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Minimum Necessary Data Rates for Accurate Track Fusion

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Abstract— **Advances in technology are making distributed networks of sensor platforms a reality for surveillance systems. Such networks provide many benefits including the ability to cover a broader area, to compute more accurate target estimates and provide a reduction in susceptibility to countermeasures. In spite of these advantages, many practical issues remain in the deployment of such networks. One such significant issue is bandwidth limitations on the communication channels between the sensor platforms. This paper examines the effect of bandwidth restrictions on a decentralised system architecture. In such a structure, each sensor tracks targets within its individual surveillance region and then transmits its track data to a global fusion centre. The minimum necessary data rates required to permit reconstruction of the global track estimates to a given level of accuracy are presented in the case when the track state can be represented by a Gauss-Markov system.**

I. INTRODUCTION

Modern surveillance systems often involve multiple sensors, possibly of different types, that are spatially distributed. Such sensor networks have the ability to provide broad coverage and improved estimation accuracy due to the diverse viewing angles and complementary information provided by heterogeneous sensors. Multisensor networks may also reduce the sensitivity of the surveillance system to electronic counter-measures. A key feature in the use of such networks is management of the network's resources such as bandwidth.

It is known that the optimal structure for such networks is a centralised architecture, when the primary objective is estimation accuracy [1]. In this type of architecture, each sensor transmits its measurements to a central processor where all the estimation is performed. However, a centralised architecture has several drawbacks, including high bandwidth requirements and high vulnerability to attack. Latency on the network links also increases the probability that measurements are received at the central processor out of order.

In a decentralised network architecture, each local sensor performs local estimation using its own measurements and then transmits these estimates, and associated information, to the central processor. Such networks avoid the problems that exist in a centralised architecture. However, care must be taken when combining or fusing estimates from the local sensors as they will be correlated if they are estimates of the same target [2].

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In spite of the reduction in bandwidth requirements in a decentralised architecture, bandwidth limitations are still a concern. The effect on estimation accuracy when the sensor data is compressed prior to transmission to the central processor in a decentralised architecture has been examined in [3] and [4]. The equivalent problem in a centralised structure is covered in [5]. The problem of bandwidth allocation in a centralised architecture was examined in [6], [7], [8] when the bandwidth is too low to transmit all the collected information. A similar problem was considered in [9].

An outstanding problem in this area is the calculation of the minimum data rates needed on each communication channel to achieve a desired level of estimation accuracy in a decentralised framework. In this paper we consider this problem when the target dynamics can be described by a Gauss-Markov system. An idealised scenario is examined where there is a single target under track, there are no false measurements and the target is always detected. The sensor data needed for optimal fusion in this scenario is derived. From this expression it is then possible to compute the minimum data rates that are required for transmitting the local estimates to the fusion centre so that the fused estimate has a desired accuracy. This result provides a strict lower bound for the bandwidth required in any practical system as the additional uncertainties in a practical system will increase the amount of information that it is necessary to transmit. This in turn will increase the bandwidth requirements. We call the data rates so computed the *minimum necessary data rates* it will not be possible to achieve the desired level of accuracy with lower data rates.

The structure of the paper is as follows. In Section II an expression for the data that needs to be transmitted to the fusion centre is derived, in the case when the target and sensor dynamics are described by a Gauss-Markov system. In Section III the rate distortion region is given when a single decoder reconstructs a signal from two correlated Gaussian sources. In Section IV it is shown how the results of the previous two sections can be combined to provide an expression for the minimum necessary data rates required to fuse local tracks at a desired level of accuracy. This answers the previously unsolved question of the minimum data rates needed to achieve sufficiently accurate track fusion. Finally, the results of Section IV are illustrated by an example of a simple tracking system in Section V.

II. FUSION METHOD

Consider the scenario shown in Fig. 1 where two non-colocated sensors are obtaining noisy measurements of the same target. Let Y_k^i be the sequence of measurements from sensor i at time k, i.e. $Y_k^1 = \{y_1^1, y_2^1, \ldots, y_k^1\}$ for sensor one and $Y_k^2 = \{y_1^2, y_2^2, \ldots, y_k^2\}$ for sensor two. With these definitions it is known that the optimal estimate of the unknown target state x is given by $p(x_k|Y_k^1, Y_k^2)$ where $p(\cdot)$ is a probability density function (pdf). A centralised estimation scheme requires that each sensor sends its measurements to the fusion centre. However, this typically requires very high bandwidth on the communication channels. In addition, it may be desirable to have local track estimates available at each local sensor.

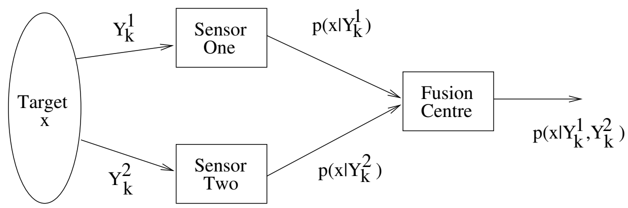

Fig. 1. A decentralised estimation architecture.

One method of avoiding these problems is to use a decentralised estimation architecture. In this type of framework, each sensor forms its local estimate of the target $p(x_k|Y_k^i)$ and transmits a sufficient statistic for that local estimate to the fusion centre. It is known that if the channels are memoryless and independent given x, and the target state can be described by a Markov process, then the optimal estimate $p(x_k|Y_k^1, Y_k^2)$ can be recovered at the fusion centre [10]. In the case of just two sensors that is considered here, the global estimate is given by

$$p(x_k|Y_k^1, Y_k^2) = c_k \frac{\prod_{i=1}^2 p(x_k|Y_k^i)}{\prod_{i=1}^2 p(x_k|Y_{k-1}^i)} p(x_k|Y_{k-1}^1, Y_{k-1}^2) \quad (1)$$

where c_k is a normalisation constant.

When the target and sensor dynamics are linear and Gaussian it is straightforward to show that $p(x_k|Y_k^1, Y_k^2)$ is also a Gaussian pdf. As a result, it is possible to calculate the global fused estimate given sufficient statistics of the local estimates. Stated formally, suppose the target and sensor models are described by

$$\begin{aligned} x_{k+1} &= F x_k + w_k && (2) \\ y_k^i &= H^i x_k + v_k^i && (3) \end{aligned}$$

where $\{w_k\}$ and $\{v_k^i\}$ are independent, identically distributed, zero mean, white noise sequences with $\mathsf{E}[w_k w_k'] = \Sigma_Q$ and $\mathsf{E}[v_k^i (v_k^i)'] = \Sigma_R^i$. If

$$x_0 \sim \mathcal{N}(\hat{x}(0|0), P(0|0)) \quad (4)$$

then $p(x_k|Y_k^1, Y_k^2)$ is Gaussian with a mean of $\hat{x}(k|k)$ and covariance $P(k|k)$. The global mean and covariance can be recovered from the local means $\hat{x}^i$ and covariances P^i using the following set of recursive equations [11, Ch. 8.6]

$$\begin{aligned} P(k|k)^{-1}\hat{x}(k|k) &= P(k|k-1)^{-1} F \hat{x}(k-1|k-1) \quad (5) \\ &+ \sum_{i=1}^{2} \left[P^i(k|k)^{-1}\hat{x}^i(k|k) - P^i(k|k-1)^{-1} F \hat{x}^i(k-1|k-1) \right] \end{aligned}$$

$$\begin{aligned} P(k|k)^{-1} &= P(k|k-1)^{-1} + \quad (6) \\ &\quad \sum_{i=1}^{2} \left[P^i(k|k)^{-1} - P^i(k|k-1)^{-1} \right] \\ P(k|k-1) &= F P(k-1|k-1) F' + \Sigma_Q \quad (7) \end{aligned}$$

where $P^i(k|k)$ and $P^i(k|k-1)$ are the filtered and predicted state error covariances respectively from each local tracking filter.

III. RATE DISTORTION REGION FOR CORRELATED GAUSSIAN SOURCES

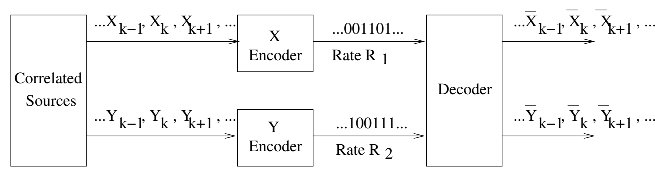

Fig. 2. Correlated source coding configuration.

Consider the scenario shown in Fig. 2 where two correlated memoryless sources, $\{X_k\}$ and $\{Y_k\}$, are encoded with separate encoder before transmission to a joint decoder. The well-known result of Slepian and Wolf gives the minimum data rate needed in order to reproduce the two source inputs with arbitrary small error probability when each of the two sources are independent samples of a discrete random variable [12].

The case when the two sources are correlated scalar Gaussian random variables is considered in [13] and a bound is given for the *admissible rate region* needed to decode the sources with a given mean expected level of distortion. Stated formally the results of [13], when extended to vector-valued Gaussian random variables, are as follows. Suppose $\{X_k\}$ and $\{Y_k\}$ are sequences of identically distributed random variables from a Gaussian distribution with the bivariate probability density function

$$p_{XY}(x,y) = \frac{1}{2\pi|\Lambda|^{\frac{1}{2}}} \exp\left(-\tfrac{1}{2}(\xi-\mu)'\Lambda^{-1}(\xi-\mu)\right) \quad (8)$$

where $\xi = [x,y]'$ and $\mu = [E[X], E[Y]]'$ and

$$\Lambda = \begin{bmatrix} \Sigma_X^2 & \rho\Sigma_X\Sigma_Y \\ \rho\Sigma_X\Sigma_Y & \Sigma_Y^2 \end{bmatrix} \quad (9)$$

where X and Y are defined on the spaces $\mathscr{X}$ and $\mathscr{Y}$ respectively. We will represent a sequence of n samples from each source as $X^n = X_1, X_2, \ldots, X_n$ and $Y^n = Y_1, Y_2, \ldots, Y_n$ respectively. The data sequences X^n and Y^n are separately encoded and sent to the common decoder which outputs the estimates $\bar{X}^n$ and $\bar{Y}^n$. The encoder functions ϕ_1 and ϕ_2 are defined as

$$\begin{aligned} \phi_1 &: \mathscr{X}^n \to \mathscr{C}_1 = \{1,2,\ldots,C_1\} && (10) \\ \phi_2 &: \mathscr{Y}^n \to \mathscr{C}_2 = \{1,2,\ldots,C_2\} && (11) \end{aligned}$$

and they satisfy the rate constraints

$$\frac{1}{n}\log C_i \le R_i + \delta \quad (12)$$

for $i = 1,2$ where $\delta > 0$. The decoders are given by two functions ψ_1 and ψ_2 where

$$\bar{X}^n = \psi_1(\phi_1(X^n), \phi_2(Y^n)) \quad (13)$$
$$\bar{Y}^n = \psi_2(\phi_1(X^n), \phi_2(Y^n)) \quad (14)$$

Define the mean expected distortions in these estimates, Δ_1 and Δ_2, as

$$\Delta_1 = \frac{1}{n}\mathsf{E}\left[\sum_{k=1}^{n}(X_k - \bar{X}_k)'(X_k - \bar{X}_k)\right] \quad (15)$$
$$\Delta_2 = \frac{1}{n}\mathsf{E}\left[\sum_{k=1}^{n}(Y_k - \bar{Y}_k)'(Y_k - \bar{Y}_k)\right] \quad (16)$$

For a given pair of positive numbers D_1 and D_2 a rate pair (R_1, R_2) is called *admissible* if for any $\delta > 0$ and any $n > n_0(\delta)$ there are encoders ϕ_i and decoders ψ_i that satisfy (10)–(14) such that

$$\Delta_i \leq D_i + \delta \quad (17)$$

for $i = 1,2$. Define the *rate distortion region* $\mathscr{R}(D_1, D_2)$

$$\mathscr{R}(D_1, D_2) \stackrel{\triangle}{=} \{(R_1, R_2) : (R_1, R_2) \text{ is admissible}\} \quad (18)$$

The boundary of this region gives the minimum necessary data rates to recover the sources with the prescribed degree of mean expected distortion.

Following [13, Theorem 2] it is straightforward to show that for vector-valued Gaussian random variables

Theorem 1: For every $D_1, D_2 > 0$

$$\mathscr{R}(D_1, D_2) \subseteq \mathscr{R}_{out}(D_1, D_2) \quad (19)$$

where

$$\mathscr{R}_{out}(D_1, D_2) = \mathscr{R}_1^*(D_1) \cap \mathscr{R}_2^*(D_2) \cap \hat{\mathscr{R}}_{12}(D_1, D_2) \quad (20)$$

and

$$\mathscr{R}_1^*(D_1) = \left\{(R_1, R_2) : R_1 \geq \tfrac{1}{2} log^+ \left(\frac{|\Sigma_X^2|}{D_1}(1 - \rho^2 + \rho^2 2^{-2R_2})\right)\right\} \quad (21)$$

$$\mathscr{R}_2^*(D_2) = \left\{(R_1, R_2) : R_2 \geq \tfrac{1}{2} log^+ \left(\frac{|\Sigma_Y^2|}{D_2}(1 - \rho^2 + \rho^2 2^{-2R_1})\right)\right\} \quad (22)$$

$$\hat{\mathscr{R}}_{12} = \left\{(R_1, R_2) : R_1 + R_2 \geq \tfrac{1}{2}\log^+ \left((1 - \rho^2)\frac{|\Sigma_X^2||\Sigma_Y^2|}{D_1 D_2}\right)\right\} \quad (23)$$

and $\log^+(x) = \max\{\log(x), 0\}$.

In addition, we conjecture that [13, Theorem 3] can be extended to the vector-valued case with the following result.

Theorem 2: For every $D_1, D_2 > 0$

$$\mathscr{R}(D_1, D_2) \supseteq \mathscr{R}_{in}(D_1, D_2) \quad (24)$$

where

$$\mathscr{R}_{in}(D_1, D_2) = \mathscr{R}_1^*(D_1) \cap \mathscr{R}_2^*(D_2) \cap \tilde{\mathscr{R}}_{12}(D_1, D_2) \quad (25)$$

and

$$\tilde{\mathscr{R}}_{12}(D_1, D_2) = \left\{(R_1, R_2) : R_1 + R_2 \geq \tfrac{1}{2}\log^+ \left((1 - \rho^2)\frac{\gamma_{max}}{2}\frac{|\Sigma_X^2||\Sigma_Y^2|}{D_1 D_2}\right)\right\} \quad (26)$$

where

$$\gamma_{max} = 1 + \sqrt{1 + \frac{4\rho^2}{(1 - \rho^2)^2}\frac{|\Sigma_X^2||\Sigma_Y^2|}{D_1 D_2}} \quad (27)$$

From these results it can be seen that the boundaries given by $\mathscr{R}_1^*(D_1)$ and $\mathscr{R}_2^*(D_2)$ are tight and that the gap between the $\hat{\mathscr{R}}_{12}$ and $\tilde{\mathscr{R}}_{12}$ regions is

$$\Delta_R \stackrel{\triangle}{=} \tfrac{1}{2}\log\left(\frac{\gamma_{max}}{2}\right) \quad (28)$$

IV. MINIMUM DATA RATES FOR TRACK FUSION

Given the results of the previous two sections it is now possible to derive the minimum data rates needed to reconstruct the optimal track estimate at the fusion centre to within a desired degree of mean expected accuracy.

Consider the case of two sensors where the initial conditions of each sensor level tracker, $\hat{x}^i(0|0)$ and $P^i(0|0)$, are known at the fusion centre, along with the time each sensor initiates its track on the target. In this case, it can be shown that it is only necessary for each sensor to transmit

$$\tilde{y}^i(k) \stackrel{\triangle}{=} y_k^i - H^i\hat{x}^i(k|k-1) \quad (29)$$

which are known as the *innovations*. With knowledge of $\tilde{y}^i(k)$ and all the necessary prior information, the fusion centre can reproduce the sensor-level track estimates since

$$\hat{x}^i(k|k) = \hat{x}^i(k|k-1) + P^i(k|k)(H^i)'(\Sigma_R^i)^{-1}\tilde{y}^i(k) \quad (30)$$
$$\hat{x}^i(k+1|k) = F\hat{x}^i(k|k) \quad (31)$$

This is due to the well-known result that the state error covariance matrices in the Kalman filter, $P^i(k|k-1)$ and $P^i(k|k)$, do not depend on the data. In effect, this converts the decentralised multisensor tracking structure into a centralised tracking scheme. While these assumptions are unlikely to be realised in practice, they represent the multisensor tracking structure that requires the minimal amount of information to be transmitted to the fusion centre. Thus minimum data rates for this idealised system provide strict lower bounds for any practical system.

It is well known that if the state space models given by (2)–(3) are controllable and observable then the Kalman filter for each system will tend to a steady-state [14]. Let $\bar{P}_+^i$ and $\bar{P}_-^i$ be the steady-state values of $P^i(k|k)$ and $P^i(k|k-1)$ respectively. Once the filters have reached steady-state the innovations from each sensor-level tracker, $\tilde{y}^i(k)$ are identically distributed Gaussian random variables with

$$\tilde{y}^i(k) \sim \mathscr{N}(0, \bar{S}^i) \quad (32)$$

where

$$\bar{S}^i = H^i\bar{P}_-^i(H^i)' + \Sigma_R^i \quad (33)$$

Thus the minimum necessary data rates needed on the communication links to transmit the innovations from the sensor platforms to the fusion centre can be derived using the results from Section III.

Now, it is a standard result of Kalman filter theory that the innovations from each filter $\tilde{y}^i(k)$ are uncorrelated over time [14]. However, as both sensors are tracking the same target $\tilde{y}^1(k)$ and $\tilde{y}^2(k)$ are correlated. It is straightforward to show that

$$\bar{S}^c \triangleq E[\tilde{y}^1(k)\tilde{y}^2(k)'] = H^1 P^c(k|k-1)(H^2)' \tag{34}$$

where the predicted and filtered cross-covariance matrices are defined as

$$P^c(k|k-1) \triangleq E[(x_k - \hat{x}^1(k|k-1))(x_k - \hat{x}^2(k|k-1))'] \tag{35}$$

$$P^c(k|k) \triangleq E[(x_k - \hat{x}^1(k|k))(x_k - \hat{x}^2(k|k))'] \tag{36}$$

Writing $\bar{P}^c_-$ and $\bar{P}^c_+$ for the steady-state predicted and filtered cross-covariance matrices respectively, we have [15]

$$\bar{P}^c_- = F\bar{P}^c_+ F' \tag{37}$$

$$\bar{P}^c_+ = (I - \bar{K}^1 H^1)F\bar{P}^c_+ F'(I - \bar{K}^2 H^2)' + \bar{Q} \tag{38}$$

where $\bar{K}^i, i = 1,2$ are the steady-state gains from each Kalman filter and

$$\bar{Q} = (I - \bar{K}^1 H^1)\Sigma_Q (I - \bar{K}^2 H^2)' \tag{39}$$

The equation for $\bar{P}^c_+$ has a unique, positive definite solution provided the systems are controllable and observable and F is invertible [16].

Now the central fused estimate $\hat{x}(k|k)$ can only be calculated exactly, even once the filters are in steady-state, if the $\tilde{y}^i(k)$ are transmitted perfectly. If instead the values $\bar{y}^i(k)$ are used, where $\bar{y}^i(k)$ are the estimates of $\tilde{y}^i(k)$ after encoding, transmission and decoding over communications channels with rates R_1 and R_2 where $(R_1, R_2) \in \mathscr{R}(D_1, D_2)$ we can use the results of Section III to calculate the additional error in fused estimate $\hat{x}(k|k)$ that is due to the bandwidth limits on the channels. An example of this for a simple tracking system is shown in the next section.

V. EXAMPLE

A. Scenario

Consider the case of tracking a target moving with near constant velocity in one dimension with measurements of position only from two non-colocated sensors. A state space model for such a scenario is given by

$$x_{k+1} = Fx_k + w_k \tag{40}$$

$$y^i_k = H^i x_k + v^i_k \tag{41}$$

where

$$F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \tag{42}$$

$$H^i = \begin{bmatrix} 1 & 0 \end{bmatrix} \tag{43}$$

$$\Sigma_Q = \begin{bmatrix} \frac{T^4}{4} & \frac{T^3}{2} \\ \frac{T^3}{2} & T^2 \end{bmatrix} \sigma_q^2 \tag{44}$$

and T is the sampling rate. For simplicity, we will assume that the sensors are registered and the measurement noise variance is the same for both, i.e. $\Sigma^i_R = \sigma_r^2$. As a consequence, the filters for both sensor are identical so the the superscript i will be neglected in the remainder of this section.

As this system is observable and controllable and F is invertible all the relevant Kalman filter matrices tend to a steady-state. Let

$$\bar{K} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \tag{45}$$

then it can be shown that [15]

$$k_1 = \frac{-1}{8}\left(\lambda^2 + 8\lambda - (\lambda+4)\sqrt{\lambda^2 + 8\lambda}\right) \tag{46}$$

$$k_2 = \frac{1}{4T}\left(\lambda^2 + 4\lambda - \lambda\sqrt{\lambda^2 + 8\lambda}\right) \tag{47}$$

$$\lambda = \frac{\sigma_q T^2}{\sigma_r} \tag{48}$$

where λ is known as the *target manoeuvring index*. With these definitions it can be shown that [16]

$$\bar{S} = \frac{\sigma_r^2}{1 - k_1} \tag{49}$$

and

$$\bar{S}^c = p^c_{11} + 2Tp^c_{12} + T^2 p^c_{22} \tag{50}$$

and the steady-state cross-covariance matrix is given by

$$P^c = \begin{bmatrix} p^c_{11} & p^c_{12} \\ p^c_{12} & p^c_{22} \end{bmatrix} \tag{51}$$

where the elements of this matrix can be computed via iteration until the steady-state values are reached.

In addition, the steady-state error covariance matrix for each sensor-level tracker is [15]

$$\bar{P}_+ = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \tag{52}$$

where

$$p_{11} = k_1 \sigma_r^2 \tag{53}$$

$$p_{12} = k_2 \sigma_r^2 \tag{54}$$

$$p_{22} = \frac{k_2(k_1 - \frac{1}{2}Tk_2)}{T(1-k_1)}\sigma_r^2 \tag{55}$$

B. Minimum Data Rates

Suppose each sensor-level tracker transmits an encoded version of its filter innovation $\tilde{y}^i(k)$ to the fusion centre at every scan. Let the decoded value of each of the innovations be denoted by $\bar{y}^i(k)$. If the rates on the communications links between the sensors and the fusion centre are in the admissible rate distortion region $\mathscr{R}(D_1, D_2)$ then it can be shown that the expected additional error in the value of $\hat{x}(k|k)$ has an upper bound of

$$\Delta_x \triangleq \sum_i \bar{P}_+(H^i)'\Sigma_R^{-2} H^i \bar{P}_+ D_i \tag{56}$$

$$= \begin{bmatrix} k_1^2 & k_1 k_2 \\ k_1 k_2 & k_2^2 \end{bmatrix}(D_1 + D_2) \tag{57}$$

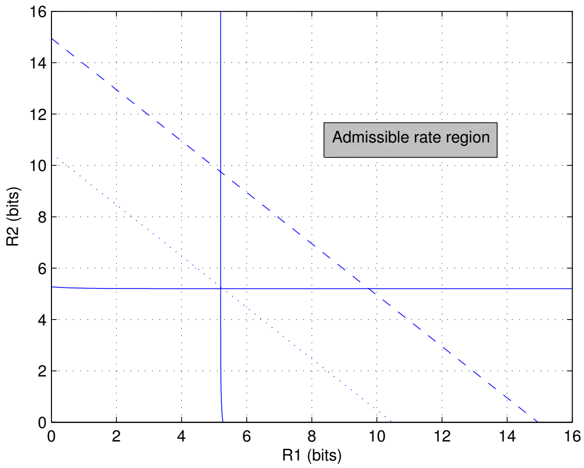

Fig. 3. Admissible rate region when the acceptable distortion on each channel is chosen so that the position error variance is increased by no more than 1%, when $\sigma_q^2 = 0.01$, $\sigma_r^2 = 1.0$ and $T = 5$. The dotted line is $\hat{\mathscr{R}}_{12}$, the dashed line is $\tilde{\mathscr{R}}_{12}$ and the solid lines are $\mathscr{R}_i^*$.

The admissible rate distortion regions can be found using (19) and (24) with

$$\Sigma_X^2 = \Sigma_Y^2 = \bar{S} = \frac{\sigma_r^2}{1-k_1} \tag{58}$$

and

$$\rho \Sigma_X \Sigma_Y = \bar{S}^c \tag{59}$$

Using the same acceptable distortion levels on both channels, i.e. $D_1 = D_2$, Fig. 3 shows the admissible rate region when the D_i values are chosen so that there is at most a 1% increase in the position error variance due to the limits on the channel capacity. In this example, $\sigma_q^2 = 0.01$, $\sigma_r^2 = 1.0$ and $T = 5$. The value of $\hat{\mathscr{R}}_{12}$ is given by the dotted line and $\tilde{\mathscr{R}}_{12}$ is given by the dashed line. From this figure it can be seen that approximately 7.5 bits per channel are needed to meet the requirements. Note, it is not necessary that both channels have the same capacity. In this particular case, any pair (R_1, R_2) that satisfy

$$R_i \geq 5.3 \text{ bits}, \quad i = 1,2 \tag{60}$$

$$R_1 + R_2 \geq 15.0 \text{ bits} \tag{61}$$

will be in the admissible rate region. Also, the channel capacities may vary with time, provided rates satisfy (60)–(61) at all times.

Table I gives the minimum necessary data rates for a variety of scenarios, assuming equal capacity on each channel. Each scenario is specified by the value of the target manoeuvring index, λ, and the acceptable percentage increase in the position error variance. The limit given in the tables is given by $\frac{1}{2}\tilde{\mathscr{R}}_{12}$ which is an upper bound on the minimum acceptable rate.

As expected, this table clearly shows that as the uncertainty in the target position increases, as measured by the target manoeuvring index λ, the required channel capacity increases. Conversely, as the acceptable distortion level increases, the minimum necessary capacity decreases.

TABLE I

MINIMUM NECESSARY DATA RATES ASSUMING EQUAL CAPACITY ON EACH CHANNEL.

Target Man. Index λ	% Increase in Pos. Error Var.	Min. Data Rate (bits)
0.1	10.0	2.0
0.1	5.0	2.6
0.1	1.0	4.3
1.0	10.0	4.0
1.0	5.0	4.7
1.0	1.0	6.5
2.5	10.0	5.0
2.5	5.0	5.7
2.5	1.0	7.5

VI. CONCLUSIONS

This paper has derived an analytical method for computing the minimum bandwidth needed to estimate the track of a target with Gauss-Markov dynamics to a specified level of accuracy in a decentralised sensor network. The resulting expression for the required bandwidth can be seen to be a function of the uncertainty in the track estimates and the acceptable level of error in the fused track state estimates. These results were derived using an idealised system, therefore this provides a lower bound on the requirements of any practical system for tracking targets with these types of dynamics.

Future work in this area includes the derivation of equivalent minimum necessary data rates when the estimation scenario includes clutter or false measurements; the probability of target detection is less than unity; and in the presence of multiple targets. Another issue is the construction of coding schemes that achieve the bound calculated in this paper. Related work is being carried out on the optimal scheduling of data transmissions from the sensors to the central processor when there is insufficient bandwidth to transmit all the required information.

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