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Robust Model-Based Fault Detection Using Adaptive Robust Observers

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***Abstract*— A goal in many applications is to combine *a priori* knowledge of the physical system with experimental data to detect faults in a system at an early enough stage as to conduct preventive maintenance. The mathematical model of the physical system is the information available before hand. One of the key issues in the design of model-based fault detection schemes is the effect of the model uncertainties such as severe parametric uncertainty and unmodeled dynamics on their performance. This paper presents the application of a nonlinear model-based adaptive robust observer design to help in the detection of faults in sensors for a class of nonlinear systems. The observer is designed by explicitly considering the nonlinear dynamics of the system under consideration. Robust filter structures are used to attenuate the effect of the model uncertainty which combined with controlled parameter adaptation helps in reducing the extent of model uncertainty and in increasing the sensitivity of the fault detection scheme to help in the detection of incipient failure. This continuous monitoring of the state estimates enables us to detect any off-nominal system behavior and detect faults even in the presence of model uncertainties.**

I. INTRODUCTION

Model-based fault detection and diagnosis systems have found extensive use because of the fast response to abrupt failure and the ease of implementation of the model-based FDD systems in real-time algorithms. The most common methods for model-based fault detection are either based on state estimation or parameter estimation. A comprehensive review of the different methods for fault detection and their applicability to a given physical system has been presented in [1]. A detailed review of the different methods for observer based fault detection is given in [2], [3].

The major challenge in the design of model-based FDD systems is that they rely on an idealized assumption that a perfect mathematical model of the physical process is available and this model is utilized in the design of state-observers to detect sensor faults. In practice however, model uncertainty is always present and hence, the fault detection algorithm should be robust to the presence of model uncertainty. Another conflicting design requirement for a FDD scheme is to be able to detect incipient failure as early as possible to help in preventive maintenance. This would require an increase in the sensitivity of the FDD scheme. Hence, the robustness of the fault detection algorithm to modeling errors without losing sensitivity to faults is the key problem in the application of model based fault detection algorithms.

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Some of the schemes presented in the literature include sensor fault detection algorithms utilizing robust observers as in [4], [5]. Unfortunately, these schemes make assumptions on the structure of the model uncertainty to help in the design of co-ordinate transformations that decouple the effect of the fault and the uncertainty. Since assuming a known structure would limit the applicability of the fault detection algorithms, the researchers in [6]–[9] designed FDD schemes in which the model uncertainty is assumed to be unstructured but, bounded by suitable constants or functions obtained through *a priori* system identification. This approach lumps various model uncertainties into an unknown term without differentiating between the different causes of model uncertainties.

In this paper we present a model-based fault detection scheme to detect sensor faults using the recently proposed nonlinear adaptive robust observer (ARO). The ARO is designed to reconstruct the signal whose sensor we are monitoring for faults and this reconstructed state is compared with the measurements from the sensor to help in detecting sensor faults. The ARO uses robust filter structures to attenuate the effect of unmodeled dynamics and combines it with on-line parameter adaptation to reduce the extent of model uncertainty. The parameters of the system are updated only when certain persistence-of-excitation conditions are satisfied and these estimates are used to improve the state estimation process and help in characterizing the health of the system.

The organization of the paper is as follows: In section II the problem is formulated and the assumptions made in the design are given. In section III, the fault detection scheme is presented describing the state estimation process. Analysis of robustness of the proposed scheme and the class of sensor faults that can be detected is given in section IV and conclusions are presented in section V.

II. PROBLEM FORMULATION

Let $x = \begin{bmatrix} x_r \\ x_m \end{bmatrix} \in \mathbf{R}^n$ be the state vector of the system being monitored. In the proposed sensor fault detection scheme we utilize the measured signals ($x_m \in \mathbf{R}^{n-1}$) to estimate the signal from the sensor being monitored ($x_r \in \mathbf{R}$). In this section we present the class of uncertain nonlinear systems for which adaptive robust observers can be designed to detect sensor faults. This class of systems are affine in the signal whose sensor is being monitored.

A. Nominal Model

The nominal model of the system dynamics being considered can be written as:

$$\begin{aligned}\dot{x}_r &= F_{x_r}(x_m,u)\theta + G_{x_r}(x_m)x_r + \Delta_{x_r}\\ \dot{x}_m &= F_{x_m}(x_m,u)\theta + \Phi(x_m,u)\theta x_r + \Delta_{x_m}\\ y &= \begin{bmatrix} y_r \\ y_m \end{bmatrix} = \begin{bmatrix} x_r + \beta(t-T_f)f_s(x,u) \\ x_m \end{bmatrix}\end{aligned} \quad (1)$$

where $x_m = y_m \in \mathbf{R}^{n-1}$ is the vector of states that is measured, $x_r \in \mathbf{R}$ is the state whose sensor is being monitored, $u \in \mathbf{R}^m$ is the input to the system and, $\theta \in \mathbf{R}^p$ is the vector of constant but unknown parameters. $F_{x_r} \in \mathbf{R}^p$, $G_{x_r} \in \mathbf{R}$, $F_{x_m}(x_m,u) \in \mathbf{R}^{n-1\times p}$ and $\Phi(x_m,u) \in \mathbf{R}^{n-1\times p}$ are matrices or vectors of known smooth functions which are used to describe the nominal model of the system. Δ_{x_r} and Δ_{x_m} represent the lumped unknown nonlinear functions such as modeling errors. $y_r = x_r + \beta(t-T_f)f_s(x,u)$ is the measurement from the sensor being monitored with T_f being the time of occurrence of the fault, β represents the time profile of the fault. The time profile is given by:

$$\beta(t-T_f) = \begin{cases} 0 & \text{if } t < T_f \\ 1 - e^{-\alpha(t-T_f)} & \text{if } t \geq T_f \end{cases} \quad (2)$$

where $\alpha > 0$ denotes the rate of fault evolution and $f_s(x,u)$ is the fault function that affects the sensor. In this paper we utilize the state estimation error ($\tilde{y}_r$) as the residual signal as an indicator for the presence of faults in the sensor. The state estimation error is given by:

$$\begin{aligned}\tilde{y}_r &= \hat{x}_r - y_r\\ &= \hat{x}_r - x_r - \beta(t-T_f)f_s(x,u)\end{aligned} \quad (3)$$

where $\hat{x}_r$ is the estimate of the state x_r using a state observer. In the absence of faults in the sensor i.e., $f_s(x,u) = 0$, the measurement $y_r = x_r$, which leads to $\tilde{y}_r = \hat{x}_r - x_r = \tilde{x}_r$. On the other hand, in the presence of a fault we have that the estimation error is:

$$\tilde{y}_r = \hat{x}_r - y_r = \tilde{x}_r - \beta(t-T_f)f_s(x,u) \quad (4)$$

where the first term is the estimation error because of the presence of model uncertainty. Therefore, by monitoring $\tilde{y}_r$ we are able to detect a fault in the sensor.

The equation (1) represents the nominal system dynamics in the presence of sensor faults. The major difficulties in the design of the sensor fault detection schemes for the system described in equation (1) are:

1. The system dynamics represented by the functions $F_{x_m}(x_m,u)$, $G_{x_r}(x_m)$, $\Phi(x_m,u)$ and $F_{x_r}(x_m,u)$ are highly nonlinear.
2. The system has model uncertainties coming from both unknown parameters θ and unmodeled dynamics and uncertain nonlinearities represented by Δ_{x_r} and Δ_{x_m}.

As opposed to the sensor fault detection schemes described in [9], the model uncertainty in the system dynamics of equation (1) has been differentiated into those present because of uncertain parameters and uncertainty because of unmodeled dynamics. By using parameter adaptation (or other learning) mechanisms we can reduce the extent of model uncertainty and hence, increase the sensitivity of the fault detection scheme without a corresponding loss of robustness of the fault detection scheme.

B. Assumptions

The following practical assumptions are made in the design of the fault detection system:

Assumption 1: The unknown but constant parameters θ_i lie in a known bounded region Ω_{θ_i}, i.e.,

$$\theta_i \in \Omega_{\theta_i} = \{\theta_i : \theta_{imin} < \theta_i < \theta_{imax}\} \quad (5)$$

Assumption 2: The unmodeled dynamics and uncertain nonlinearities Δ_{x_m} and Δ_{x_r} are bounded by known constants, i.e.,

$$\Delta_i \in \Omega_{\Delta_i} = \{\Delta_i : |\Delta_i(x,u,t)| \leq \delta_i\} \quad (6)$$

where δ_i, $i = x_m, x_r$, are known constants.

Assumption 3: The system states and the control input to the system remain bounded before and after the occurrence of the fault; i.e., $x(t) \in \mathscr{L}_\infty$ and $u(t) \in \mathscr{L}_\infty$.

Remark 1: The above assumption is made because no fault accommodation is considered in this paper. Therefore, the feedback controller is such that the measured signals $x(t)$ and $u(t)$ remain bounded $\forall\, t \geq 0$.

Assumption 4: There exists a function $\omega(x_m, \theta_\omega) \in \mathbf{R}$ that can be linearly parameterized by a set of unknown parameters θ_ω and satisfies

$$\frac{\partial \omega}{\partial x_m}\Phi(x_m,u)\theta = \psi(x_m,u) \quad (7)$$

where $\psi(x_m,u)$ is a known function of x_m, u and independent of θ. Furthermore, the resulting function

$$A_\xi(x_m,u) = G_{x_r}(x_m) - \psi(x_m,u) \quad (8)$$

is exponentially stable i.e., $A_\xi < 0$.

Remark 2: From the above assumption (4) we know that the function $\omega(x,\theta_\omega)$ is linear with respect to the parameters θ_ω. Hence, we can write the function as

$$\omega(x_m,\theta_\omega) = \sigma(x_m)\theta_\omega \quad (9)$$

where $\sigma(x_m) \in \mathbf{R}^{1\times p_\omega}$ is the vector of functions of the measured states only.

III. Fault Detection Architecture Based on Adaptive Robust Observers

To address the problems in the design of the fault detection scheme, the following strategies are employed:

1. The nonlinear model of the plant is used in the design of the observers which generate the residual signals. This reduces the effect of unmodeled dynamics which would increase if a linearized model of the system is used.
2. The observer uses robust filter structures to attenuate the effect of model uncertainties. The use of parameter adaptation improves the performance of the observer

while providing extra signals i.e., the parameter estimates to help in the detection of incipient failure.

3. By updating the parameter estimates only when certain persistence of excitation conditions are met enables us to compute hard bounds on the parameter estimates in the presence of unmodeled dynamics leading to improved thresholds which help in improving the sensitivity of the scheme while still maintaining robustness.

In order to monitor n sensors in a system, the state estimation scheme proposed in this paper can be used to build a bank of n adaptive robust observers each of which monitors one sensor. A block diagram depicting the flow of information and the overall architecture of the proposed fault detection scheme is shown in figure 1.

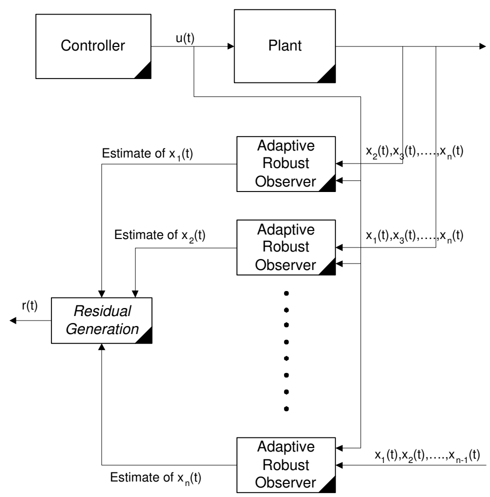

Fig. 1. Flow of Information in the ARO based Fault Detection Scheme

Remark 3: It should be noted that the proposed sensor fault detection scheme is applicable to subsystems whose dynamics are as specified in equation (1).

A. State Estimation for Residual Generation

Using the design procedure described in [10], [11], the adaptive robust observer is designed to reconstruct the state x_r from the measurements x_m based on the system dynamics in (1). For brevity we only present the main design steps:

Define a co-ordinate transformation of the form:

$$\xi = x_r - \omega(x_m, \theta_\omega) \tag{10}$$

where $\omega(x_m,\theta_\omega)=\sigma(x_m)\theta_\omega$ is a design function satisfying Assumption (4).Hence, the dynamics of the transformed state can be written as:

$$\begin{aligned}\dot{\xi} &= \dot{x_r} - \dot{\omega}(x_m, \theta_\omega)\\ &= A_\xi(x_m,u)x_r + (F_{x_r}(x_m,u) - \frac{\partial \omega}{\partial x_m}F_{x_m}(x_m,u))\theta + \Delta_\xi \end{aligned} \tag{11}$$

where, $\Delta_\xi = \Delta_{x_r} - \frac{\partial\omega}{\partial x_m}\Delta_{x_m}$.

Utilizing the results in [10], [11] and the co-ordinate transformation in (10), the dynamics of the transformed state are given as:

$$\dot{\xi} = A_\xi(x_m,u)\xi + \Upsilon\theta_0 + \Delta_\xi \tag{12}$$

where,

$$\Upsilon = \left[(F_{x_r}(x_m,u), -\varphi(x_m,u), A_\xi(x_m,u)\sigma(x_m)\right] \in \mathbf{R}^{1\times(p+pp_\omega+p_\omega)} \tag{13}$$

and

$$\theta_0 = \begin{bmatrix}\theta\\ \theta_{new}\\ \theta_\omega\end{bmatrix} \in \mathbf{R}^{p+pp_\omega+p_\omega} \tag{14}$$

with $\varphi_i(x_m,u) = \frac{\partial\sigma(x_m,u)}{\partial x_i} \otimes F_{x_i}^T(x_m,u)$ and $\theta_{new} = \theta_\omega \otimes \theta$.

Using the dynamics of the transformed state in equation (12), the observer can be designed as:

$$\dot{\hat{\xi}} = A_\xi(x_m,u)\hat{\xi} + \Upsilon\theta_0 \tag{15}$$

Hence, the error dynamics of the transformed state ($\tilde{\xi} = \hat{\xi} - \xi$) in the presence of model uncertainties can be written as:

$$\begin{aligned}\dot{\tilde{\xi}} &= \dot{\hat{\xi}} - \dot{\xi}\\ &= A_\xi(x_m,u)\tilde{\xi} - \Delta_\xi\end{aligned} \tag{16}$$

Since the above observer described in equation (15) is not implementable due to the lack of information about the parameter vector θ_0, we design the following filters:

$$\dot{\tau}_{\theta_0} = A_\xi(x_m,u)\tau_{\theta_0} + \Upsilon \tag{17}$$

The estimate of the transformed state can then be written as:

$$\hat{\xi} = \tau_{\theta_0}\theta_0 \tag{18}$$

From equations (17) and (18) it can be verified that the dynamics of the transformed state estimation error are still represented by (16). Therefore, the equivalent expression for the estimated state x_r is:

$$x_r = \tau_{\theta_0}\theta_0 + \sigma(x_m)\theta_\omega - \tilde{\xi} \tag{19}$$

B. Parameter Estimation Scheme

Unfortunately, the parameters θ and θ_ω are uncertain and contribute to the model uncertainty in the system dynamics. In order to reduce the extent of the model uncertainty, online parameter adaptation is utilized and estimates of the parameters $\hat{\theta}$ and $\hat{\theta}_\omega$ are used to estimate x_r i.e.,

$$\begin{aligned}\hat{x}_r &= \tau_{\theta_0}\hat{\theta}_0 + \sigma(x_m)\hat{\theta}_\omega\\ &= \Lambda\hat{\theta}_\Lambda\end{aligned} \tag{20}$$

where, $\Lambda = \begin{bmatrix}\tau_{\theta_0} & \sigma(x_m)\end{bmatrix} \in \mathbf{R}^{1\times(p+pp_\omega+2p_\omega)}$ and $\hat{\theta}_\Lambda = \begin{bmatrix}\hat{\theta}_0\\ \hat{\theta}_\omega\end{bmatrix} \in \mathbf{R}^{p+pp_\omega+2p_\omega}$.

An adaptation law is now designed to estimate the system parameters so that these estimates can be used in the implementation of the adaptive robust observer (20) and reduce the extent of model uncertainty. Consider the system dynamics of the x_m subsystem in equation (1),using the results in [10], the x_m dynamics can be rewritten in the following form:

$$\begin{aligned}\dot{x}_m &= F_{x_m}(x_m,u)\theta + \Phi(x_m,u)\theta(\theta_\Lambda^T\Lambda^T - \tilde{\xi}) + \Delta_{x_m}\\ &= F_{x_m}(x_m,u)\theta + \Xi(x_m,u)\theta_\Xi - \Phi(x_m,u)\theta\tilde{\xi} + \Delta_{x_m}\end{aligned} \tag{21}$$

where, $\Xi(x_m,u) = (\Phi(x_m,u) \otimes \Lambda) \in \mathbf{R}^{(n-1)\times p(p+pp_\omega+2p_\omega)}$ and $\theta_\Xi = (\theta \otimes \theta_\Lambda) \in \mathbf{R}^{p(p+pp_\omega+2p_\omega)}$. Hence, the equivalent system dynamics of x_m are linear in the unknown parameters given by:

$$\dot{x}_m = \Theta(x_m,u)\theta_\Theta - \Phi(x_m,u)\theta\tilde{\xi} + \Delta_{x_m} \tag{22}$$

where, $\Theta(x_m,u) = [F_{x_m}(x_m,u), \Xi(x_m,u)]$ and $\theta_\Theta = \begin{bmatrix}\theta \\ \theta_\Xi\end{bmatrix}$.

Since the dynamics are linear in the unknown parameter vector θ_Θ and to by-pass the need for derivatives of the unmeasured states, the following filters are proposed:

$$\dot{\Omega}^T = A\Omega^T + \Theta(x_m,u) \tag{23}$$

$$\dot{\Omega}_0 = A(\Omega_0 + x_m) \tag{24}$$

where A is any exponentially stable matrix, $\Omega^T \in \mathbf{R}^{(n-1)\times(p+p(p+pp_\omega+2p_\omega))}$ and $\Omega_0 \in \mathbf{R}^{n-1}$. Define $z = x_m + \Omega_0$, then using equations (22), (24):

$$\dot{z} = Az + \Theta(x_m,u)\theta_\Theta + \Delta_z \tag{25}$$

where, $\Delta_z = \Phi(x_m,u)\theta\tilde{\xi} + \Delta_{x_m}$. Let $\varepsilon = x_m + \Omega_0 - \Omega^T\theta_\Theta = z - \Omega^T\theta_\Theta$. From (25), (23) the dynamics of ε is:

$$\dot{\varepsilon} = A\varepsilon + \Delta_z \tag{26}$$

which has stable dynamics in the absence of unmodeled dynamics (i.e., $\Delta_{x_r} = \Delta_{x_m} = 0$). The model used for the estimation of the unknown parameter vector (θ_Θ) is:

$$z = \Omega^T\theta_\Theta + \varepsilon \tag{27}$$

where ε represents the effect of the unmodeled dynamics. The parameters are updated only when the regressor is rich enough. Let $R(kT) = \int_{(k-1)T}^{kT}\Omega(\tau)\Omega^T(\tau)d\tau$ where T is the time window over which the regressor is monitored to estimate the its richness and $k = 1, 2, \cdots$ is an integer. The parameter adaptation law is then given as:

$$\hat{\theta}_\Theta(kT) = \begin{cases} R(kT)^{-1}\int_{(k-1)T}^{kT}\Omega(\tau)z(\tau)d\tau & \text{if } R(kT) \geq \alpha(kT)I \\ \hat{\theta}_\Theta((k-1)T) & \text{otherwise} \end{cases} \tag{28}$$

where $\alpha(kT) > 0$ is a positive number with the parameter estimate being given by:

$$\hat{\theta}_\Theta(t) = \hat{\theta}_\Theta((k-1)T), \ \forall \ t \in [(k-1)T, kT) \tag{29}$$

In matrix terminology $A \geq \alpha I$ implies that the $\lambda_{min}(A) \geq \alpha$. The design variable α quantifies the richness of the signal.

The fault detection scheme that monitors the sensor measuring x_r consists of the observer (20) and the parameter estimation scheme (28).

C. Residual Evaluation Scheme

In model-based fault detection schemes, residual signals are utilized to detect the presence of faults. In this dissertation, the state estimation error $\tilde{y}_r$ is continuously monitored to help detect faults. Utilizing equations (19), (20) and (4) we get that the state estimation error satisfies:

$$\tilde{y}_r = \Lambda\tilde{\theta}_\Lambda + \tilde{\xi} - \beta(t - T_f)f_s(x,u) \tag{30}$$

Hence, for the sensor that measures the state x_r, the residual evaluation scheme is given as:

$$r = \begin{cases} \text{fault} & \text{if } |\tilde{y}_r(t)| \geq \bar{x}_r^0(t) \\ \text{fault-free} & \text{otherwise} \end{cases} \tag{31}$$

where $\bar{x}_r^0(t)$ is a suitable threshold on the state estimation error $\tilde{y}_r$ that will be specified in the next section. As can be seen from equation (30), the presence of model uncertainties leads to non-zero state estimation error even in the absence of a sensor fault. The proper choice of the threshold $\bar{x}_r^0$ would prevent any false alarms while still being able to detect certain incipient faults in sensors.

D. Parameter Estimation Scheme for Observer Design

In this paper we utilize on-line parameter adaptation using the batched least-squares approach as described in equation (28). The model used for the estimation of the unknown parameter vector θ_Θ is given by equation (27) where the model uncertainty ε is governed by (26). The parameter adaptation scheme that is utilized in the adaptive state estimation process is given by equation (28), where $\alpha(kT)$ can be described as the degree of excitation and it determines the amount of information needed to obtain a reasonably good estimate of the unknown parameter vector θ_Θ.

Consider the model used for the parameter estimation in (27). If the parameters are updated according to the estimation scheme in (28), then the parameter estimation error is given as:

$$\tilde{\theta}_\Theta(t) = R(kT)^{-1}\int_{(k-1)T}^{kT}\Omega(\tau)\varepsilon(\tau)d\tau, \ \forall \ t \in [kT, (k+1)T) \tag{32}$$

where the dynamics of ε are:

$$\dot{\varepsilon} = A\varepsilon + \Delta_z \tag{33}$$

with $\Delta_z = \Phi(x_m,u)\theta\tilde{\xi} + \Delta_{x_m}$ where the dynamics of the transformed state are given by:

$$\dot{\tilde{\xi}} = A_\xi(x_m,u)\tilde{\xi} - \Delta_\xi \tag{34}$$

As can be seen from the equations, in the absence of unmodeled dynamics (i.e., $\Delta_{x_r} = \Delta_{x_m} = 0$), ε is asymptotically stable which makes the parameter estimation error in equation (32) converge to zero. However, in the presence of bounded uncertain nonlinearities as in assumption (2), the dynamics of ε has bounded solutions with a proper choice of the Hurwitz matrix A and observer gain matrix $A_\xi(x_m,u)$.

Since the observer gain $A_\xi(x_m,u)$ is chosen to be exponentially stable it is easy to prove the following lemma.

Lemma 1: With the observer in equation (20), and the dynamics of the transformed state in equation (16), the transformed state estimation error $\tilde{\xi}$ is always bounded i.e., $\tilde{\xi} \in \mathscr{L}_\infty[0,\infty)$.

Since $\tilde{\xi}$ is bounded and since $\Phi(x_m,u)$ is continuous in a bounded interval [kT,(k+1)T) in addition to assumptions (1) and (2), we have that:

$$|\Delta_z| \leq \delta_z \tag{35}$$

Since, the filter matrix A is chosen to be stable, $\exists$ a positive constant ν_A such that:

$$\eta^T(A^T+A)\eta \le -\nu_A \eta^T \eta, \forall \eta \in R^{n\times n} \tag{36}$$

Using the above equation, we can prove the following lemma.

Lemma 2: With the system of filters given in equations (23) and (24), the model mismatch is bounded i.e.,

$$|\varepsilon(t)| \le \frac{2|\delta_z|}{\nu_A} \tag{37}$$

Looking at the above results and the bounded solutions of the estimation error ($\varepsilon(t)$), if the parameters are updated only when the regressor is rich enough, then the parameter estimation error is bounded and is given by the following result:

Theorem 1: If the parameters are adapted using the model in equation (27) and the adaptation scheme described in (28) with the regressor being rich enough, then the parameter estimation error is bounded and is given by:

$$|\tilde{\theta}_\Theta(t)| \le \tilde{\theta}_{\Theta_{max}}(kT), \ \forall\ t \in [kT,(k+1)T) \tag{38}$$

where,

$$\tilde{\theta}_{\Theta_{max}}(0) = |\theta_{\Theta_{max}} - \theta_{\Theta_{min}}| \tag{39}$$

and

$$\tilde{\theta}_{\Theta_{max}}(kT) = \begin{cases} \frac{2|\delta_z|}{\alpha(kT)\nu_A} \int_{(k-1)T}^{kT} |\Omega(\tau)| d\tau, & \text{if } R(kT) \ge \alpha(kT) I \\ \tilde{\theta}_{\Theta_{max}}((k-1)T), & \text{otherwise} \end{cases} \tag{40}$$

with $k=1,2,\cdots$.

Proof: Since the parameter estimation is carried out only when $\lambda_{min}(R(kT)) \ge \alpha(kT)$ i.e., all eigen values of $R(kT)^{-1} \le \frac{1}{\alpha(kT)}$. From equations (32) and (37), the parameter estimation error then satisfies $\forall\ t \in [kT,(k+1)T)$:

$$|\tilde{\theta}_\Theta(t)| \le \frac{1}{\alpha(kT)} \frac{2|\delta_z|}{\nu_A} \int_{(k-1)T}^{kT} |\Omega(\tau)| d\tau \tag{41}$$

Therefore, the maximum parameter estimation error is given by equation (38) with $\tilde{\theta}_{\Theta_{max}}(0) = |\theta_{\Theta_{max}} - \theta_{\Theta_{min}}|$. $\triangle$

Remark 4: From equation (22) we have $\theta_\Theta = \begin{bmatrix} \theta \\ \theta_\Xi \end{bmatrix}$ with $\theta_\Xi = \theta \otimes \theta_\Lambda$. Hence, θ_Θ is an overparameterized vector of θ_Λ, i.e., θ_Λ is a subset of the θ_Θ vector and using equation (38) we can compute the maximum parameter estimation error bound $\tilde{\theta}_{\Lambda_{max}}$ such that:

$$|\tilde{\theta}_{\Lambda_{max}}(t)| \le \tilde{\theta}_{\Lambda_{max}}(kT), \ \forall\ t \in [kT,(k+1)T) \tag{42}$$

IV. Performance Results

The equations (20), (28) and (31) describe the sensor fault detection scheme that consists of the residual generation and residual evaluation scheme. In this section we present results on the robustness and sensitivity of the proposed scheme.

A. Robustness

Robustness in a fault detection scheme helps avoid false alarms in the presence of model uncertainties. In order to detect the presence of faults in the sensor measuring the signal x_r, we use the state estimation error as the residual signal. Therefore, the threshold for the error $\tilde{x}_r^0$ has to be chosen such that

$$|\tilde{y}_r(t)| \le \tilde{x}_r^0(t) \tag{43}$$

i.e., the estimation error should be less than the selected threshold for all possible values of the model uncertainty.

Theorem 2: Consider the fault detection scheme that monitors the sensor measuring x_r consisting of the observer (20), the parameter estimation scheme (28) and the residual evaluation scheme (31). If the threshold for the sensor fault detection scheme $\tilde{x}_r^0$ is chosen such that:

$$\tilde{x}_r^0(t) = |\Lambda| \tilde{\theta}_{\Lambda_{max}}(kT) + \frac{|\delta_\xi|}{\nu_{A_\xi}}, \ \forall t \in [kT,(k+1)T) \tag{44}$$

then the proposed sensor fault detection scheme is robust to model uncertainty and avoids false alarms.

Proof: Consider the state estimation scheme using parameter adaptation as shown in equation (20):

$$\hat{x}_r = \Lambda \hat{\theta}_\Lambda \tag{45}$$

In model-based fault detection schemes, the residual signal is monitored to detect the presence of faults. In this dissertation we utilize the estimation error ($\tilde{y}_r = \hat{x}_r - y_r$) as the residual signal. Hence, the residual is given as:

$$\tilde{y}_r = \hat{x}_r - y_r = \hat{x}_r - x_r - \beta(t-T) f_s(x,u) \tag{46}$$

where using (19),

$$\begin{aligned} \tilde{y}_r &= \Lambda\hat{\theta}_\Lambda - \Lambda\theta_\Lambda + \tilde{\xi} - \beta(t-T) f_s(x,u) \\ &= \Lambda\tilde{\theta}_\Lambda + \tilde{\xi} - \beta(t-T) f_s(x,u) \end{aligned} \tag{47}$$

In the absence of a fault in the sensor (i.e., $f_s(x,u)=0$), the state estimation error is given as:

$$\tilde{y}_r = \Lambda\tilde{\theta}_\Lambda + \tilde{\xi} \tag{48}$$

By proper choice of the observer gain $A_\xi(x_m,u)$ the effect of the initial condition $\tilde{\xi}(0)$ can be attenuated and the transformed state estimation error is given by:

$$|\tilde{\xi}(t)| \le \frac{|\delta_\xi|}{\nu_{A_\xi}} \tag{49}$$

Using equation (42), the parameter estimation error is bounded as:

$$|\tilde{\theta}_\Lambda(t)| \le \tilde{\theta}_{\Lambda_{max}}(kT), \ \forall\ t \in [kT,(k+1)T) \tag{50}$$

Hence, the state estimation error in the absence of faults but, in the presence of model uncertainties is given as $\forall\ t \in [kT,(k+1)T)$:

$$\begin{aligned} |\tilde{y}_r(t)| &= |\Lambda\tilde{\theta}_\Lambda + \tilde{\xi}| \\ &\le |\Lambda| \tilde{\theta}_{\Lambda_{max}}(kT) + |\tilde{\xi}| \\ &\le |\Lambda| \tilde{\theta}_{\Lambda_{max}}(kT) + \frac{|\delta_\xi|}{\nu_{A_\xi}} \end{aligned} \tag{51}$$

Therefore, if the threshold for detecting sensor faults in chosen such that:

$$\bar{x}_r^0(t)=|\Lambda|\tilde{\theta}_{\Lambda_{max}}(kT)+\frac{|\delta_\xi|}{\nu_{A_\xi}} \tag{52}$$

then in the absence of sensor faults and in the presence of model uncertainties the state estimation error satisfies:

$$|\tilde{y}_r(t)|\leq\bar{x}_r^0(t),\ \forall\ t\in[kT,(k+1)T) \tag{53}$$

Therefore the state estimation error is always less than that of the threshold in the absence of faults and is therefore robust to model uncertainty. △

Hence, to prevent false alarms and be robust to model uncertainty, if the threshold $\bar{x}_r^0$ is chosen as in equation (44) then the proposed scheme is robust. As we can see the threshold before the adaptation is chosen so as to take into account the presence of parametric uncertainty. After the adaptation this uncertainty is reduced which leads to a more sensitive bound without a corresponding loss of robustness.

B. Fault Function Analysis

In this section of the paper we characterize the class of faults that the proposed fault detection scheme would be able to detect. The faults that are detected occur at an unknown time T_f and develop at an unknown rate α. The following result gives the class of faults that can be detected by the proposed scheme:

Theorem 3: If there exists an instance of time $t\in[kT,(k+1)T)$ such that the sensor fault function $f_s(x,u)$ satisfies the condition:

$$|\beta(t-T_f)f_s(x,u)|\geq 2|\Lambda|\tilde{\theta}_{\Lambda_{max}}(kT)+2\frac{|\delta_\xi|}{\nu_{A_\xi}} \tag{54}$$

then a fault in the sensor will be detected, i.e., $|\tilde{y}_r(t)|\geq\bar{x}_r^0(t)$.
Proof: The state estimation error after the occurrence of a fault in the sensor satisfies $\forall\ t\in[kT,(k+1)T)$:

$$\tilde{y}_r(t)=\hat{x}_r-y_r=\tilde{x}_r-\beta(t-T_f)f_s(x,u) \tag{55}$$

where the last term represents the effect of a sensor fault. Utilizing the triangle inequality, we get that $\forall\ t\in[kT,(k+1)T)$:

$$\begin{aligned}|\tilde{y}_r(t)|&=|\Lambda\tilde{\theta}_\Lambda+\tilde{\xi}-\beta(t-T_f)f_s(x,u)|\\ &\geq-|\Lambda\tilde{\theta}_\Lambda|-|\tilde{\xi}|+|\beta(t-T_f)f_s(x,u)|\\ &\geq-|\Lambda|\tilde{\theta}_{\Lambda_{max}}(kT)-\frac{|\delta_\xi|}{\nu_{A_\xi}}+|\beta(t-T_f)f_s(x,u)|\end{aligned} \tag{56}$$

In order to be able to detect a sensor fault in the system, the state estimation error should satisfy:

$$|\tilde{y}_r(t)|\geq\bar{x}_r^0(t) \tag{57}$$

where,

$$\bar{x}_r^0(t)=|\Lambda|\tilde{\theta}_{\Lambda_{max}}(kT)+\frac{|\delta_\xi|}{\nu_{A_\xi}},\ \forall\ t\in[kT,(k+1)T) \tag{58}$$

Therefore, if the sensor fault function satisfies the following equation, a fault is detected in the sensor:

$$|\beta(t-T_f)f_s(x,u)|\geq 2|\Lambda|\tilde{\theta}_{\Lambda_{max}}(kT)+2\frac{|\delta_\xi|}{\nu_{A_\xi}} \tag{59}$$

Hence, the above result shows that if the sensor fault function satisfies equation (59) at some point of time $t\in[kT,(k+1)T)$, then the sensor fault is detected at time t. △

As can be seen from equation (59) the class of sensor faults that is detected by the proposed fault detection scheme is characterized by the size of the sensor fault i.e., a fault is detected when the effect of the fault is greater than the effect of the model uncertainties. This is a consequence of the assumption that the size of the model uncertainties in known. Hence, a proper choice of the parameter adaptation scheme would lead to a scheme that is not only robust to model uncertainty but also sensitive.

V. Conclusions

In this paper we present a fault detection scheme to detect incipient faults in sensors. An adaptive robust observer (ARO) is designed to reconstruct the state and the estimate of the state from the ARO is compared with the sensor measurement to detect faults in the sensor. The ARO utilizes robust filter structures to attenuate the effect of model uncertainty and uses on-line parameter adaptation to reduce the extent of the model uncertainty. This use of the adaptation leads to an increase in the sensitivity of the detection scheme which helps in the detection of incipient failure of the sensors.

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