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A New Architecture for Robust Model Reference Control

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***Abstract*— A new controller configuration for Robust Model Reference Control is presented in this paper. The structure is based on a right coprime factorization of the plant and makes use of an observer-based feedback control scheme combined with a prefilter controller. It is designed first to guarantee stability robustness and some levels of performance in terms of disturbance rejection. This is done by solving a constrained $\mathcal{H}_\infty$ optimization problem using the right coprime factorization of the plant in an *active* way. Secondly, the prefilter controller is designed to guarantee robust open-loop processing of the reference commands. This is done by assuming a Reference Model and by solving a Model Matching Problem.**

I. INTRODUCTION

Standard feedback control is based on processing the difference between the reference inputs and the actual outputs. It is well known that, in such a case, the design problem has one degree of freedom (1-DOF) which may be described in terms of the stable Youla parameter [10]. Two degree-of-freedom (2-DOF) compensators are characterized by allowing a separate processing of the reference inputs and the controlled outputs and may be stated by means of two stable Youla parameters. The 2-DOF compensators present the advantage of a complete separations between feedback and reference tracking properties [12]: the feedback properties of the control system are assured by a feedback controller, i.e., the first degree of freedom; the reference tracking specifications are addressed by a prefilter controller, i.e., the second degree of freedom, which determines the open-loop processing of the reference commands.

As is is pointed out in [11], classical control approaches tend to stress the use of feedback to modify the systems' response to commands. A clear example, widely used in the literature of linear control, is the usage of Reference Models to specify the desired properties on a control system [1]. What is specified through a Reference Model is the desired closed-loop system response. Therefore, as the system responds to commands is an open-loop property and robustness properties are associated with the feedback [9], no stability margins are necessarily guaranteed when achieving the desired closed-loop response behavior.

A 2-DOF control configuration may be used in order to achieve a control system with both a performance specification, e.g., through a Reference Model, and some guaranteed stability margins. Nevertheless, the lack of methodologies to design the two compensators may be the reason for the 2-DOF compensators not to be widely used. Since the best way of allocating the gain between the two controllers is not so clear, the approaches found in the literature are mainly based on optimization problems. Basically, these optimization procedures represent different ways of setting the Youla parameters to represent the controllers [10], [12], [6].

The approach presented in [6] expands the role of $\mathcal{H}_\infty$ optimization tools in 2-DOF system design. The 1-DOF loop-shaping design procedure [7] is extended to a 2-DOF control configuration by means of a parametrization in terms of two stable, but otherwise free, Youla parameter. A feedback controller is designed to guarantee robust stability and disturbance rejection requirements in a manner similar to the 1-DOF loop-shaping design procedure. A prefilter controller is then introduced to force the response of the closed-loop system to follow that of a specified Reference Model. The approach is carried out by assuming uncertainty in the normalized coprime factors plant descriptions [5]. Such uncertainty description allows a formulation of the $\mathcal{H}_\infty$ robust stabilization problem providing explicit formulas for the corresponding controller. Nevertheless, the translation of physical parameters uncertainty into normalized coprime factors plant uncertainty is not straightforward.

In this paper we present a new 2-DOF control configuration based a right coprime factorization of the plant. Within the factorization framework, the partial state is used as controlled variable. The presented approach is not based on setting the two controllers arbitrarily, with internal stability as the only restriction, and parameterize the controller in terms of the Youla parameter. Instead,

1) An observer-based feedback control scheme is designed to guarantee some levels of stability robustness. This is done by solving a constrained $\mathcal{H}_\infty$ optimization problem using partial state as controlled variable and the right coprime factorization of the plant in an *active* way.
2) A prefilter controller is computed to guarantee the robust open-loop processing of the reference commands. This is done by assuming a Reference Model with the desired relations from the reference signals and by solving a model matching problem imposed on the prefilter controller such that the response of the overall close-loop system match that of the reference model.

II. COPRIME FACTORIZATION OVER $\mathcal{RH}_\infty$

The central idea of the so called *factorization approach* is to factor a transfer (matrix) function of a system, not necessarily stable, as a ratio of two stable transfer (matrix)

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functions. (See [10] or [4] among others). Within this framework, a doubly coprime factorization of a transfer (matrix) function P_o is written as

$$P_o = N_r M_r^{-1} = M_\ell^{-1} N_\ell \quad (1)$$

where the pairs (N_r, M_r) and (M_ℓ, N_ℓ) represent, respectively, a Right Coprime Factorization (RCF) and Left Coprime Factorization (LCF) of P_o over $\mathcal{RH}_\infty$. This fact means that there exist stable transfer functions X_r, Y_r, X_ℓ, $Y_\ell \in \mathcal{RH}_\infty$ such that the following Bezout identity holds:

$$\begin{bmatrix} X_r & Y_r \\ -N_\ell & M_\ell \end{bmatrix} \begin{bmatrix} M_r & -Y_\ell \\ N_r & X_\ell \end{bmatrix} = I \quad (2)$$

The following theorem arises to provide right and left coprime factorizations for a proper real rational matrix $P_o(s)$, where

$$P_o(s) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \quad (3)$$

is a minimal stabilisable and detectable state-space realization.

Theorem 1: Define

$$\begin{bmatrix} M_r & -Y_\ell \\ N_r & X_\ell \end{bmatrix} \doteq \left[\begin{array}{c|cc} A+BF & B & -L \\ \hline \mathrm{F} & I & 0 \\ C+DF & D & I \end{array}\right] \quad (4)$$

$$\begin{bmatrix} X_r & Y_r \\ -N_\ell & M_\ell \end{bmatrix} \doteq \left[\begin{array}{c|cc} A+LC & -(B+LD) & L \\ \hline F & I & 0 \\ C & -D & I \end{array}\right] \quad (5)$$

where F and L are such that $A+BF$ and $A+LC$ are asymptotically stable. Then, $P_o = N_r M_r^{-1}$ ($P_o = M_\ell^{-1} N_\ell$) is a RCF (LCF).

Proof: The theorem is demonstrated by substituting (4) and (5) in to equation (2). □

A new control configuration based on the fractional representation framework is addressed next.

III. Controller configuration

The proposed configuration is shown in Figure 2 and it is concerned on using the so-called partial state ξ which is the fictitious signal that appears after the right coprime factorization of P_o. The resulting structure provides a genuine way of building the error signal, e_1: instead of comparing the output signals with the references, as it is usually done in standard feedback control, the error signal results form the difference between a *desired* partial state and the *actual* partial state, i.e. $e_1 = \xi^d - \xi$. So, the feedback controller K_1 could be designed to minimize the error signal e_1, say to force the actual partial state, ξ, to match the desired partial state, ξ^d.

By feeding the partial state back, the final achieved controller K_1 avoids dealing with RHP-zeros of the plant. Effectively, if we assume that $P_o = M_r^{-1} N_r$ is a right coprime factorization for the nominal model of the plant and no perturbations interfere the control system system, i.e. $d_i = d_o = 0$, the simplified feedback control structure shown in Figure 1 is obtained.

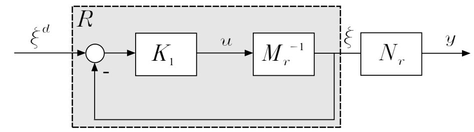

Fig. 1. Ideal representation of the feedback scheme.

This situation is, in fact, unrealistic since the partial state is an artificial signal appearing in the context of coprime factorization and direct access to ξ is meaningless when one is faced with the real plant. Nevertheless, the structure in Figure 1 illustrates the advantage in which the possible RHP-zeros of the plant are not introduced into the feedback loop and this prevents to cause closed-loop instability. It should be noted that, within a right coprime factorization of the plant, the poles of P_o are absorbed into M_r^{-1} as poles and the zeros of P_o are absorbed into N_r as zeros. While a standard output feedback controller is in charge of the nominal plant P_o, the controller K_1 faces just one of the coprime transfer matrices in which the nominal plant is split up, that is M_r^{-1}.

Remark 1: Assuming a nominal situation in which no disturbances d_i and d_o enter the control scheme as it is illustrated in Figure 1, the relation from the desired partial state ξ^d and the output signal y, is $y = N_r R\, \xi^d$, where N_r is the transfer matrix that contains the (possibly RHP) zeros of the plant and R is

$$R \doteq M_r^{-1} K_1 (I + M_r^{-1} K_1)^{-1} \quad (6)$$

As long as R relates the desired partial state, ξ^d, and the actual partial state, ξ, it should have, ideally, an all-pass shape with constant magnitude equal to one.

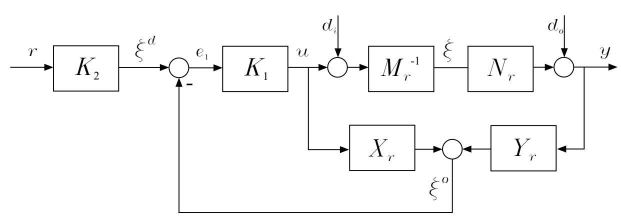

Fig. 2. Overall 2-DOF control configuration.

To observe the partial state, the Bezout components are introduced to the control system. So, we have $\xi^o = (X_r M_r + Y_r N_r)\xi - X_r d_i + Y_r d_o$ and, as long as the two observer transfer matrix functions X_r, Y_r, satisfies the Bezout identity (2), the scheme allows the reconstruction of the partial state, i.e., $\xi^o = \xi - X_r d_i + Y_r d_o$.

It is seen that such a reconstructed signal ξ^o incorporates a measure of the disturbance signals, d_i and d_o. In this way, the error signal e_1 can be seen as a measure of how the *actual* partial state differs from the *desired* partial state due to the disturbance signals d_i and d_o.

To provide the desired value for the partial state, ξ^d, a prefilter controller K_2 is incorporated. The prefilter is designed to force the response of the closed-loop system to follow that of a specified Reference Model, T_{ref}.

The 2-DOF scheme shown in Figure 2 presents a concise, and well suited for design, separation between feedback and open-loop properties. Therefore, since robustness is, in fact, a feedback property [9] we will setup the control problem as the design of the feedback controller K_1 in order to achieve robust stability with some levels of nominal regulation specification and the reference controller K_2 in order to achieve robust tracking specifications.

IV. DESIGN PRODEDURE

Let us assume, without lost of generality, that multiplicative input uncertainty is considered. In such an uncertainty description, the plant P to be controlled is unknown but belonging to a set of plants, $\mathcal{P}$, built around a nominal model, P_o,

$$\mathcal{P} = \{P : P = P_o(I + W_2 \Delta W_1)\}, \quad \bar{\sigma}(\Delta) \leq 1 \; \forall \omega \tag{7}$$

where $W_1 = w_I I$, $W_2 = I$.

Let us also assume that the disturbance rejection performance for P_o is specified by means of a weighting matrix, W_p. Finally, let us assume that an input/output ideal response is specified by means of a Reference Model, T_{ref}. Therefore, the control objective can be stated as follows:

> *Given a nominal system P_o, an uncertainty description that gives rise to the family of plants $\mathcal{P}$ in (7), a weighing matrix W_p and a Reference Model, T_{ref}, design a control system so that the relations from the reference signals for all possible plants $P \in \mathcal{P}$ behave as close as possible to T_{ref}, providing the desired stability margins and the specified nominal regulation properties.*

This problem could be called the design of *Model Reference Robust Controller*. The way to the solution will comprise the application of the following steps to the control scheme depicted in Figure 2.

Step 1: Feedback controller design. Find a coprime factorization for the nominal model, $P_o = M_r^{-1} N_r$, the associated Bezout components, X_r and Y_r and design the feedback controller K_1 so that the nominal disturbances rejection specifications are kept and the resulting closed-loop system remains stable for all $P \in \mathcal{P}$.

Step 2: Reference controller design. Design a prefilter controller K_2 so that relations from the references to the outputs for all $P \in \mathcal{P}$ behave as close as possible to that of the Reference Model, T_{ref}. The design of the reference controller will depend upon the performance criterion chosen for the desired closeness to the reference model T_{ref}.

A. Feedback controller design

One of the most distinctive features of the proposed approach is the usage of the partial state ξ as the controlled variable. Another distinguishing feature is that of giving the right coprime factors of the plant and its associated Bezout complements an *active* role in the overall control system.

Effectively, from Theorem 1 we know that a right coprime factorization of the nominal model, $P_o = M_r^{-1} N_r$, and the associated Bezout components, X_r and Y_r can be found by means of the state feedback gain, F, and the state observation gain, L. They are entirely free assignable matrices with the only restriction that $A + BF$ and $A + LC$ must be asymptotically stable. We can see, from equation (4), that the eigenvalues of $A + BF$ determine the resultant poles of M_r and N_r. Analogously from equation (5), the eigenvalues of $A + LC$ establish the poles of X_r and Y_r.

Obviously, for a right coprime factorization, $P_o = M_r^{-1} N_r$, the Bezout equation $X_r M_r + Y_r N_r = I$ must be fulfilled independently of the place that the eigenvalues of $A + BF$ and the eigenvalues of $A + LC$ are assigned. Nevertheless, depending on such a pole placement, each one of the involved factors, M_r, N_r, X_r and Y_r, will have different poles and different zeros. As long as $M_r(s)$, $N_r(s)$, $X_r(s)$ and $Y_r(s)$ contribute individually in the relevant input/output transfer matrix functions, the pole placement procedure is used to provide an extra degree of freedom.

The uncertainty description assumed in (7) and nominal regulation requirements defined in terms of weighted sensitivity allow us to redraw the observer-based feedback control configuration as shown in Figure 3.

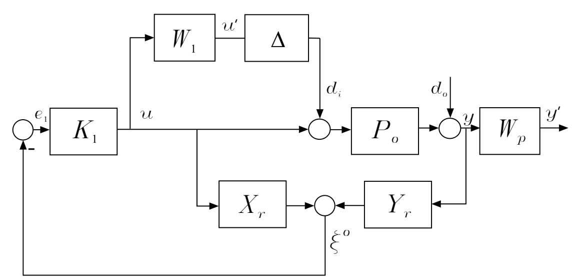

Fig. 3. Observer-based feedback control scheme with the uncertain plant, $\mathcal{P}$

From the resultant control scheme in Figure 3 we can compute the transfer matrix function, say $\mathcal{N}$, that relates the input signals $[d_i \; d_o]^T$ with the weighed output signals, $[u' \; y']^T$,

$$\mathcal{N} = \begin{bmatrix} \mathcal{N}_{11} & \mathcal{N}_{12} \\ \mathcal{N}_{21} & \mathcal{N}_{22} \end{bmatrix} \tag{8}$$

where

$$\begin{aligned} \mathcal{N}_{11} &= -W_1 M_r R Y_r N_r M_r^{-1} \\ \mathcal{N}_{12} &= -W_1 M_r R Y_r \\ \mathcal{N}_{21} &= W_p (N_r (I - R) M_r^{-1} + N_r R X_r) \\ \mathcal{N}_{22} &= W_p (I - N_r R Y_r) \end{aligned} \tag{9}$$

and R is defined in (6).

The weighing matrix, W_p, for the Sensitivity transfer function may be represented by

$$W_p = \frac{s/\mathrm{g} + \omega_b}{s + \omega_b \varepsilon} \tag{10}$$

where it is observed that $|W_p^{-1}(j\omega)|$ is equal to $\varepsilon \ll 1$ at low frequencies and it is equal to $\mathrm{g} \geq 1$ at high frequencies.

The asymptote crosses 1 at ω_b, which is approximately the bandwidth requirement. An asymptotic plot of W_p^{-1} over the frequency is illustrated in Figure 4.

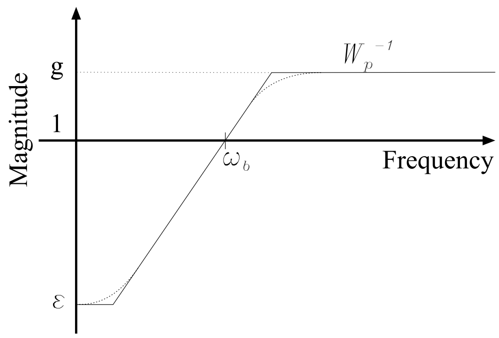

Fig. 4. Exact and asymptotic plot of $W_p^{-1}(\omega)$.

On the other hand, considering unstructured uncertainty (i.e., Δ is a full complex matrix of appropriate dimensions) and a multiplicative input uncertainty description as the one considered in (7), the Small Gain Theorem [8] imposes a condition on the ∞-norm of the transfer matrix function from the input disturbance, d_i, to the weighted control signal, u', in order for robust stability to be guaranteed, i.e.,

$$\|\mathcal{N}_{11}\|_\infty < 1 \tag{11}$$

This fact restricts the design of the feedback controller K_1 since, form (8) and condition (11), we have robust stability iff

$$\left\|-W_1 M_r R Y_r N_r M_r^{-1}\right\|_\infty < 1 \tag{12}$$

In this point, we could face step 1 by finding an optimal feedback controller K_1 minimizing $\mathcal{N}_{21}$ in (8) subject to the constraint (12). Nevertheless, the proposed design strategy suggests to make the most on the pole placement procedure carried to find a coprime factorization of the nominal plant.

The procedure starts by choosing an initial high-pass shape for $I - R$. For instance, we may choose

$$I - R \doteq W_p^{-1}, \tag{13}$$

yielding to $R \doteq I - W_p^{-1}$. Once R and $I - R$ are set, the following constrained optimization problem is posed:

$$\begin{aligned} &\min_{p_i} \|W_p(I - N_r R Y_r)\|_\infty \\ &\text{subject to} \\ &\left\|-W_1 M_r R Y_r N_r M_r^{-1}\right\|_\infty < 1 \end{aligned} \tag{14}$$

being $p_i = [p_{F_i}\ p_{L_i}]^T \in \mathcal{C}^-$, being p_{F_i} the assigned eigenvalues of $A + BF$ and p_{L_i} the assigned eigenvalues of $A + LC$. In other words, p_{F_i} are the placed poles for N_r and M_r, and p_{L_i} the placed poles for X_r and Y_r.

Actually, N_r and also M_r are unknowns but they are not entirely free since the coprime factorization in equation (4) and (5) constrains them. The same happens with the Bezout complements Y_r and X_r, which are linked by means of the Bezout identity (2).

The *indirect* design procedure is completed by recovering the feedback controller K_1 from equation (6). The design equation for K_1 is

$$K_1 = M_r R (I - R)^{-1} \tag{15}$$

It should be noted that the initial shape $I - R$ is the target for the optimization problem (14). Nevertheless we can say the following:

Remark 2: The high-pass shape $I - R$ can not be chosen arbitrarily: if $I - R$ is such that $\bar{\sigma}(R) \geq 1/\underline{\sigma}(W_1)$, the optimization problem would have to try very hard to fulfill the constraint $\left\|-W_1 M_r R Y_r N_r M_r^{-1}\right\|_\infty < 1$ and this fact would be in expenses of $\|W_p(I - N_r R Y_y)\|_\infty$ which would not be, probably, minimized to a sufficient degree. Therefore, the high-pass shape S^d may be chosen such that

$$\bar{\sigma}(R) < 1/\underline{\sigma}(W_1) \tag{16}$$

Next, we briefly describe the important points in the feedback design procedure.

- **1.1. Uncertainty description:** Assume that the plant P to be controlled is unknown but belonging to a set of plants $\mathcal{P}$ build around the nominal model P_o, as in (7).
- **1.2. Initial factorization:** Provide $p_{F_{i0}} \in \mathcal{C}^-$ and $p_{L_{i0}} \in \mathcal{C}^-$, i.e., initial stable eigenvalues for the right coprime factorization of the nominal model of the plant, $P_o = M_r^{-1} N_r$, and initial stable eigenvalues for the associated Bezout components X_r, Y_r, respectively.
- **1.3. Desired sensitivity:** Set the *desired* high-pass shape $I - R$ using equation (13) and equation (10).
- **1.4. Constrained optimization problem:** Solve the constrained optimization problem (14) with $p_{F_{i0}}$ and $p_{L_{i0}}$ as initial guesses.
- **1.5. Design of K_1:** Recover the feedback controller K_1 by means of the design equation (15), with the established R and the *optimum* solution M_r .
- **1.6. Analysis:** Analyze and redesign, if necessary, making adjustments on $I - R$, $p_{F_{i0}}$ and $p_{L_{i0}}$ as initial guesses and, possibly, W_1.
- **1.7. Implementation:** Implement the feedback part of the controller scheme shown in Figure 2 with the recovered K_1 and the *optimum* right coprime solutions N_r, M_r, X_r and Y_r.

B. Reference controller design

The reference controller K_2 is in charge of adapting the reference signal r and providing the *desired* partial state ξ^d. A simple constant prefilter allows for the commands, r, to enter the feedback control scheme keeping the steady-state accuracy. The resultant step response dynamics are those fixed by the feedback controller structure. Note that in the nominal case, i.e., $P = P_o$, the prefilter controller K_2 sees just $N_r R$. Therefore, the relation from the references r to the outputs y reads as $y = N_r R K_2\, r$.

It should be noted that the right coprime factor N_r contains the zeros of P_o and the poles p_{F_i} achieved with the *optimum* pole placement; R has been fixed as a design parameter. However, for many tracking problems this will not be sufficient and a dynamic design for the prefilter K_2 is required.

To include different and independent dynamics for the step response, we have to take advantage of the second degree of freedom that K_2 provides. Therefore, the prefilter controller K_2 is designed to force the response of the closed-loop system for all plants $P \in \mathcal{P}$ described in (7) to follow that of a specified model, T_{ref}, often called the reference model. The overall design problem is shown in Figure 6.

Let us assume that T_{ref} is the desired closed-loop transfer matrix function selected to introduce the desired step response characteristics into the design process (time-domain specifications). Then, the design problem is to find a reference controller K_2 so that the relation form the reference, r, to the output, y, behave as close as possible to that of the reference model, T_{ref}. The design of the reference controller will depend upon the performance criterion chosen for the desired closeness to the reference model T_{ref}. We address the problem to optimal Model Reference controller specifications in an ∞-norm sense. This way, the design of the reference controller turns out to be a Model Matching Problem [4].

From the overall scheme in Figure 6, we can compute the transfer matrix function $\tilde{\mathcal{N}}$ that relates the inputs $[d_i\ r]^T$, i.e., the disturbances d_i and the command signal r, with the outputs $[u'\ e_2']^T$, i.e., the weighted control signal u' and the weighted Model Matching error, e_2',

$$\tilde{\mathcal{N}}=\begin{bmatrix} -W_1 M_r R Y_r N_r M_r^{-1} & \alpha W_1 M_r R K_2 \\ \alpha(N_r(I-R)M_r^{-1}+N_r R X_r) & \alpha^2(N_r R K_2 - T_{ref}) \end{bmatrix} \tag{17}$$

Remark 3: The relations from the disturbances signal, d_i, are not dependent on the prefilter controller K_2. Moreover, if $r=0$ and $\alpha=1$ we have that $e_2'=y$. Then, the relations from d_i are the same as those computed in the first step of the design, i.e. the feedback controller structure, as it can be seen in equation (8).

The scaling parameter α is used to place more attention in model matching, i.e., the (2,2) block, rather than in transfer matrix function from r to u', i.e., the (1,2) block.

In view of Remark 3, the $\mathcal{H}_\infty$ norm of the complete transfer matrix function (17) is minimized to find K_2 without modifying the robust stability margins provided by the feedback controller scheme in the first step of the design, i.e., the (1,2) block.

The 2-DOF design problem shown in Figure 6 can be easily cast into the general control configuration [3] shown in Figure 5(a).

The interconnection matrix G in Figure 5(a) contains the feedback control scheme with the *optimum* right coprime factors N_r, M_r^{-1}, X_r and Y_r; the controller K_1; the uncertainty weights W_1, W_2 and the scaling factors αI. Comparing

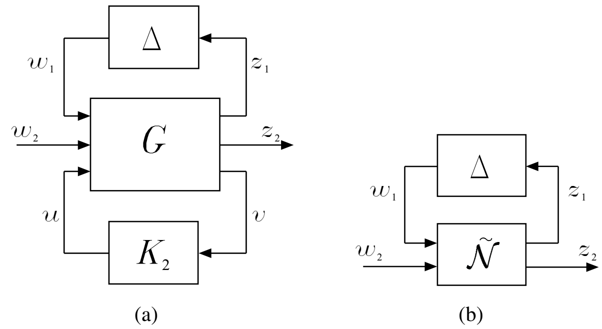

Fig. 5. General interconnection of system with uncertainty.

Figure 6 with the scheme in Figure 5(a) we define $w_1=d_i$, $w_2=r$, $z_1=u'$, $z_2=e_2'$, $v=\beta$ and $u=\xi^d$. The augmented plant G and the controller K_2 are related by the following lower linear fractional transformation:

$$\mathcal{F}_\ell(G,K_2) \doteq G_{11}+G_{12}K_2(I-G_{22}K_2)^{-1}G_{21} \tag{18}$$

The lower LFT (18) is represented in Figure 5(b) by the matrix transfer function $\tilde{\mathcal{N}}$, calculated in (17).

The partitioned generalized plant G is:

$$\begin{bmatrix} u' \\ e_2' \\ \hline \beta \end{bmatrix} = \left[\begin{array}{c|c} G_{11} & G_{12} \\ \hline G_{21} & G_{22} \end{array}\right] \begin{bmatrix} d_i \\ r \\ \hline \xi^d \end{bmatrix} = \tag{19}$$

$$= \left[\begin{array}{cc|c} -W_1 M_r R Y_r N_r M_r^{-1} & 0 & W_1 M_r R \\ N_r(I-R)M_r^{-1}+N_r R X_r & -\alpha^2 T_{ref} & \alpha N_r R \\ \hline 0 & \alpha I & 0 \end{array}\right] \begin{bmatrix} d_i \\ r \\ \hline \xi^d \end{bmatrix}$$

The generalized plant (19) can to be solved suboptimally for the prefilter controller K_2 using standard algorithms and γ-iteration [2]. The purpose of K_2 is to ensure that

$$\|N_r R K_2 - T_{ref}\|_\infty < \gamma \alpha^{-2} \tag{20}$$

Remark 4: The reference signals r must be scaled by a constant matrix W_r to make the closed-loop transfer function from r to the controlled output y match the desired reference model T_{ref} exactly at steady-state. This is not guaranteed by the optimization problem which aims to minimize the ∞-norm of the error, i.e., $\left\|\alpha^2(N_r R K_2 - T_{ref})\right\|_\infty$. The required scaling is given by

$$W_r \doteq [K_2(0)N_r(0)R(0)]^{-1}T_{ref}(0) \tag{21}$$

Therefore, the resulting reference controller is $K_2 W_r$.

The important points in the reference controller design procedure are briefly described.

2.1. Reference Model: Select a desired closed-loop transfer function, T_{ref}, between the reference, r, and the output controlled signals, y .

2.2. α **selection:** Select this scalar tuning parameter to place more emphasis on the model matching problem, block (2,2) on equation (17).

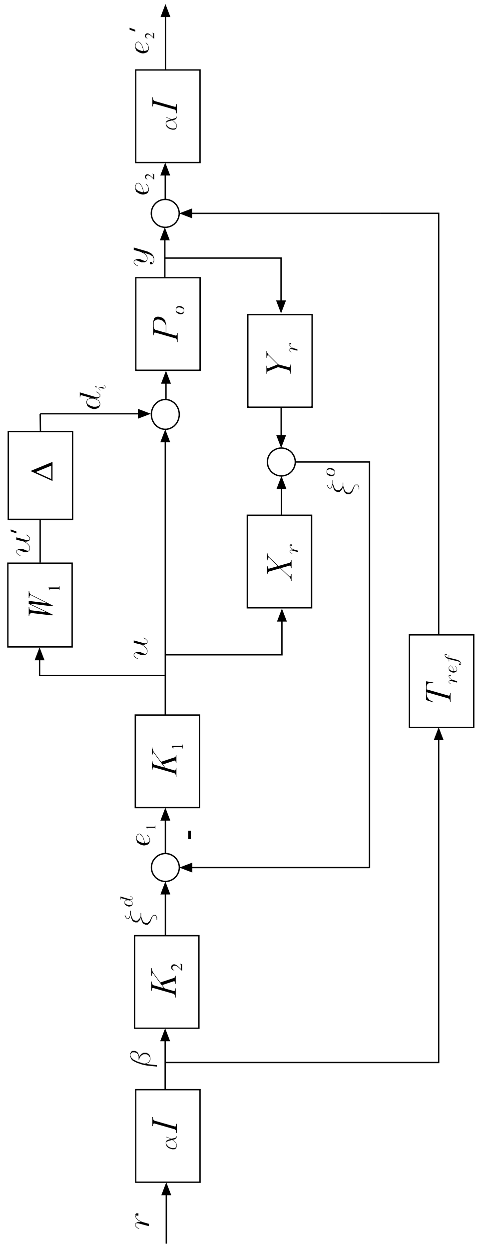

Fig. 6. The 2-DOF design problem.

2.3. General Control Configuration: Put de 2-DOF design problem into the general control interconnection and define the generalized plant G as equation (19).

2.4. Design of K_2: Synthesize K_2 by solving the uncertain generalized plant $\mathcal{F}_u(G, \Delta)$.

2.5. Steady-state gain matching: Replace the prefilter controller by K_2W_r to have exact model-matching at steady-state.

2.6. Analysis: Analyze and redesign, if necessary, making use of α and/or T_{ref}.

2.7. Implementation: Build the prefilter part of the controller scheme shown in Figure 2.

V. CONCLUSIONS

A new 2-DOF control configuration based on a right coprime factorization of the model of the plant has been presented in this paper. The approach can be seen as an alternative to the common strategy of setting the two controllers arbitrarily, with internal stability the only restriction, and parameterizing the controller in terms of the Youla parameter.

The resultant two-step design procedure, starts with an observer-based feedback control scheme which is designed first to guarantee robust stability and nominal disturbance rejection. The final feedback controller K_1 is designed in an indirect way by solving an $\mathcal{H}_\infty$ constrained optimization problem for the eigenvalues of the right coprime factors N_r, M_r, X_r and Y_r. Such an uncommon feedback structure give an important benefit in which the possible RHP-zeros of the plant are not introduced into the feedback loop, preventing to cause closed-loop instability.

The commands dynamics provided by the feedback control configuration, $y = N_r R \xi^d$, can be assumed whereas an static prefilter controller is found to adapt the reference command, i.e., $\xi^d = K_2 r$, and to provide input/output unity steady-state gain. Nevertheless, we have designed a prefilter controller to provide extra input/output dynamics to those given by the feedback control structure and it has been designed to provide robust model reference responses.

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