATIC MODELS

A six-link, rigid, revolute robot manipulator can be described by the following dynamic model [14]

$$M(\theta)\ddot{\theta} + V_m(\theta,\dot{\theta})\dot{\theta} + G(\theta) + F_d\,\dot{\theta} = \tau. \tag{1}$$

In (1), $\theta(t) \in \mathbb{R}^6$ is the joint position[1], $M(\theta) \in \mathbb{R}^{6\times 6}$ represents the inertia matrix, $V_m(\theta,\dot{\theta}) \in \mathbb{R}^{6\times 6}$ is the centripetal-Coriolis matrix, $G(\theta) \in \mathbb{R}^6$ is the gravity vector, $F_d \in \mathbb{R}^{6\times 6}$ is a constant diagonal matrix which represents the viscous friction coefficients, and $\tau(t) \in \mathbb{R}^6$ represents the input torque vector. The dynamic model given in (1) has the following properties [14], which are utilized in the subsequent control design and analysis:

Property 1: The inertia matrix is symmetric and positive-definite, and satisfies the following inequalities

$$m_1 \|x\|^2 \le x^T M(\theta)x \le m_2 \|x\|^2 \qquad \forall\, x \in \mathbb{R}^6 \tag{2}$$

where $m_1, m_2 \in \mathbb{R}$ are positive constants and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: The inertia and centripetal-Coriolis matrices satisfy the following skew-symmetric relationship

$$x^T\left(\frac{1}{2}\dot{M}(\theta) - V_m(\theta,\dot{\theta})\right)x = 0 \qquad \forall\, x \in \mathbb{R}^6. \tag{3}$$

Property 3: The centripetal-Coriolis matrix satisfies the following skew-symmetric relationship

$$V_m(\theta,x)y = V_m(\theta,y)x \qquad \forall\, x, y \in \mathbb{R}^6. \tag{4}$$

Property 4: The norm of the centripetal-Coriolis matrix and the norm of the friction matrix, can be upper bounded as follows:

$$\|V_m(\theta,x)\|_{i\infty} \le \zeta_c \|x\| \qquad \forall\, x \in \mathbb{R}^6, \quad \|F_d\| \le \zeta_f \tag{5}$$

where $\zeta_c, \zeta_f \in \mathbb{R}$ are positive constants, and $\|\cdot\|_{i\infty}$ denotes the induced-infinity norm of a matrix.

Property 5: Parametric uncertainty in $M(\theta)$, $V_m(\theta,\dot{\theta})$, $G(\theta)$ and F_d, is linearly parameterizable.

Let $\mathcal{E}$ and $\mathcal{B}$ be orthogonal coordinate frames attached to the manipulator's end-effector and fixed base, respectively. The position and orientation of $\mathcal{E}$ relative to $\mathcal{B}$ can be represented through the following forward kinematic model [15]

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} h_p(\theta) \\ h_q(\theta) \end{bmatrix}. \tag{6}$$

In (6), $h_p(\cdot) : \mathbb{R}^6 \to \mathbb{R}^3$ denotes a function that maps $\theta(t)$ to the measurable[2] task-space position coordinates of the end-effector, denoted by $p(\cdot) \in \mathbb{R}^3$, and $h_q(\cdot) : \mathbb{R}^6 \to \mathbb{R}^4$ denotes a function that maps $\theta(t)$ to the measurable[3] unit quaternion and is denoted by $q(t) \in \mathbb{R}^4$. The unit quaternion vector, denoted by $q(t) = \left[q_o(t),\, q_v^T(t)\right]^T$ with $q_o(t) \in \mathbb{R}$ and $q_v(t) \in \mathbb{R}^3$ [12], [14], provides a global nonsingular parameterization of the end-effector orientation, and is subject to the constraint, $q^T q = 1$. Several algorithms exist to determine the orientation of $\mathcal{E}$ relative to $\mathcal{B}$ from a rotation matrix that is a function of $\theta(t)$. Conversely, a rotation matrix, denoted by $R(q) \in SO(3)$, can be determined from a given $q(t)$ by the formula [15]

$$R(q) = \left(q_o^2 - q_v^T q_v\right) I_3 + 2q_v q_v^T + 2q_o q_v^{\times} \tag{7}$$

where I_3 is the 3×3 identity matrix, and the notation $a^{\times}$, $\forall\, a = [a_1,\, a_2,\, a_3]^T$, denotes the following skew-symmetric matrix

$$a^{\times} \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \tag{8}$$

The time derivative of (6) is given by the following expression

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} J_p \\ J_q \end{bmatrix}\dot{\theta} \tag{9}$$

where $J_p(\theta) : \mathbb{R}^6 \to \mathbb{R}^{3\times 6}$ and $J_q(\theta) : \mathbb{R}^6 \to \mathbb{R}^{4\times 6}$ denote the unknown position and orientation Jacobian matrices, respectively, defined as $J_p(\theta) = \partial h_p/\partial\theta$ and $J_q(\theta) = \partial h_q/\partial\theta$. To facilitate the subsequent development, (9) is expressed as follows:

$$\begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(\theta)\dot{\theta} \text{ where } J(\theta) = \begin{bmatrix} J_p \\ 2B^T J_q \end{bmatrix} \in \mathbb{R}^{6\times 6}. \tag{10}$$

Remark 1: *The dynamic and kinematic terms for a general revolute robot manipulator, denoted by $M(\theta)$, $V_m(\theta,\dot{\theta})$, $G(\theta)$ and $J(\theta)$, are assumed to depend on $\theta(t)$ only as arguments of trigonometric functions and hence, remain bounded for all possible $\theta(t)$. During the control development, the assumption will be made that if $p(t) \in \mathcal{L}_\infty$, then $\theta(t) \in \mathcal{L}_\infty$ (Note that $q(t)$ is always bounded, since $q^T q = 1$).*

The expression in (10) is obtained by exploiting the fact that $q(t)$ is related to the angular velocity of the end-effector, denoted by $\omega(t) \in \mathbb{R}^3$, via the following differential equation

$$\omega = B^T \dot{q} \tag{11}$$

where the known Jacobian-like matrix $B(q) : \mathbb{R}^4 \to \mathbb{R}^{4\times 3}$ is defined as follows:

$$B = \frac{1}{2}\begin{bmatrix} -q_v^T \\ q_o I_3 - q_v^{\times} \end{bmatrix}. \tag{12}$$

Property 6: The kinematic system in (10) can be linearly parameterized as follows:

$$J\dot{\theta} = W_j \phi_j \tag{13}$$

where $W_j(\theta,\dot{\theta}) \in \mathbb{R}^{6\times n_1}$ denotes a regression matrix of measurable signals, and $\phi_j \in \mathbb{R}^{n_1}$ denotes a vector of n_1 unknown constants.

[1] It is assumed that the actuated manipulator joint is rigidly connected to the links, so that the link-space and joint-space are equivalent. Hence, the words joint and link can be used interchangeably.

[2] The task-space position of $\mathcal{E}$ relative to $\mathcal{B}$ is assumed to be measurable, as in [3]-[8], [9], [10], and [20]. For example, a camera system could be utilized.

[3] The task-space orientation of $\mathcal{E}$ relative to $\mathcal{B}$ is also assumed to be measurable through the use of a camera system.

Property 7: There exists upper and lower bounds for the parameter ϕ_j such that $J(\theta, \phi_j)$ is always invertible. We will assume that the bounds for each parameter can be calculated as follows

$$\underline{\phi}_{ji} \leq \phi_{ji} \leq \overline{\phi}_{ji} \tag{14}$$

where, $\phi_{ji} \in \mathbb{R}$ denotes the i^{th} component of $\phi_j \in \mathbb{R}^{n_1}$ and $\underline{\phi}_{ji}, \overline{\phi}_{ji} \in \mathbb{R}$ denote the i^{th} components of $\underline{\phi}_j, \overline{\phi}_j \in \mathbb{R}^{n_1}$, which are defined as follows

$$\begin{aligned} \underline{\phi}_j &= \left[\underline{\phi}_{j1}, \underline{\phi}_{j2}, \cdots, \underline{\phi}_{jn_1}\right]^T \\ \overline{\phi}_j &= \left[\overline{\phi}_{j1}, \overline{\phi}_{j2}, \cdots, \overline{\phi}_{jn_1}\right]^T. \end{aligned} \tag{15}$$

III. Problem Statement

The objective is to the design the control input $\tau(t)$ to ensure end-effector position and orientation tracking for the robot model given by (1) and (10) despite parametric uncertainty in the kinematic and dynamic models. We will assume that the only measurable signals are the joint position, joint velocity, and end-effector position. To mathematically quantify this objective, a desired position and orientation of the robot end-effector is defined by a desired orthogonal coordinate frame $\mathcal{E}_d$. The vector $p_d(t) \in \mathbb{R}^3$ denotes the position of the origin of $\mathcal{E}_d$ relative to the origin of $\mathcal{B}$, while the rotation matrix from $\mathcal{E}_d$ to $\mathcal{B}$ is denoted by $R_d(t) \in SO(3)$.

The end-effector position tracking error $e_p(t) \in \mathbb{R}^3$ is defined as

$$e_p = p_d - p \tag{16}$$

where $p_d(t)$, $\dot{p}_d(t)$, and $\ddot{p}_d(t)$ are assumed to be known bounded functions of time. If the orientation of $\mathcal{E}_d$ relative to $\mathcal{B}$ is specified in terms of a desired unit quaternion $q_d(t) = \left[q_{od}(t),\, q_{vd}^T(t)\right]^T \in \mathbb{R}^4$, with $q_{od}(t) \in \mathbb{R}$ and $q_{vd}(t) \in \mathbb{R}^3$. Then similarly to (7), $R_d(q_d)$ can be calculated from $q_d(t)$ as follows

$$R_d(q_d) = \left(q_{od}^2 - q_{vd}^T q_{vd}\right) I_3 + 2q_{vd}q_{vd}^T + 2q_{od}q_{vd}^{\times} \tag{17}$$

where it is assumed that $R_d, \dot{R}_d, \ddot{R}_d \in \mathcal{L}_\infty$. As in (11), the time derivative of $q_d(t)$ is related to the desired angular velocity of the end-effector (i.e., the angular velocity of $\mathcal{E}_d$ relative to $\mathcal{B}$), denoted by $\omega_d(t) \in \mathbb{R}^3$, through the known kinematic equation

$$\dot{q}_d = B(q_d)\omega_d. \tag{18}$$

To quantify the difference between the actual and desired end-effector orientations, we define the rotation matrix $\tilde{R} \in SO(3)$ from $\mathcal{E}$ to $\mathcal{E}_d$ as follows

$$\tilde{R} \triangleq R_d^T R = \left(e_o^2 - e_v^T e_v\right) I_3 + 2e_v e_v^T + 2e_o e_v^{\times} \tag{19}$$

where $e_q(t) \triangleq \left[e_o(t),\, e_v^T(t)\right]^T \in \mathbb{R}^4$ represents unit quaternion tracking error that satisfies the constraint

$$e_q^T e_q = e_o^2 + e_v^T e_v = 1. \tag{20}$$

The quaternion tracking error $e_q(t)$ can be explicitly calculated from $q(t)$ and $q_d(t)$ via quaternion algebra by noticing that the quaternion equivalent of $\tilde{R} = R_d^T R$ is the following quaternion product [13], [21]

$$e_q = q q_d^* \tag{21}$$

where $q_d^*(t) \triangleq \left[q_{od}(t),\, -q_{vd}^T(t)\right]^T \in \mathbb{R}^4$ is the unit quaternion representing the rotation matrix $R_d^T(q_d)$. After using quaternion algebra, the quaternion tracking error can be derived as follows (see [21] and Theorem 5.3 of [13])

$$\begin{bmatrix} e_o \\ e_v \end{bmatrix} = \begin{bmatrix} q_o q_{od} + q_v^T q_{vd} \\ q_{od} q_v - q_o q_{vd} + q_v^{\times} q_{vd} \end{bmatrix}. \tag{22}$$

Based on (11), (18), and (22), the unit quaternion error system can be formulated as follows [11]

$$\begin{bmatrix} \dot{e}_o \\ \dot{e}_v \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} e_v^T \tilde{\omega} \\ \frac{1}{2}\left(e_o I_3 - e_v^{\times}\right) \tilde{\omega} \end{bmatrix}. \tag{23}$$

The angular velocity of $\mathcal{E}$ with respect to $\mathcal{E}_d$ with coordinates in $\mathcal{E}_d$, denoted by $\tilde{\omega}(t) \in \mathbb{R}^3$, can be calculated from (19) as follows [18]

$$\tilde{\omega} = R_d^T\left(\omega - \omega_d\right). \tag{24}$$

The end effector tracking errors are then written using (10), (16), and (24) as

$$\begin{bmatrix} \dot{e}_p \\ \tilde{\omega} \end{bmatrix} = \Lambda\left(\begin{bmatrix} -\dot{p}_d \\ -\omega_d \end{bmatrix} + J\dot{\theta}\right) \tag{25}$$

where $\Lambda \in \mathbb{R}^{6\times 6}$ is defined as

$$\Lambda = \begin{bmatrix} -I_3 & 0_{3\times 3} \\ 0_{3\times 3} & R_d^T \end{bmatrix}, \tag{26}$$

where $0_{3\times 3}$ represents a 3×3 matrix of zeros. Based on the above definitions, the tracking objective defined in terms of the end-effector position and unit quaternion error is to design the control input $\tau(t)$ such that

$$\|e_p(t)\| \to 0 \quad \text{and} \quad \|e_v(t)\| \to 0 \quad \text{as} \quad t \to \infty. \tag{27}$$

The orientation tracking objective given in (27) can also be stated in terms of $e_q(t)$. Specifically, (20) implies that

$$0 \leq \|e_v(t)\| \leq 1 \qquad \text{and} \qquad 0 \leq |e_o(t)| \leq 1 \tag{28}$$

for all time and if $\|e_v(t)\| \to 0$ as $t \to \infty$ then $e_o(t) \to 1$ as $t \to \infty$. Thus, if $\|e_v(t)\| \to 0$ as $t \to \infty$ then (19) along with the previous statement can be used to conclude that $\tilde{R}(t) \to I_3$ as $t \to \infty$, and hence, the orientation tracking objective has been achieved.

IV. Tracking Error System Development

To facilitate the development of the open-loop error system, an auxiliary variable $\eta(t) \in \mathbb{R}^6$ is defined as follows:

$$\eta = \left(\Lambda\hat{J}\right)^{-1} \begin{bmatrix} \dot{p}_d + k_1 e_p \\ -R_d^T\omega_d + k_2 e_v \end{bmatrix} + \dot{\theta} \tag{29}$$

where $k_1, k_2 \in \mathbb{R}^{3\times 3}$ are positive, constant, diagonal matrices, and $\hat{J}(\cdot) \in \mathbb{R}^{6\times 6}$ is an estimated manipulator Jacobian matrix. After adding and subtracting the terms $\Lambda\hat{J}(\cdot)\dot{\theta}(t)$

and $\Lambda\hat{J}(\cdot)\,\eta(t)$ to (25) and utilizing (26), the following kinematic error system can be developed

$$\begin{bmatrix} \dot{e}_p \\ \tilde{\omega} \end{bmatrix} = -\begin{bmatrix} k_1 e_p \\ k_2 e_v \end{bmatrix} + \Lambda\left(\hat{J}\eta + W_j\tilde{\phi}_j\right) \qquad (30)$$

where $W_j(\cdot) \in \mathbb{R}^{6\times n_1}$ was introduced in (13) and the parameter estimation error term $\tilde{\phi}_j(t) \in \mathbb{R}^{n_1}$ is defined as

$$\tilde{\phi}_j = \phi_j - \hat{\phi}_j. \qquad (31)$$

The adaptive estimate $\hat{\phi}_j(t) \in \mathbb{R}^{n_1}$ introduced in (31) is designed as follows:

$$\dot{\hat{\phi}}_j = proj\,\{y\} \qquad (32)$$

where the auxiliary term $y \in \mathbb{R}^{n_1}$ is defined as

$$y = \Gamma_1 W_j^T \Lambda^T \begin{bmatrix} e_p \\ e_v \end{bmatrix} \qquad (33)$$

where $\Gamma_1 \in \mathbb{R}^{n_1\times n_1}$ is a constant positive diagonal matrix and the function $proj\{y\}$ is defined as follows

$$proj\{y_i\} \triangleq \begin{cases} y_i & \text{if } \hat{\phi}_{ji} > \underline{\phi}_{ji} \\ y_i & \text{if } \hat{\phi}_{ji} = \underline{\phi}_{ji} \quad \text{and } y_i > 0 \\ 0 & \text{if } \hat{\phi}_{ji} = \underline{\phi}_{ji} \quad \text{and } y_i < 0 \\ 0 & \text{if } \hat{\phi}_{ji} = \overline{\phi}_{ji} \quad \text{and } y_i > 0 \\ y_i & \text{if } \hat{\phi}_{ji} = \overline{\phi}_{ji} \quad \text{and } y_i \leq 0 \\ y_i & \text{if } \hat{\phi}_{ji} < \overline{\phi}_{ji} \end{cases} \qquad (34)$$

$$\underline{\phi}_{ji} \leq \hat{\phi}_{ji}(0) \leq \overline{\phi}_{ji} \qquad (35)$$

where y_i denotes the i^{th} component of y, and $\hat{\phi}_{ji}(t)$ denotes the i^{th} component of $\hat{\phi}_j(t)$ (Note that the above projection algorithm ensures that $\underline{\phi}_j \leq \hat{\phi}_j(t) \leq \overline{\phi}_j$. For further details the reader is referred [16]).

To obtain the closed loop error system for $\eta(t)$, we first take the time derivative of (29) to obtain the following expression

$$\dot{\eta} = \frac{d}{dt}\left\{\left(\Lambda\hat{J}\right)^{-1}\begin{bmatrix} \dot{p}_d + k_1 e_p \\ -R_d^T\omega_d + k_2 e_v \end{bmatrix}\right\} + \ddot{\theta}. \qquad (36)$$

After pre-multiplying (36) by $M(\theta)$, substituting (1) into the resulting expression for $M(\theta)\ddot{\theta}(t)$, and utilizing (29), the following simplified expression can be obtained

$$M\dot{\eta} = -V_m\eta + \tau + W_y\phi_y \qquad (37)$$

where $W_y(p_d, \dot{p}_d, \ddot{p}_d, q_d, \omega_d, \dot{\omega}_d, p, q, \theta, \dot{\theta}) \in \mathbb{R}^{6\times n_2}$ is a regression matrix of known and measurable quantities, and $\phi_y \in \mathbb{R}^{n_2}$ is a vector of n_2 unknown constant parameters. The product $W_y(\cdot)\,\phi_y$ introduced in (37) is defined as

$$\begin{aligned} W_y\phi_y \;=\;& M\frac{d}{dt}\left\{\left(\Lambda\hat{J}\right)^{-1}\begin{bmatrix} \dot{p}_d + k_1 e_p \\ -R_d^T\omega_d + k_2 e_v \end{bmatrix}\right\} \\ &+V_m\left(\Lambda\hat{J}\right)^{-1}\begin{bmatrix} \dot{p}_d + k_1 e_p \\ -R_d^T\omega_d + k_2 e_v \end{bmatrix} \\ &-G(\theta) - F_d\,\dot{\theta}. \end{aligned} \qquad (38)$$

Based on (37) and the subsequent stability analysis, the control input $\tau(t)$ is designed as

$$\tau = -W_y\hat{\phi}_y - k_r\eta - \left(\Lambda\hat{J}\right)^T\begin{bmatrix} e_p \\ e_v \end{bmatrix}. \qquad (39)$$

where $k_r \in \mathbb{R}^{6\times 6}$ is a constant positive diagonal matrix and $\hat{\phi}_y(t) \in \mathbb{R}^{n_2}$ denotes an adaptive estimate which is generated by the following differential expression

$$\dot{\hat{\phi}}_y = \Gamma_2 W_y^T\eta \qquad (40)$$

where $\Gamma_2 \in \mathbb{R}^{n_2\times n_2}$ is a positive constant diagonal matrix. After substituting (39) into (37), the following closed-loop error system is obtained

$$M\dot{\eta} = -V_m\eta + W_y\tilde{\phi}_y - k_r\eta - \left(\Lambda\hat{J}\right)^T\begin{bmatrix} e_p \\ e_v \end{bmatrix} \qquad (41)$$

where the adaptive estimation error is defined as

$$\tilde{\phi}_y = \phi_y - \hat{\phi}_y. \qquad (42)$$

V. Stability Analysis

Theorem 1: Given the robotic system described by (1), the control input (39) along with the adaptive laws defined in (32) and (40) guarantee asymptotic regulation of the end-effector position error and the unit quaternion error in the sense that $\|e_p(t)\| \to 0$ as $t \to \infty$ and $\|e_v(t)\| \to 0$ as $t \to \infty$.

Proof: Let $V(t) \in \mathbb{R}$ denote the following non-negative scalar function

$$\begin{aligned} V \;=\;& \frac{1}{2}e_p^T e_p + (1-e_0)^2 + e_v^T e_v \\ &+\frac{1}{2}\tilde{\phi}_j^T\Gamma_1^{-1}\tilde{\phi}_j + \frac{1}{2}\tilde{\phi}_y^T\Gamma_2^{-1}\tilde{\phi}_y + \frac{1}{2}\eta^T M\eta. \end{aligned} \qquad (43)$$

After taking the time derivative of (43) and utilizing (23), (31) and (42), the following expression is obtained

$$\begin{aligned} \dot{V} \;=\;& e_p^T\dot{e}_p + (1-e_0)\left(e_v^T\tilde{\omega}\right) + e_v^T\left(e_0 I_3 - e_v^\times\right)\tilde{\omega} \\ &+\frac{1}{2}\eta^T\dot{M}\eta + \eta^T M\dot{\eta} \\ &-\tilde{\phi}_y^T\Gamma_2^{-1}\dot{\hat{\phi}}_y - \tilde{\phi}_j^T\Gamma_1^{-1}\dot{\hat{\phi}}_j. \end{aligned} \qquad (44)$$

Upon further simplification of equation (44) by canceling common terms, and substituting for $M\dot{\eta}$ from (41), the following expression for $\dot{V}(t)$ can be obtained

$$\begin{aligned} \dot{V} \;=\;& \begin{bmatrix} e_p^T & e_v^T \end{bmatrix}\begin{bmatrix} \dot{e}_p \\ \tilde{\omega} \end{bmatrix} - \eta^T k_r\eta \\ &-\eta^T V_m\eta + \eta^T W_y\tilde{\phi}_y - \eta^T\left(\Lambda\hat{J}\right)^T\begin{bmatrix} e_p \\ e_v \end{bmatrix} \\ &+\frac{1}{2}\eta^T\dot{M}\eta - \tilde{\phi}_j^T\Gamma_1^{-1}\dot{\hat{\phi}}_j - \tilde{\phi}_y^T\Gamma_2^{-1}\dot{\hat{\phi}}_y. \end{aligned} \qquad (45)$$

After using Property 3, substituting from (30), (32), and (40) and canceling terms, $\dot{V}(t)$ can be expressed as

$$\begin{aligned} \dot{V} \;=\;& \begin{bmatrix} e_p^T & e_v^T \end{bmatrix}\left(-\begin{bmatrix} k_1 e_p \\ k_2 e_v \end{bmatrix} + \Lambda W_j\tilde{\phi}_j\right) \\ &-\eta^T k_r\eta - \tilde{\phi}_j\Gamma_1^{-1}proj\{y\}. \end{aligned} \qquad (46)$$

Substituting for y from (33) and using the definition of the projection function, (34), the expression for $\dot{V}(t)$ can be upper bounded as follows

$$\dot{V} \leq -\lambda_{min}\{k_1\} \|e_p\|^2 - \lambda_{min}\{k_2\} \|e_v\|^2 - \lambda_{min}\{k_r\} \|\eta\|^2 . \quad (47)$$

where λ_{min} is the minimum Eigenvalue of the matrix.

The expressions in (43) and (47) can be used to prove that $e_p(t)$, $e_v(t)$, $\eta(t)$, $\tilde{\phi}_j(t)$, $\tilde{\phi}_y(t) \in \mathcal{L}_\infty$ and that $e_p(t)$, $e_v(t)$, $\eta(t) \in \mathcal{L}_2$. Using (16) and the assumption that $p_d(t) \in \mathcal{L}_\infty$, it is clear that $p(t) \in \mathcal{L}_\infty$. From (31) and (42) it can be concluded that $\hat{\phi}_j(t)$, $\hat{\phi}_y(t) \in \mathcal{L}_\infty$. Utilizing Property 7, the definition of $\eta(t)$ in (29) and the fact that $e_p(t)$, $e_v(t)$, $\eta(t) \in \mathcal{L}_\infty$, we can show that $\dot{\theta}(t) \in \mathcal{L}_\infty$. Moreover, (9), (16) and the fact that $J(\theta) \in \mathcal{L}_\infty$ can be used to show that $\dot{p}(t)$, $\dot{e}_p(t) \in \mathcal{L}_\infty$. From (20), (23), (25) and (28) we can show that $e_0(t)$, $\dot{e}_0(t)$, $\dot{e}_v(t) \in \mathcal{L}_\infty$. From the definition of $W_j(\cdot)$ and $W_y(\cdot)$ in (13) and (38) respectively and the preceding arguments, it is clear that $W_y(\cdot), W_j(\cdot) \in \mathcal{L}_\infty$. Utilizing (32), (33), (34) and (40), we can show that $\dot{\hat{\phi}}_j(t)$, $\dot{\hat{\phi}}_y(t) \in \mathcal{L}_\infty$. The definition of $\tau(t)$ in (39) can be used to show that $\tau(t) \in \mathcal{L}_\infty$; hence, $\theta(t)$, $\dot{\theta}(t)$, $\ddot{\theta}(t) \in \mathcal{L}_\infty$ and from (36) we can conclude that $\dot{\eta}(t) \in \mathcal{L}_\infty$. Since $\dot{e}_p(t)$, $\dot{e}_v(t)$, $\dot{\eta}(t) \in \mathcal{L}_\infty$ and $e_p(t)$, $e_v(t)$, $\eta(t) \in \mathcal{L}_2$, Barbalat's Lemma [17] can be used to show that, $\|e_p(t)\| \to 0$, $\|e_v(t)\| \to 0$ and $\|\eta(t)\| \to 0$ as $t \to \infty$. ■

VI. Conclusion

A task-space, adaptive tracking controller for robot manipulators with uncertainty in both the kinematic and the dynamic models was proposed. The controller yields asymptotic regulation of the end-effector position and orientation tracking errors. The advantages of the proposed controller are that singularities associated with the three parameter representation are avoided.

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