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Remote Stabilization of Networked Control Systems with Unknown Time Varying Delays by LMI Techniques

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Abstract— **In this paper, the stabilization of one class of remote control systems with unknown time varying delays is analyzed and discussed using LMI techniques. A discrete time state space model under a static control law for remote control systems is first introduced based on some assumptions on the uncertain term. The time delay is unknown, time varying, and can be decomposed into two parts: one fixed part which is unknown and is an integer multiple of the sampling time; the other part which is randomly varying but bounded by one sampling time. Static controller designs based on delay dependent stability conditions are presented. This system is then extended to a more general case when the randomly varying part of the time delay is not limited to one sampling time. The derivative of the time delay is not limited to be bounded. Hence, the contributions are as follows: i) for a given controller, we can use these stability criteria to test stability of the resulted system; ii) we can design a remote controller to stabilize an unstable system. Finally, a simulation example is presented to demonstrate the remote stabilization of open loop unstable systems.**

I. INTRODUCTION

Over the past few years, increasing attention has been drawn to the control of time delay systems Stability problem is one of the most important issues in time delay systems since delays caused by transmission may result in instability, especially when there are uncertainties [1]-[3].

Many delay independent and delay dependent stability criteria, which are mainly concerning continuous systems, have been presented [4]. However, most practical applications of remote controllers through network transmission are implemented digitally. Up until now, little attention has been paid to discrete time systems with delays and controlled in a remote way. One possible reason is that, for a known delay, the delay difference equation can be rewritten to be a high order system without delay by augmentation [5]. However, in case with large known delay, this augmentation approach can lead to high dimensional systems; and moreover, it is not applicable for systems with unknown delays.

In several references [6]-[7], no input delay was considered in the problem formulation. This simplifies the design of complex controllers for robust stabilization. In [8], two delay dependent conditions were developed for discrete time systems, but no uncertainties were considered in the system parameters. Based on the above analysis, in this paper, we further look into the stability problem of remote control systems with unknown time varying delay in the control input in the discrete time domain. In this case, the existing theory using augmentation cannot be applied because of the unknown time varying delay. One approach is to decompose the time delay into two parts: one fixed part which is unknown and is an integer multiple of the sampling time; the other part which is randomly varying but bounded by one sampling time. Then the stability condition of the discrete time state space model under a static control law for the remote control system is first introduced based on a certain assumption on the uncertain term.

This system is then extended to a more general case when the randomly varying part of the time delay has a bound which is more than one sampling time. Delay dependent stability conditions are presented for the static controller design by using LMI techniques. Delay dependent conditions based on linear matrix inequalities (LMIs) are derived by introducing some slack matrix variables which are less conservative than the conventional approaches [9]. A similar approach for continuous time systems is presented in [10]. Accordingly, we can design a static remote controller to stabilize a given open loop unstable system if there exists solutions from the derived LMIs. There is no assumption on the variation rate of the time delay, i.e., the derivative of the time delay is not necessary to be bounded. Another advantage is that for a given controller, we can use these stability criteria to test stability of the resulted closed loop systems. In this paper, the analysis on the discrete time system and the static controller design when the delay variation part is less than one sampling time is presented. In Section IV, the system description of the general case and the controller design are analyzed when the delay variation part is longer than one sampling time. Notations: $\mathcal{R}^n$ denotes an n-dimensional real vector space; $\|\cdot\|$ is the Euclidean norm or induced matrix norm.

II. PROBLEM FORMULATION

The following system with uncertain delay is considered

$$\dot{\mathbf{x}}(t) = A_p\mathbf{x}(t) + B_p\mathbf{u}(t - \tau_2(t)), \tag{1}$$

where $\mathbf{x} \in \mathcal{R}^n$ is the measurable state vector, $\mathbf{u} \in \mathcal{R}^m$ denotes the remote control signal, $A_p \in \mathcal{R}^{n\times n}$ and $B_p \in \mathcal{R}^{n\times m}$ are known constant matrices. The system in (1) can be illustrated as in $Fig.1$, where $\tau_1(t)$ is the time delay from the system sensor to the remote controller and $\mathbf{u}_c \in \mathcal{R}^m$ is

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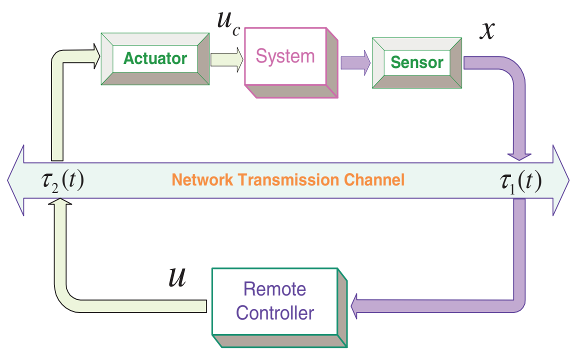

Fig. 1. Block diagram of the remote control systems

the control input to the physical system. In this paper, the sensor is assumed to be time driven. The controller and the actuator are assumed to be event driven. The assumption on the time delays is as follows.

Assumption 1: The time delays in the two channels can be represented as $\tau_i(t) = h_i T_s + \varepsilon_i(t)$, $i = 1, 2$, where T_s is the sampling time and $\varepsilon_i(t)$ is unknown but is bounded as $0 \leq \varepsilon_i(t) < L_i T_s$ with $L_i \ll h_i$, where L_i is a known integer. h_i is an unknown integer constant and is assumed to be: $h_i \in [0, \bar{h}_i]$, where $\bar{h}_i$ is known.

Remark 1: Note that the controller and the actuator are event driven; thus there is no holding-up on the controller (pure gain) and actuator side. The total delay from the sensor to the actuator can be represented as $\tau(t) = \tau_2(t) + \tau_1(t - \tau_2(t))$. According to Assumption 1, similarly $\tau(t)$ can be rewritten as $\tau(t) = hT_s + \varepsilon(t)$, with $h \in [h_1 + h_2, h_1 + h_2 + 1]$ being an unknown integer and $\varepsilon(t)$ being unknown but bounded with $0 \leq \varepsilon(t) < LT_s$ and $L < h \leq \bar{h}$, where L and $\bar{h}$ are known integers.

Here we first consider the case when $L = 1$ which means that the randomly varying part of the time delay is bounded by one sampling time. The case when $h > L > 1$ is extended in Section IV.

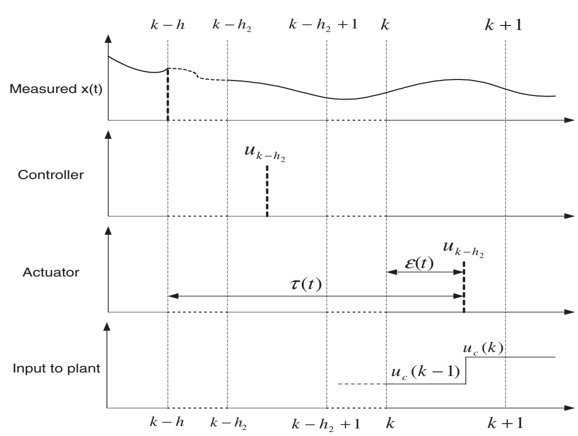

Fig. 2. Timing flow of the control system

The timing flow of the control system is shown in Fig.2. Suppose the measured state $\mathbf{x}_{k-h}$ is sent at time $(k-h)T_s$ from the sensor, it arrives at the controller in the sampling interval $[(k-h_2)T_s, (k-h_2+1)T_s)$ according to the delay structure in Assumption 1. At the controller side, the control signal is then denoted as $\mathbf{u}_{k-h_2} = K\mathbf{x}_{k-h}$ if a static gain is designed and the sampled state arrives at the controller at time $t \in [(k-h_2)T_s, (k-h_2+1)T_s)$. This control signal arrives at the actuator at time $t = kT_s + \varepsilon(t)$.

Integrating the open loop system in the interval $[kT_s, (k+1)T_s]$, and using Fig.2, we have

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + (B + \Delta B_0)\mathbf{u}_{k-h_2} + (B + \Delta B_1)\mathbf{u}_{k-h_2-1} \tag{2}$$

where $A = e^{A_p T_s}$, $B = \int_0^{T_s} e^{A_p s} ds B_p$, $\Delta B_0 = -\int_{\varepsilon(k)}^{T_s} e^{A_p s} ds B_p$, $\Delta B_1 = -\int_0^{\varepsilon(k)} e^{A_p s} ds B_p$ and $\Delta B_0 + \Delta B_1 = -B$.

Because there are uncertainties in the time delay, we cannot use the prediction method to stabilize the system which requires accurate information of the delay in order to predict the future states. Hence a static controller is designed as $\mathbf{u}(t) = K\mathbf{x}(t - \tau_1(t))$ where $\tau_1(t)$ is the delay from the sensor to the controller in the continuous time domain. The main task here is how to design a proper K such that the closed loop control system is stable. Furthermore, as previously discussed in Fig.2, in discrete time domain we have $\mathbf{u}_{k-h_2} = K\mathbf{x}_{k-h}$. Then (2) becomes

$$\begin{aligned}\mathbf{x}_{k+1} &= A\mathbf{x}_k + (B + \Delta B_0)K\mathbf{x}_{k-h} \\ &\quad + (B + \Delta B_1)K\mathbf{x}_{k-h-1},\end{aligned} \tag{3}$$

where A and B can be derived directly from A_p and B_p in (1). Based on the above analysis, though ΔB_i, $i = 0, 1$, are uncertain, embedding them into standard uncertain terms would allow us to treat the problem as a robust stabilization problem. Hence in this paper, we assume that ΔB_i satisfies the following assumption, which is a commonly used condition for most of the existing approaches for uncertain time delay systems [1] [9].

Assumption 2: ΔB_i can be expressed as $\Delta B_i = E\Gamma_i(k)F_i$, $i = 0, 1$, where E, F_i are known with appropriate dimensions and $\Gamma_i(k)^T \Gamma_i(k) < I$.

Note that in Assumption 2, there always exist E and F_i such that $\Gamma_i(k)^T \Gamma_i(k) < I$ for ΔB_i, $i = 0, 1$. However, for a specific system pair (A_p, B_p) and sampling time T_s, we can use numerical methods to get the matrices E and F_i which are required to be known for the controller design.

Lemma 1: [9] Let Σ_1 and Σ_2 be real constant matrices of compatible dimensions, and $H(t)$ be a real matrix function satisfying $H(t)^T H(t) \leq I$. Then the inequality holds

$$\Sigma_1^T H(t) \Sigma_2 + \Sigma_2^T H(t)^T \Sigma_1 \leq \varepsilon \Sigma_1^T \Sigma_1 + \varepsilon^{-1} \Sigma_2^T \Sigma_2, \tag{4}$$

where ε is a positive constant.

III. CONTROLLER DESIGN

Rewrite the system in (3) as

$$\mathbf{x}_{k+1} = A\mathbf{x}_k + A_{0\Delta}\mathbf{x}_{k-h} + A_{1\Delta}\mathbf{x}_{k-h-1}, \tag{5}$$

where $A_{0\Delta} \triangleq BK + \Delta B_0 K = A_0 + \Delta_0$ with $A_0 = BK$ and $\Delta_0 = \Delta B_0 K$, and $A_{1\Delta} \triangleq BK + \Delta B_1 K = A_1 + \Delta_1$ with

$A_1 = BK$ and $\Delta_1 = \Delta B_1 K$. For any matrices N_i, S_i and M_i $(i = 1, 2, 3, 4)$ of appropriate dimensions, the following equations hold:

$$\begin{aligned} \Psi_1 &= 2\left[\mathbf{x}_k^T N_1 + \mathbf{x}_{k-h}^T N_2 + \mathbf{x}_{k-h-1}^T N_3 + \mathbf{x}_{k+1}^T N_4\right] \\ &\quad \left[\mathbf{x}_k - \mathbf{x}_{k-h} - \sum_{j=k-h+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})\right] = 0, \end{aligned} \quad (6)$$

$$\begin{aligned} \Psi_2 &= 2\left[\mathbf{x}_k^T S_1 + \mathbf{x}_{k-h}^T S_2 + \mathbf{x}_{k-h-1}^T S_3 + \mathbf{x}_{k+1}^T S_4\right] \\ &\quad \left[\mathbf{x}_k - \mathbf{x}_{k-h-1} - \sum_{j=k-h}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})\right] = 0, \end{aligned} \quad (7)$$

$$\begin{aligned} \Psi_3 &= 2\left[\mathbf{x}_k^T M_1 + \mathbf{x}_{k-h}^T M_2 + \mathbf{x}_{k-h-1}^T M_3 + \mathbf{x}_{k+1}^T M_4\right] \\ &\quad [\mathbf{x}_{k+1} - A\mathbf{x}_k - A_{0\Delta}\mathbf{x}_{k-h} - A_{1\Delta}\mathbf{x}_{k-h-1}] = 0. \end{aligned} \quad (8)$$

Based on the Lyapunov functional analysis while combining the equations (6)-(8) in the derivation, first we have the following proposition.

Proposition 1: For a given control gain K and a given upperbound $\bar{h}$ of h, the system in (3) is asymptotically stable if there exist symmetric positive definite matrices $P, Q_1, Q_2, Z_1, Z_2 \in \mathcal{R}^{n\times n}$, and matrices N_i, S_i, M_i, $i = 1, 2, 3, 4$ with appropriate dimensions such that the following inequality holds,

$$\begin{bmatrix} D_{11} & * & * & * & * & * \\ D_{21} & D_{22} & * & * & * & * \\ D_{31} & D_{32} & D_{33} & * & * & * \\ D_{41} & D_{42} & D_{43} & D_{44} & * & * \\ \bar{h}N_1^T & \bar{h}N_2^T & \bar{h}N_3^T & \bar{h}N_4^T & -\bar{h}Z_1 & * \\ S_1^T & S_2^T & S_3^T & S_4^T & 0 & -(\bar{h}+1)^{-1}Z_2 \end{bmatrix} < 0, \quad (9)$$

where

$$\begin{aligned} D_{11} &= -P + \bar{h}Z_1 + Q_1 + (\bar{h}+1)Z_2 + Q_2 + N_1 \\ &\quad + N_1^T + S_1 + S_1^T - M_1 A - A^T M_1^T, \\ D_{21} &= N_2 - N_1^T + S_2 - M_2 A - A_{0\Delta}^T M_1^T, \\ D_{31} &= N_3 - S_1^T + S_3 - M_3 A - A_{1\Delta}^T M_1^T, \\ D_{32} &= -N_3 - S_3^T - M_3 A_{0\Delta} - A_{1\Delta}^T M_2^T, \\ D_{22} &= -Q_1 - N_2 - N_2^T - M_2 A_{0\Delta} - A_{0\Delta}^T M_2^T, \\ D_{33} &= -Q_2 - S_3 - S_3^T - M_3 A_{1\Delta} - A_{1\Delta}^T M_3^T, \\ D_{41} &= -\bar{h}Z_1 - (\bar{h}+1)Z_2 + N_4 + S_4 - M_4 A + M_1^T, \\ D_{42} &= -N_4 - M_4 A_{0\Delta} + M_2^T, \\ D_{43} &= -S_4 - M_4 A_{1\Delta} + M_3^T, \\ D_{44} &= P + \bar{h}Z_1 + (\bar{h}+1)Z_2 + M_4 + M_4^T. \end{aligned}$$

Proof: Consider the following Lyapunov function for the discrete system in (3),

$$\begin{aligned} V_k &= V_{1,k} + V_{2,k} + V_{3,k} + V_{4,k} + V_{5,k} \\ &\triangleq \mathbf{x}_k^T P \mathbf{x}_k + \sum_{i=-h}^{-1} \sum_{j=k+i+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})^T Z_1 (\mathbf{x}_j - \mathbf{x}_{j-1}) \\ &\quad + \sum_{j=k-h}^{k-1} \mathbf{x}_j^T Q_1 \mathbf{x}_j + \sum_{j=k-h-1}^{k-1} \mathbf{x}_j^T Q_2 \mathbf{x}_j \\ &\quad + \sum_{i=-h-1}^{-1} \sum_{j=k+i+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})^T Z_2 (\mathbf{x}_j - \mathbf{x}_{j-1}). \end{aligned}$$

Then we have

$$\begin{aligned} \Delta V_{1,k} &= \mathbf{x}_{k+1}^T P \mathbf{x}_{k+1} - \mathbf{x}_k P \mathbf{x}_k, \\ \Delta V_{2,k} &\le \bar{h}(\mathbf{x}_{k+1} - \mathbf{x}_k)^T Z_1 (\mathbf{x}_{k+1} - \mathbf{x}_k) \\ &\quad - \sum_{j=k-h+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})^T Z_1 (\mathbf{x}_j - \mathbf{x}_{j-1}), \\ \Delta V_{3,k} &= \mathbf{x}_k^T Q_1 \mathbf{x}_k - \mathbf{x}_{k-h}^T Q_1 \mathbf{x}_{k-h}, \\ \Delta V_{4,k} &= \mathbf{x}_k^T Q_2 \mathbf{x}_k - \mathbf{x}_{k-h-1}^T Q_2 \mathbf{x}_{k-h-1}, \\ \Delta V_{5,k} &\le (\bar{h}+1)(\mathbf{x}_{k+1} - \mathbf{x}_k)^T Z_2 (\mathbf{x}_{k+1} - \mathbf{x}_k) \\ &\quad - \sum_{j=k-h}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})^T Z_2 (\mathbf{x}_j - \mathbf{x}_{j-1}). \end{aligned}$$

Defining the following new variables

$$\begin{aligned} \mathbf{z} &\triangleq [\mathbf{x}_k, \mathbf{x}_{k-h}, \mathbf{x}_{k-h-1}, \mathbf{x}_{k+1}]^T, N \triangleq [N_1, N_2, N_3, N_4]^T, \\ S &\triangleq [S_1, S_2, S_3, S_4]^T, M \triangleq [M_1, M_2, M_3, M_4]^T, \end{aligned}$$

and combining (6)-(8), we have that ΔV_k becomes

$$\begin{aligned} \Delta V_k &= \Delta V_{1,k} + \Delta V_{2,k} + \Delta V_{3,k} + \Delta V_{4,k} \\ &\quad + \Delta V_{5,k} + \Psi_1 + \Psi_2 + \Psi_3 \\ &\le \mathbf{x}_{k+1}^T P \mathbf{x}_{k+1} - \mathbf{x}_k P \mathbf{x}_k + \mathbf{x}_k^T Q_1 \mathbf{x}_k \\ &\quad + \mathbf{x}_k^T Q_2 \mathbf{x}_k - \mathbf{x}_{k-h-1}^T Q_2 \mathbf{x}_{k-h-1} - \mathbf{x}_{k-h}^T Q_1 \mathbf{x}_{k-h} \\ &\quad + \bar{h}(\mathbf{x}_{k+1} - \mathbf{x}_k)^T Z_1 (\mathbf{x}_{k+1} - \mathbf{x}_k) \\ &\quad - \sum_{j=k-h+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})^T Z_1 (\mathbf{x}_j - \mathbf{x}_{j-1}) \\ &\quad + (\bar{h}+1)(\mathbf{x}_{k+1} - \mathbf{x}_k)^T Z_2 (\mathbf{x}_{k+1} - \mathbf{x}_k) \\ &\quad - \sum_{j=k-h}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})^T Z_2 (\mathbf{x}_j - \mathbf{x}_{j-1}) \\ &\quad + 2\mathbf{z}^T N \left[\mathbf{x}_k - \mathbf{x}_{k-h} - \sum_{j=k-h+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})\right] \\ &\quad + 2\mathbf{z}^T S \left[\mathbf{x}_k - \mathbf{x}_{k-h-1} - \sum_{j=k-h}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})\right] \\ &\quad + 2\mathbf{z}^T M [\mathbf{x}_{k+1} - A\mathbf{x}_k - A_{0\Delta}\mathbf{x}_{k-h} \\ &\quad - A_{1\Delta}\mathbf{x}_{k-h-1}]. \end{aligned} \quad (10)$$

Moreover,

$$\begin{aligned} -2\mathbf{z}^T N \sum_{j=k-h+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1}) &\le \bar{h}\mathbf{z}^T N Z_1^{-1} N^T \mathbf{z} \\ + \sum_{j=k-h+1}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1})^T Z_1 (\mathbf{x}_j - \mathbf{x}_{j-1}), & \end{aligned} \quad (11)$$

$$-2\mathbf{z}^T S \sum_{j=k-h}^{k} (\mathbf{x}_j - \mathbf{x}_{j-1}) \le (\bar{h}+1)\mathbf{z}^T S Z_2^{-1} S^T \mathbf{z}$$

$$+\sum_{j=k-h}^{k}(\mathbf{x}_j-\mathbf{x}_{j-1})^T Z_2(\mathbf{x}_j-\mathbf{x}_{j-1}). \tag{12}$$

Using (11) and (12), (10) can be written as

$$\begin{aligned}
\Delta V_k \leq\ & \mathbf{x}_{k+1}^T P\mathbf{x}_{k+1}-\mathbf{x}_k P\mathbf{x}_k-\mathbf{x}_{k-h-1}^T Q_2\mathbf{x}_{k-h-1} \\
& +\mathbf{x}_k^T Q_2\mathbf{x}_k+\mathbf{x}_k^T Q_1\mathbf{x}_k-\mathbf{x}_{k-h}^T Q_1\mathbf{x}_{k-h} \\
& +2\mathbf{z}^T N(\mathbf{x}_k-\mathbf{x}_{k-h})+2\mathbf{z}^T S(\mathbf{x}_k-\mathbf{x}_{k-h-1}) \\
& +\bar{h}(\mathbf{x}_{k+1}-\mathbf{x}_k)^T Z_1(\mathbf{x}_{k+1}-\mathbf{x}_k) \\
& +(\bar{h}+1)(\mathbf{x}_{k+1}-\mathbf{x}_k)^T Z_2(\mathbf{x}_{k+1}-\mathbf{x}_k) \\
& +2\mathbf{z}^T M\left[\mathbf{x}_{k+1}-A\mathbf{x}_k-A_{0\Delta}\mathbf{x}_{k-h}\right. \\
& \left.-A_{1\Delta}\mathbf{x}_{k-h-1}\right] \\
& +\bar{h}\mathbf{z}^T N Z_1^{-1}N^T\mathbf{z}+(\bar{h}+1)\mathbf{z}^T S Z_2^{-1}S^T\mathbf{z} \\
=\ & \mathbf{z}^T\left\{\begin{bmatrix} D_{11} & * & * & * \\ D_{21} & D_{22} & * & * \\ D_{31} & D_{32} & D_{33} & * \\ D_{41} & D_{42} & D_{43} & D_{44}\end{bmatrix}\right. \\
& \left.+\bar{h}NZ_1^{-1}N^T+(\bar{h}+1)SZ_2^{-1}S^T\right\}\mathbf{z},
\end{aligned}$$

where $\bar{h}$ is the upper bound of h. By using the Schur complement, then $\Delta V_k<0$ holds if the following condition is satisfied

$$\begin{bmatrix} D_{11} & * & * & * \\ D_{21} & D_{22} & * & * \\ D_{31} & D_{32} & D_{33} & * \\ D_{41} & D_{42} & D_{43} & D_{44}\end{bmatrix} +\bar{h}NZ_1^{-1}N^T+(\bar{h}+1)SZ_2^{-1}S^T<0,$$

As a result, (13) means that (9) holds. Furthermore by using the Lyapunov-Krasovskii stability theorem [11], $\Delta V_k<0$ means that the system in (3) is asymptotically stable. ■

Equation (9) contains the uncertainties Δ_0 and Δ_1. Thus we cannot use it directly to check the system stability. The following proposition is given as a sufficient condition for the feasibility of (9) by dealing with the uncertainties in the inequality. First we can represent the uncertainties as

$$\begin{bmatrix}\Delta_0 & \Delta_1\end{bmatrix}=E\begin{bmatrix}\Gamma_0 & \Gamma_1\end{bmatrix}\begin{bmatrix}F_0K & 0 \\ 0 & F_1K\end{bmatrix}\triangleq E\Gamma G \tag{13}$$

with $\Gamma=\begin{bmatrix}\Gamma_0 & \Gamma_1\end{bmatrix}$ and $\Gamma^T\Gamma<I$, and $G=\begin{bmatrix}G_0 & G_1\end{bmatrix}=\begin{bmatrix}F_0K & 0 \\ 0 & F_1K\end{bmatrix}$.

Proposition 2: For a given control gain K and a given upperbound $\bar{h}$ of h, the system (3) is asymptotically stable if there exist symmetric positive definite matrices $P,Q_1,Q_2,Z_1,Z_2\in\mathcal{R}^{n\times n}$, matrices N_i,S_i,M_i, $i=1,2,3,4$ with appropriate dimensions and constant $\rho>0$ such that the following inequality holds,

$$\begin{bmatrix}
D'_{11} & * & * & * & * & * & * & * & * \\
D'_{21} & D'_{22} & * & * & * & * & * & * & * \\
D'_{31} & D'_{32} & D'_{33} & * & * & * & * & * & * \\
D'_{41} & D'_{42} & D'_{43} & D'_{44} & * & * & * & * & * \\
N_1^T & N_2^T & N_3^T & N_4^T & \frac{-Z_1}{\bar{h}} & * & * & * & * \\
S_1^T & S_2^T & S_3^T & S_4^T & 0 & \frac{-Z_2}{\bar{h}+1} & * & * & * \\
H_1^T & H_2^T & H_3^T & H_4^T & 0 & 0 & -\rho I & * & * \\
0 & \rho F_0K & 0 & 0 & 0 & 0 & 0 & -\rho I & * \\
0 & 0 & \rho F_1K & 0 & 0 & 0 & 0 & 0 & -\rho I
\end{bmatrix}<0, \tag{14}$$

where

$$\begin{aligned}
D'_{11} &= -P+\bar{h}Z_1+Q_1+(\bar{h}+1)Z_2+Q_2+N_1 \\
&\quad +N_1^T+S_1+S_1^T-M_1A-A^TM_1^T, \\
D'_{21} &= N_2-N_1^T+S_2-M_2A-K^TB^TM_1^T, \\
D'_{31} &= N_3-S_1^T+S_3-M_3A-K^TB^TM_1^T, \\
D'_{32} &= -N_3-S_3^T-M_3BK-K^TB^TM_2^T, \\
D'_{22} &= -Q_1-N_2-N_2^T-M_2BK-K^TB^TM_2^T, \\
D'_{33} &= -Q_2-S_3-S_3^T-M_3BK-K^TB^TM_3^T, \\
D'_{41} &= -\bar{h}Z_1-(\bar{h}+1)Z_2+N_4+S_4-M_4A+M_1^T, \\
D'_{42} &= -N_4+M_2^T-M_4BK, \\
D'_{43} &= -S_4+M_3^T-M_4BK, \\
D'_{44} &= D_{44}=P+\bar{h}Z_1+(\bar{h}+1)Z_2+M_4+M_4^T, \\
H_1 &= M_1E,\ H_2=M_2E,\ H_3=M_3E,\ H_4=M_4E.
\end{aligned}$$

Proof: The uncertainty part in (9) can be dealt with by using the property in Lemma 1. The left side of the inequality in (9) can be further represented as

$$\begin{bmatrix}
D'_{11} & * & * & * & * & * \\
D'_{21} & D'_{22} & * & * & * & * \\
D'_{31} & D'_{32} & D'_{33} & * & * & * \\
D'_{41} & D'_{42} & D'_{43} & D'_{44} & * & * \\
N_1^T & N_2^T & N_3^T & N_4^T & -\bar{h}^{-1}Z_1 & * \\
S_1^T & S_2^T & S_3^T & S_4^T & 0 & -(\bar{h}+1)^{-1}Z_2
\end{bmatrix}+\Psi_4,$$

where

$$\begin{aligned}
\Psi_4 &= -\begin{bmatrix}0 \\ A_{0\Delta}^T \\ A_{1\Delta}^T \\ 0\end{bmatrix}\begin{bmatrix}M_1^T & M_2^T & M_3^T & M_4^T\end{bmatrix} \\
&\quad -\begin{bmatrix}M_1 \\ M_2 \\ M_3 \\ M_4\end{bmatrix}\begin{bmatrix}0 & A_{0\Delta} & A_{1\Delta} & 0\end{bmatrix} \\
&\leq \rho^{-1}ME(ME)^T+\rho\begin{bmatrix}0 \\ G_0^T \\ G_1^T \\ 0\end{bmatrix}\begin{bmatrix}0 & G_0 & G_1 & 0\end{bmatrix}.
\end{aligned} \tag{15}$$

Using (15) and Schur complement, it is obvious to obtain that the following condition is a sufficient condition of (9),

$$\begin{bmatrix}
D'_{11} & * & * & * & * & * & * & * & * \\
D'_{21} & D'_{22} & * & * & * & * & * & * & * \\
D'_{31} & D'_{32} & D'_{33} & * & * & * & * & * & * \\
D'_{41} & D'_{42} & D'_{43} & D'_{44} & * & * & * & * & * \\
N_1^T & N_2^T & N_3^T & N_4^T & \frac{-Z_1}{\bar{h}} & * & * & * & * \\
S_1^T & S_2^T & S_3^T & S_4^T & 0 & \frac{-Z_2}{\bar{h}+1} & * & * & * \\
H_1^T & H_2^T & H_3^T & H_4^T & 0 & 0 & -\rho I & * & * \\
0 & \rho F_0K & 0 & 0 & 0 & 0 & 0 & -\rho I & * \\
0 & 0 & \rho F_1K & 0 & 0 & 0 & 0 & 0 & -\rho I
\end{bmatrix}<0, \tag{16}$$

where $H_1=M_1E, H_2=M_2E, H_3=M_3E, H_4=M_4E$. ■

Based on the analysis in Propositions 1 and 2, we are now able to design the gain K which can ensure the asymptotical stability of the networked control system in (3).

Theorem 1: For given scalars $\theta_i, i = 1, 2, 3, 4$, and a given upperbound $\bar{h}$ of h, if there exist symmetric positive definite matrices $\hat{P}, \hat{Q}_1, \hat{Q}_2, \hat{Z}_1, \hat{Z}_2 \in \mathcal{R}^{n\times n}$, matrices $Y, \hat{N}_i, \hat{S}_i, i = 1, 2, 3, 4$, nonsingular matrix X with appropriate dimensions and constant $\sigma > 0$ such that the following inequality holds,

$$\begin{bmatrix} \Phi_{11} & * & * & * & * & * & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * & * & * & * & * & * \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & * & * & * & * & * & * \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} & * & * & * & * & * \\ \hat{N}_1^T & \hat{N}_2^T & \hat{N}_3^T & \hat{N}_4^T & -\frac{\hat{Z}_1}{\bar{h}} & * & * & * & * \\ \hat{S}_1^T & \hat{S}_2^T & \hat{S}_3^T & \hat{S}_4^T & 0 & -\frac{\hat{Z}_2}{\bar{h}+1} & * & * & * \\ J_1^T & J_2^T & J_3^T & J_4^T & 0 & 0 & -\sigma I & * & * \\ 0 & F_0 Y & 0 & 0 & 0 & 0 & 0 & -\sigma I & * \\ 0 & 0 & F_1 Y & 0 & 0 & 0 & 0 & 0 & -\sigma I \end{bmatrix}$$

$$< 0, \tag{17}$$

where

$$\begin{aligned}
\Phi_{11} &= -\hat{P} + \bar{h}\hat{Z}_1 + \hat{Q}_1 + (\bar{h}+1)\hat{Z}_2 + \hat{Q}_2 + \hat{N}_1 + \hat{N}_1^T \\
&\quad + \hat{S}_1 + \hat{S}_1^T - \theta_1 A X^T - \theta_1 X A^T, \\
\Phi_{21} &= \hat{N}_2 - \hat{N}_1^T + \hat{S}_2 - \theta_2 A X^T - \theta_1 Y^T B^T, \\
\Phi_{31} &= \hat{N}_3 - \hat{S}_1^T + \hat{S}_3 - \theta_3 A X^T - \theta_1 Y^T B^T, \\
\Phi_{32} &= -\hat{N}_3 - \hat{S}_3^T - \theta_3 B Y - \theta_2 Y^T B^T, \\
\Phi_{22} &= -\hat{Q}_1 - \hat{N}_2 - \hat{N}_2^T - \theta_2 B Y - \theta_2 Y^T B^T, \\
\Phi_{33} &= -\hat{Q}_2 - \hat{S}_3 - \hat{S}_3^T - \theta_3 B Y - \theta_3 Y^T B^T, \\
\Phi_{41} &= -\bar{h}\hat{Z}_1 - (\bar{h}+1)\hat{Z}_2 + \hat{N}_4 + \hat{N}_4^T - \theta_4 A X^T + \theta_1 X, \\
\Phi_{42} &= -\hat{N}_4 + \theta_2 X - \theta_4 B Y, \\
\Phi_{43} &= -\hat{S}_4 + \theta_3 X - \theta_4 B Y, \\
\Phi_{44} &= \hat{P} + \bar{h}\hat{Z}_1 + (\bar{h}+1)\hat{Z}_2 + \theta_4 X^T + \theta_4 X, \\
J_1 &= \sigma\theta_1 E, J_2 = \sigma\theta_2 E, J_3 = \sigma\theta_3 E, J_4 = \sigma\theta_4 E,
\end{aligned}$$

then with the control law

$$\mathbf{u} = K\mathbf{x}(\cdot), \qquad K = Y X^{-T}, \tag{18}$$

the system in (3) is asymptotically stable for all admissible network-induced delays.

Proof: In order to transform the nonconvex LMI in (14) into a solvable LMI, at first we assume that we have some relations in M_i's, $i = 1, 2, 3, 4$. One possibility is that $M_i = \theta_i M_0$ where M_0 is nonsingular and θ_i is known and given. Define $X = M_0^{-1}$, $W = diag(X, X, X, X, X, X, I, I)$, $\sigma = \rho^{-1}$ and $Y = K X^T$. Then pre-multiplying the inequality in (14) by W and post-multiplying by W^T, the inequality in (17) can be obtained. Note that the inequality in (17) is only a sufficient condition for the solvability of (14) based on these derivations. ■

IV. EXTENSION TO THE GENERAL CASE

The system in (3) can be extended to a more general case based on different assumption of the network induced delay. In the following, the system description and the corresponding controller design are presented.

In this section, we further extend the case in Section II to the case when $h > L > 1$, i.e., the varying part of the time delay has a bound which is more than one sampling time. When $L > 1$, the discretized system becomes more complicated. It can be derived by the same procedure in Section II and is represented as follows:

$$\begin{aligned}
\mathbf{x}_{k+1} &= A\mathbf{x}_k + A_{0\Delta}\mathbf{x}_{k-h} + A_{1\Delta}\mathbf{x}_{k-h-1} \\
&\quad + \cdots + A_{L\Delta}\mathbf{x}_{k-h-L},
\end{aligned} \tag{19}$$

where $A_{i\Delta} \triangleq BK + \Delta B_i K = A_i + \Delta_i$ with $A_i = BK$ and $\Delta_i = \Delta B_i K$, $i = 0, 1, \cdots, L$. A and B are the same as in (3). The ΔB_i term is similar as in Assumption 2 but with $i = 0, 1, \cdots, L$.

The corresponding results on the control gain design can be extended from Theorem 1. The remarks and proofs are similar and hence they are omitted here.

Theorem 2: For given scalars θ_i, $i = 1, \cdots, L+3$, and a given upperbound $\bar{h}$ of h, if there exist symmetric positive definite matrices $\hat{P}, \hat{Q}_j, \hat{Z}_j \in \mathcal{R}^{n\times n}$, $j = 1, \cdots, L+1$, matrices Y, $\hat{N}_{j,i}$, nonsingular matrix X with appropriate dimensions and constant $\sigma > 0$ such that the following inequality holds,

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{21}^T \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} < 0, \tag{20}$$

$$\Gamma_{11} = \begin{bmatrix} \Phi_{1,1} & * & * & * & * \\ \Phi_{2,1} & \Phi_{2,2} & * & * & * \\ \Phi_{3,1} & \Phi_{3,2} & \Phi_{3,3} & * & * \\ \vdots & \vdots & \vdots & \ddots & * \\ \Phi_{L+3,1} & \Phi_{L+3,2} & \cdots & \Phi_{L+3,L+2} & \Phi_{L+3,L+3} \end{bmatrix},$$

$\Gamma_{21} =$

$$\begin{bmatrix} \bar{h}\hat{N}_{1,1}^T & \bar{h}\hat{N}_{1,2}^T & \cdots & \bar{h}\hat{N}_{1,L+2}^T & \bar{h}\hat{N}_{1,L+3}^T \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \bar{h}_L\hat{N}_{L+1,1}^T & \bar{h}_L\hat{N}_{L+1,2}^T & \cdots & \bar{h}_L\hat{N}_{L+1,L+2}^T & \bar{h}_L\hat{N}_{L+1,L+3}^T \\ \sigma\theta_1 E^T & \sigma\theta_2 E^T & \cdots & \sigma\theta_{L+2} E^T & \sigma\theta_{L+3} E^T \\ 0 & F_0 Y & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & F_L Y & 0 \end{bmatrix},$$

$\Gamma_{22} = diag(-\bar{h}\hat{Z}_1, \cdots, -\bar{h}_L\hat{Z}_{L+1}, -\sigma I, \cdots, -\sigma I)$,

where $\bar{h}_L = \bar{h} + L$ and

$$\begin{aligned}
\Phi_{11} &= -\hat{P} + \sum_{i=0}^{L}(\bar{h}+i)\hat{Z}_{i+1} - \theta_1 A X^T \\
&\quad -\theta_1 X A^T + \sum_{i=1}^{L+1}\left[\hat{Q}_i + \hat{N}_{i,1} + \hat{N}_{i,1}^T\right], \\
\Phi_{2,1} &= \sum_{i=1}^{L+1}\hat{N}_{i,2} - \hat{N}_{1,1}^T - \theta_2 A X^T - \theta_1 Y^T B^T, \\
\Phi_{3,1} &= \sum_{i=1}^{L+1}\hat{N}_{i,3} - \hat{N}_{2,1}^T - \theta_3 A X^T - \theta_1 Y^T B^T, \\
\Phi_{3,2} &= -\hat{N}_{1,3} - \hat{N}_{2,3}^T - \theta_3 B Y - \theta_2 Y^T B^T, \\
\Phi_{2,2} &= -\hat{Q}_1 - \hat{N}_{1,2} - \hat{N}_{1,2}^T - \theta_2 B Y - \theta_2 Y^T B^T, \\
\Phi_{3,3} &= -\hat{Q}_2 - \hat{N}_{2,3} - \hat{N}_{2,3}^T - \theta_3 B Y - \theta_3 Y^T B^T, \\
&\vdots \\
\Phi_{L+3,1} &= -\sum_{i=0}^{L}(\bar{h}+i)\hat{Z}_{i+1} + \hat{N}_{L+3,L+3}^T
\end{aligned}$$

$$
\begin{aligned}
& \quad +\hat{N}_{L+3,L+3} - \theta_{L+3} A X^T + \theta_1 X, \\
\Phi_{L+3,2} &= -\hat{N}_{L+3,L+3} + \theta_2 X - \theta_{L+3} B Y, \\
&\vdots \\
\Phi_{L+3,L+2} &= -\hat{S}_{L+3} + \theta_{L+2} X - \theta_{L+3} B Y, \\
\Phi_{L+3,L+3} &= \hat{P} + \sum_{i=0}^{L} (\bar{h} + i) \hat{Z}_{i+1} + \theta_{L+3} X^T + \theta_{L+3} X,
\end{aligned}
$$

then with the control law

$$
\mathbf{u} = K\mathbf{x}(\cdot), \qquad K = Y X^{-T}, \tag{21}
$$

the system in (3) is asymptotically stable for all admissible network-induced delays.

V. ILLUSTRATIVE EXAMPLES

Consider the following open loop unstable continuous system with

$$
A_p = \begin{bmatrix} 0.01999 & 3.048 \\ 0 & -13.86 \end{bmatrix}, B_p = \begin{bmatrix} -1.698 & 27.73 \end{bmatrix}.
$$

The time delay profiles of the two channels: from sensor to controller and from controller to actuator, are set to be same as shown in Fig.3. Hence the time delay parameters

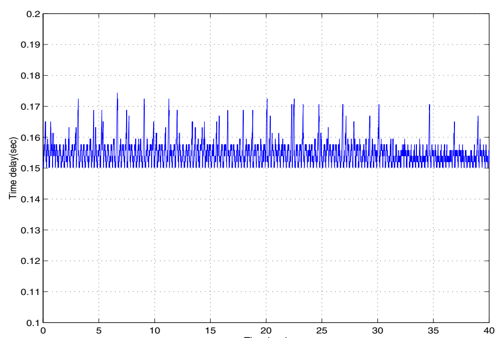

Fig. 3. The time delay profile applied in the simulation

are $\bar{h} = 6$, $L = 1$ with the sampling time $T_s = 0.05$ *sec*. Then with the same time delay parameters and sampling time as in Part A, the corresponding unstable discrete system is

$$
A = \begin{bmatrix} 1.001 & 0.11 \\ 0 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
$$

where the initial condition is $\mathbf{x}(0) = [0.5, 0.5]^T$. Choosing $E = \begin{bmatrix} 0.1 & 0.02 \\ 0.01 & 0.1 \end{bmatrix}$ and $F_0 = F_1 = [1,1]^T$, $\Gamma_k^T \Gamma_k < I$ is proved to be satisfied by calculating the terms ΔB_0 and ΔB_1, $\forall \varepsilon(k) \in [0, T_s]$ in Matlab. The initial condition is $\mathbf{x}(0) = [0.5, 0.5]^T$. Without a proper controller, the system will be definitely unstable.

Through solving the corresponding LMI (17) in Theorem 1 on the delay dependent analysis, and with given values $\theta_1 = \theta_2 = \theta_3 = 1$, $\theta_4 = 50$ and $\bar{h} = 6$, we have $K = YX^{-T} = [-0.0114, 0.0111]^T$. The resulted system response is stable as in Fig.4. Thus the unstable system is stabilized by the remote controller designed according to Theorem 1 which is based on discrete time domain analysis.

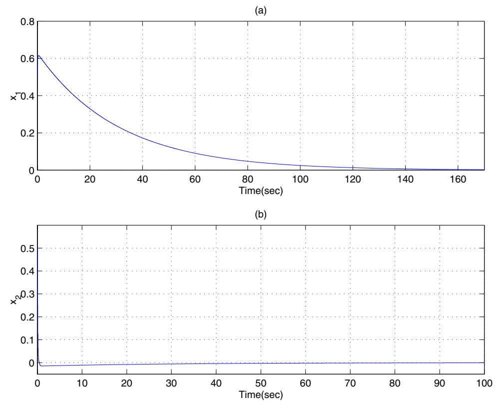

Fig. 4. The evolution of the system states

VI. CONCLUSIONS

The controller designs based on delay dependent stability conditions for discrete time remote control systems have been proposed in this paper by using LMI techniques. Two cases classified according to the knowledge of the time delay bound have been discussed. Future work may be in the following areas: i) on the robust stabilization of the remote control systems with uncertain external disturbances; ii) on the remote control problem of a class of cascade systems.

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