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Time–Optimal Path Planning of Moving Sensors for Parameter Estimation of Distributed Systems

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Abstract— **A general approach to the optimization of the observation horizon of moving sensor trajectories for parameter estimation of distributed systems is presented. Two problems are formulated here. The first consists in maximizing the determinant of the information matrix for a specified duration of observations and an open initial time, and the other constitutes its generalization to the case where the elapsed observation time is minimized subject to a guaranteed D-efficiency of the experiment. The approach is to convert the problem to an optimal control one in Mayer form, in which both the control forces of the sensors and the initial sensor positions are optimized in addition to the limits of the observation horizon.**

I. INTRODUCTION

One of the most challenging problems in the design of measurement for a distributed parameter system (DPS) is to select sensor locations so as to collect the most valuable information for estimating unknown parameters or system states. This topic has been widely investigated for the last forty years, cf. surveys in [1]–[4], and has found special interest in many areas, e.g., air quality monitoring systems, groundwater-resources management, recovery of valuable minerals and hydrocarbon, model calibration in meteorology and oceanography, hazardous environments and emerging smart material systems. Although the details of the approaches differ, in essence the underlying idea is to select those locations that lead to the best estimates of the process states and/or parameters. The optimality of the locations is judged by an appropriate measure of the estimate-error covariance matrix.

The sensor location problem was attacked from various angles, but the results are rather limited to the selection of stationary sensor positions. A generalization which imposes itself is to apply sensors which are capable of tracking points providing at a given time moment best information about the parameters. A possibility of using moving observations does arise in a variety of applications, e.g., air pollutants in the environment are often measured using data gathered by monitoring cars moving in an urban area and atmospheric variables are measured using instruments carried in a satellite. However, communications in this field are rather limited. Rafajłowicz [5] considers the determinant of the Fisher Information Matrix (FIM) associated with the parameters to be estimated as a measure of the identification accuracy and looks for an optimal time-dependent measure, rather than for the trajectories themselves. On the other hand, Uciński [2], [3], [6], [7] develops some computational algorithms based on the FIM. He reduces the problem to a state-constrained optimal-control one for which solutions are obtained via the methods of successive linearizations which is capable of handling various constraints imposed on sensor motions.

The optimal design of moving sensor trajectories is increasingly attracting attention in the context of sensor networks which play a role of importance in the research community [8], [9]. Technological advances in communication systems and the growing ease in making small, low power and inexpensive mobile systems now make it feasible to deploy a group of networked vehicles in a number of environments [10]. Applications in various fields of research are being developed and interesting ongoing projects include extensive experimentation based on testbeds. Our work on one of such experimental platforms, namely the MAS-net lab testbed being a distributed system equipped with two-wheeled differentially driven mobile robots capable of sensing the states of DPSs described by diffusion and wave equations [11], [12], revealed numerous deficiencies of the existing techniques of sensor location and commanded attention to aspects which, on one hand, are of paramount practical importance and, on the other hand, have been neglected in the literature so far. These, in turn, lead to nontrivial theoretical problems which still call for solutions.

This work is intended as an attempt to properly formulate and solve one of such problems, namely the time-optimal problem for moving sensors which observe the state of a DPS so as to estimate some of its parameters. Motivations come from technical limitations imposed on the time span of the measurements. These are inherent to mobile platforms carrying sensors, which are supplied with power from batteries. A common approach to the problem of optimum experimental design for dynamic systems is to fix the initial and terminal times for observations, since additional measurements can only improve the accuracy of the estimates. Consequently, the researchers' attention is predominantly focused only on the achieved precision while neglecting the problem of a proper selection of the observation horizon which is considered difficult. Our main objective has been to produce results which can be useful when the duration of the observations is a crucial factor. Two formulations are thus proposed: the first for a specified duration and an open initial time, and the other for the case in which the elapsed observation time constitutes a decision variable.

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II. OPTIMAL SENSOR LOCATION PROBLEM

Consider the scalar (possibly non-linear) DPS

$$\frac{\partial y}{\partial t} = \mathcal{F}\left(x, t, y, \frac{\partial y}{\partial x_1}, \frac{\partial y}{\partial x_2}, \frac{\partial^2 y}{\partial x_1^2}, \frac{\partial^2 y}{\partial x_2^2}, \theta\right), \quad x \in \Omega, \quad t \in [\alpha, \beta], \tag{1}$$

with initial and boundary conditions of the general form

$$y(x, 0) = y_0(x), \quad x \in \Omega, \tag{2}$$

$$\mathcal{E}(x, t, y, \theta) = 0, \qquad x \in \partial\Omega, \quad t \in [\alpha, \beta], \tag{3}$$

where $\Omega \subset \mathbb{R}^2$ is a fixed, bounded, open set with sufficiently smooth boundary $\partial\Omega$, the points of which will be denoted by $x = (x_1, x_2)$, $\mathcal{F}$ and $\mathcal{E}$ are some known functions, y_0 is a given initial state, t stands for time, α and β are fixed numbers, and $y = y(x, t)$ signifies the state variable with values in $\mathbb{R}$.

In what follows we assume that y depends on the vector $\theta \in \mathbb{R}^m$ of unknown parameters to be estimated from measurements made by N moving pointwise sensors over an observation horizon $[t_0, t_0 + T] \subseteq [\alpha, \beta]$. Let $x^j : [t_0, t_0 + T] \longrightarrow \Omega_{\rm ad}$ be the trajectory of the j-th sensor, where $\Omega_{\rm ad} \subset \Omega$ stands for the region where measurements can be made. The observations are assumed to be of the form

$$z(t) = y_{\rm m}(t) + \varepsilon_{\rm m}(t), \quad t \in [t_0, t_0 + T], \tag{4}$$

where

$$y_{\rm m}(t) = \operatorname{col}[y(x^1(t), t), \dots, y(x^N(t), t)],$$
$$\varepsilon_{\rm m}(t) = \operatorname{col}[\varepsilon(x^1(t), t), \dots, \varepsilon(x^N(t), t)],$$

$z(t)$ is the N-dimensional observation vector and $\varepsilon = \varepsilon(x, t)$ is a zero-mean white Gaussian noise process.

In this framework, the parameter identification problem is usually formulated as follows: Given the model (1)–(3) and the outcomes of the measurements z along the trajectories x^j, $j = 1, \dots, N$, determine an estimate $\hat{\theta} \in \Theta_{\rm ad}$ ($\Theta_{\rm ad}$ being the set of admissible parameters) which minimizes the output least-squares fit-to-data functional given by [13], [14]

$$\mathcal{J}(\theta) = \frac{1}{2}\int_{t_0}^{t_0+T} \|z(t) - \hat{y}_{\rm m}(t; \theta)\|^2 \, \mathrm{d}t \tag{5}$$

where $\hat{y}(x, t; \theta)$ denotes the solution to eqns. (1)–(3) corresponding to a given parameter θ and $\|\cdot\|$ stands for the Euclidean norm.

Clearly, the parameter estimate $\hat{\theta}$ depends on the sensors' trajectories x^j and we wish to select them so as to obtain best estimates of the system parameters. Thus a logical approach is to choose a quantitative measure related to the expected accuracy of the parameter estimates to be obtained from the data collected. Such a measure is usually based on the concept of the *Fisher Information Matrix* (FIM) [5], [15] whose inverse constitutes an approximation of the covariance matrix for the estimate of θ [16]–[18].

For simplicity of notation, let us write

$$s(t) = (x^1(t), x^2(t), \dots, x^N(t)), \quad \forall\, t \in [t_0, t_0 + T] \tag{6}$$

and set $n = \dim(s(t))$. Here, the FIM is [19]

$$M(s) = \sum_{j=1}^{N} \int_{t_0}^{t_0+T} g(x^j(t), t) g^{\mathsf{T}}(x^j(t), t) \, \mathrm{d}t, \tag{7}$$

where $g(x, t) = \nabla_\theta y(x, t; \theta)\big|_{\theta=\theta^0}$ denotes the vector of the so-called *sensitivity coefficients*, θ^0 being a prior estimate to the unknown parameter vector θ [2], [6]. In the sequel, we require g to be continuously differentiable.

Optimal sensor trajectories can be found by choosing s so as to minimize some scalar function Ψ of the information matrix. Several choices exist for such a function [16]–[18] and the most popular one is the D-optimality criterion

$$\Psi(M) = -\log\det(M), \tag{8}$$

which minimizes the volume of the confidence ellipsoid for the estimates. It will be adopted here as the measure of the information content of the observations.

III. CONSTRAINTS ON SENSOR MOVEMENTS

We assume that the sensors are conveyed by vehicles which are described by equations of motion of the form

$$\frac{\mathrm{d}s}{\mathrm{d}t}(t) = f(s(t), u(t)) \quad \text{a.e. on } [t_0, t_0 + T], \quad s(0) = s_0 \tag{9}$$

where a given function $f : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$ is required to be continuously differentiable, $s_0 \in \mathbb{R}^n$ defines an initial sensor configuration, and $u : [t_0, t_0 + T] \to \mathbb{R}^r$ is a measurable control function which satisfies

$$u_l \le u(t) \le u_u \quad \text{a.e. on } [t_0, t_0 + T] \tag{10}$$

for some constant vectors u_l and u_u.

We assume that all sensors should stay within an admissible region $\Omega_{\rm ad}$ (a given compact set) where measurements can be made. In what follows, it is convenient to choose a quite general form

$$\Omega_{\rm ad} = \{x \in \Omega : b_i(x) \le 0, \, i = 1, \dots, I\} \tag{11}$$

where the b_i's are given continuously differentiable functions. Accordingly, the conditions

$$h_{ij}(s(t)) = b_i(x^j(t)) \le 0, \quad \forall\, t \in [t_0, t_0 + T] \tag{12}$$

must be fulfilled, where $1 \le i \le I$ and $1 \le j \le N$. To shorten notation, after relabelling, we rewrite constraints (12) in the form

$$\gamma_\ell(s(t)) \le 0, \quad \forall\, t \in [t_0, t_0 + T], \tag{13}$$

where γ_ℓ, $\ell = 1, \dots, \nu$ tally with (12), $\nu = IN$.

IV. PROPOSED TIME-OPTIMAL FORMULATIONS

A. Trajectory Design with Open Initial Time and Fixed Duration of Observations

In some situations we are interested in constraining the time for making measurements to have a prescribed value and choosing the remaining factors, including the initial time t_0, so that the information provided by the experiment is maximized. The goal in such an optimal measurement problem is to determine, in addition to t_0, the forces (controls) applied to each vehicle conveying a sensor, which minimize a design criterion $\Psi[M(s)]$ defined on the set of all real-valued information matrices of the form (7) under the constraints (10) on the magnitude of the controls and induced state constraints (13). In order to increase the degree of optimality, in our approach we will regard s_0 as a control parameter vector to be chosen in addition to the control function u.

Since sensor trajectories s are unequivocally determined as solutions to the state equation (9), the above control problem can be interpreted as an optimization problem over the set of feasible triples

$$\begin{aligned}\mathcal{P}_1 = \big\{(s_0,u,t_0) \mid s_0 \in \Omega_{\text{ad}}^N, \quad \alpha \le t_0 \le \beta - T,\\ u:[t_0,t_0+T] \to \mathbb{R}^r \text{ is measurable},\\ u_l \le u(t) \le u_u \quad \text{a.e. on } [t_0,t_0+T]\big\} \quad (14)\end{aligned}$$

This leads to the following formulation:

Problem 1: Given a duration T, find the triple $(s_0,u,t_0) \in \mathcal{P}_1$ which minimizes

$$J_1(s_0,u,t_0) = \Psi[M(s)] \tag{15}$$

subject to the constraints (13).

B. Minimum-Time Trajectories with Guaranteed Efficiency

Intuitively, it is clear that the longer observation horizon, the more information we get about the estimated parameters. This is perfectly reflected by the performance index Ψ. Given an initial time t_0 and duration T producing an FIM $M(s)$, an extension of the observations beyond the limit $t = t_0 + T$ results in a perturbed FIM $M + \Delta M$, where ΔM is nonnegative definite. But for all performance indices of practical importance we then have $\Psi[M + \Delta M] > \Psi[M]$ provided that $\Delta M \neq 0$. Thus the naive modification of Problem 1 by just making T an additional design parameter would inevitably result in the optimal values $t_0^\star = \alpha$ and $T^\star = \beta - \alpha$.

On the other hand, the time duration is sometimes a critical factor. It is so especially in automated inspection in static and active environments, or in hazardous environments, where trial-and-error sensor planning cannot be used and the time allowed for observations must be as short as possible. It is also of concern in the context of a cooperative mobile sensor network formed from a number of wheeled mobile robots (e.g., differential drives, synchronous drives, etc.), if a major problem in the design is the power consumption by the robots. Such a limited energy budget raises the question of when to start and terminate sensor motions so as the elapsed time be short while guaranteeing an acceptable level of the information content of the collected observations.

The compromise proposed here relies on the notion of the D-efficiency which quantifies the suboptimality of given trajectories. Just as in the classical optimum experimental design [16], [18], we define it here as follows:

$$E_D(s) = \left\{\frac{\det(M(s))}{\det(M(\hat{s}))}\right\}^{1/m}, \tag{16}$$

where $\hat{s}$ stands for the D-optimal trajectories obtained for the observations over the entire feasible time interval $[\alpha,\beta]$, i.e., for $t_0 = \alpha$ and $T = \beta - \alpha$. The value of $\det(M(\hat{s}))$ can be determined beforehand, and setting a reasonable positive threshold $\eta < 1$, we can introduce the constraint relation

$$E_D(s) \ge \eta, \tag{17}$$

which guarantees a suboptimal yet reasonable solution. It is easily seen that (17) is equivalent to the constraint

$$\Psi[M(s)] \le C, \tag{18}$$

where $C = \Psi[M(\hat{s})] - m\log(\eta)$.

Consequently, defining the set of feasible quadruples

$$\begin{aligned}\mathcal{P}_2 = \big\{(s_0,u,t_0,T) \mid s_0 \in \Omega_{\text{ad}}^N, \quad [t_0,t_0+T] \subseteq [\alpha,\beta],\\ u:[t_0,t_0+T] \to \mathbb{R}^r \text{ is measurable},\\ u_l \le u(t) \le u_u \quad \text{a.e. on } [t_0,t_0+T]\big\}, \quad (19)\end{aligned}$$

we can now formulate the following minimum-time generalization of Problem 1:

Problem 2: Find the quadruple $(s_0,u,t_0,T) \in \mathcal{P}_2$ which minimizes

$$J_2(s_0,u,t_0,T) = T \tag{20}$$

subject to the constraints (13) *and* (18).

The above problem in which both initial and terminal times are free is slightly more difficult than Problem 1. Therefore, in what follows, we shall fix our attention on Problem 2 only, as Problem 1 can be addressed in much the same way.

V. EQUIVALENT CANONICAL OPTIMAL CONTROL PROBLEM ON A FIXED TIME INTERVAL

This section discusses the conversion of Problem 2 into a canonical optimal control problem with a fixed time interval, inequality-constrained trajectories and an endpoint cost [20]. Such a transcription makes it possible to employ existing software packages for numerically solving dynamic optimization and optimal control problems.

To simplify notation, consider the function $\text{svec} : \text{Sym}(m) \to \mathbb{R}^{m(m+1)/2}$, where $\text{Sym}(m)$ denotes the subspace of all symmetric matrices in $\mathbb{R}^{m\times m}$, that takes the lower triangular part (the elements only on the main diagonal and below) of a symmetric matrix A and stacks them into a vector a:

$$\begin{aligned}a &= \text{svec}(A)\\ &= \text{col}[A_{11},A_{21},\dots,A_{m1},A_{22},A_{32},\dots,A_{m2},\dots,A_{mm}].\end{aligned} \tag{21}$$

Similarly, let $A = \text{Smat}(a)$ be the symmetric matrix such that $\text{svec}(\text{Smat}(a)) = a$ for any $a \in \mathbb{R}^{m(m+1)/2}$.

To set forth our basic idea, define first the matrix-valued function

$$\Pi(s(t),t) = \sum_{j=1}^{N} g(x^j(t),t)g^{\mathsf{T}}(x^j(t),t). \tag{22}$$

Setting $r : [t_0, t_0 + T] \to \mathbb{R}^{m(m+1)/2}$ as the solution of the differential equations

$$\frac{\mathrm{d}r}{\mathrm{d}t}(t) = \operatorname{svec}(\Pi(s(t), t)), \quad r(t_0) = 0, \tag{23}$$

we have

$$M(s) = \operatorname{Smat}(r(t_0 + T)), \tag{24}$$

i.e., minimization of $\Psi[M(s)]$ thus reduces to minimization of a function of the terminal value of the solution to (23).

Introducing a new variable τ, called the nominal time, which is related to the real time t by $t(\tau) = t_0 + T\tau$, where $\tau \in [0, 1]$, we obtain an optimal control problem with the fixed time interval $[0, 1]$, and the initial-moment term t_0 and the duration scale factor T treated as parameters to be adjusted during optimization. Consequently, the equation of sensor dynamics (9) now becomes

$$\frac{\mathrm{d}\varsigma}{\mathrm{d}\tau}(\tau) = Tf(\varsigma(\tau), \upsilon(\tau)), \quad \varsigma(0) = s_0, \tag{25}$$

where $\varsigma(\tau) = s(t_0 + T\tau)$ and $\upsilon(\tau) = u(t_0 + T\tau)$. Similarly, the equation (23) which is used to construct the information matrix becomes

$$\frac{\mathrm{d}\varrho}{\mathrm{d}\tau}(\tau) = T \operatorname{svec}(\Pi(\varsigma(\tau), t(\tau))), \quad \varrho(0) = 0, \tag{26}$$

where $\varrho(\tau) = r(t_0 + T\tau)$.

Parameters t_0 and T are easily incorporated into the usual optimal control formulation by augmenting the system dynamics with two additional states being the solutions of the Cauchy problems

$$\frac{\mathrm{d}t}{\mathrm{d}\tau}(\tau) = T, \quad t(0) = t_0, \tag{27}$$

and

$$\frac{\mathrm{d}\bar{T}}{\mathrm{d}\tau}(\tau) = 0, \quad \bar{T}(0) = T, \tag{28}$$

respectively.

Define

$$q(\tau) = \begin{bmatrix} \varsigma(\tau) \\ \varrho(\tau) \\ t(\tau) \\ \bar{T}(\tau) \end{bmatrix}, \quad q_0 = q(0) = \begin{bmatrix} s_0 \\ 0 \\ t_0 \\ T \end{bmatrix}. \tag{29}$$

Then the equivalent fixed time interval optimal control problem consists in finding a pair $(q_0, \upsilon) \in \bar{\mathcal{P}}_2$ which minimizes the performance index

$$\bar{J}_2(q_0, \upsilon) = T \tag{30}$$

subject to

$$\begin{cases} \dfrac{\mathrm{d}q}{\mathrm{d}\tau}(\tau) = \varphi(q(\tau), \upsilon(\tau)), \quad q(0) = q_0, \\ \phi(q(1)) \le C, \\ \bar{\gamma}_\ell(q(\tau)) \le 0, \quad \forall\, \tau \in [0, 1], \end{cases} \tag{31}$$

where

$$\begin{aligned} \bar{\mathcal{P}}_2 = \big\{ (q_0, \upsilon) \mid s_0 \in \Omega_{\mathrm{ad}}^N, \quad [t_0, t_0 + T] \subseteq [\alpha, \beta], \\ \upsilon : [0, 1] \to \mathbb{R}^r \text{ is measurable}, \\ u_l \le \upsilon(t) \le u_u \quad \text{a.e. on } [0, 1] \big\}, \end{aligned} \tag{32}$$

and

$$\varphi(q, \upsilon) = \begin{bmatrix} Tf(\varsigma(\tau), \upsilon(\tau)) \\ T \operatorname{svec}(\Pi(\varsigma(\tau), t(\tau))) \\ T \\ 0 \end{bmatrix}, \tag{33}$$

$$\phi(q(\tau)) = \Psi[\operatorname{Smat}(\varrho(\tau))], \quad \bar{\gamma}_\ell(q(\tau)) = \gamma_\ell(\varsigma(\tau)). \tag{34}$$

The problem formulated in this way can be solved using existing packages for numerically solving dynamic optimization problems, such as RIOTS_95 [21], DIRCOL [22] or MISER [23]. We employed the first of them, i.e., RIOTS_95, which is designed as a MATLAB toolbox. The implemented numerical methods are supported by the theory outlined in [20]. The software automatically computes gradients for all functions with respect to the controls (they are represented as splines) and any free initial conditions. There are three main optimization routines, and the most general is based on SQP methods (it was also used in our computations reported in the next section).

VI. NUMERICAL EXAMPLE

To validate the proposed approach, consider the two-dimensional diffusion equation

$$\frac{\partial y}{\partial t} = \nabla \cdot (\kappa \nabla y) + F \tag{35}$$

for $x \in \Omega = (0, 1)^2$ and $t \in [0, 1]$, subject to subject to homogeneous initial and Dirichlet boundary conditions, where $F(x, t) = 20 \exp(-50(x_1 - t)^2)$. The optimum experimental design in context consists in optimally recovering the constant coefficients θ_1, θ_2 and θ_3 in the diffusion coefficient

$$\kappa(x) = \theta_1 + \theta_2 x_1 + \theta_3 x_2. \tag{36}$$

As regards the forcing term in our model, it approximates the action of a line source whose support is constantly oriented along the x_2-axis and moves with constant speed from the left to the right boundary of Ω. Our purpose is to estimate κ (i.e. the parameters θ_1, θ_2 and θ_3) as accurately as possible based on the measurements made by three moving sensors.

The determination of the information matrix requires the knowledge of vector of the sensitivity coefficients $g = \operatorname{col}[g_1, g_2, g_3]$ which is computed at nominal values $\theta_1^0 = 0.1$, $\theta_2^0 = -0.05$ and $\theta_3^0 = 0.2$. It can be obtained by solving the following system of PDEs [2], [3]:

$$\begin{cases} \dfrac{\partial y}{\partial t} = \nabla \cdot (\kappa \nabla y) + F, \\ \dfrac{\partial g_1}{\partial t} = \nabla \cdot \nabla y + \nabla \cdot (\kappa \nabla g_1), \\ \dfrac{\partial g_2}{\partial t} = \nabla \cdot (x_1 \nabla y) + \nabla \cdot (\kappa \nabla g_2), \\ \dfrac{\partial g_3}{\partial t} = \nabla \cdot (x_2 \nabla y) + \nabla \cdot (\kappa \nabla g_3), \end{cases} \tag{37}$$

in which the first equation constitutes the original state equation and the second, third and fourth equations result from its differentiation with respect to θ_1, θ_2 and θ_3, respectively. The initial and Dirichlet boundary conditions for all the four equations are homogeneous. The solution to (37) is found numerically using the MATLAB PDE toolbox [24], cf. Appendix I in [2] for details.

In the optimal control problem which is going to be solved in the sequel, we will need values of g along sensor trajectories. Therefore, we store their values interpolated at the nodes of a rectangular grid in a four-dimensional array (we applied uniform partitions using 21 grid points per each spatial dimension and 31 points in time). Since the solving of (23) requires values of g at points which are not necessarily nodes of that grid, the relevant interpolation is thus performed using cubic splines in space (to this end, MATLAB's procedure `interp2` was used) and linear splines in time. Since, additionally, the derivatives of g with respect to spatial variables and time are required, these derivatives are approximated numerically using the central-difference formula.

The next step consists in using RIOTS_95 to determine time-optimal sensor trajectories. We adopt the simple model

$$\frac{\mathrm{d}s}{\mathrm{d}t}(t) = u(t), \quad s(t_0) = s_0$$

Moreover, we impose the following constraints on u:

$$|u_i(t)| \leq 0.7, \quad \forall\, t \in [t_0, t_0 + T], \quad i = 1, \ldots, 6$$

As for technicalities, in order to numerically solve the measurement location problem, a partition was formed on the nominal time interval $[0, 1]$ by choosing 41 evenly spaced points and then considering the components of the control v in the class of piecewise linear splines. The system dynamics was integrated using the classical fourth-order Runge-Kutta method.

The computations were performed using a low-cost PC (Pentium 4, 2.40 GHz, 512 MB RAM) running Windows 2000 and Matlab 7 (R14). In order to avoid getting stuck in a local minimum, computations were repeated several times from different initial solutions. Each run took from three to thirty five minutes. Figures 1 and 2 present the results obtained for Problems 1 and 2. For comparison, Fig. 1(a) shows the D-optimum trajectories obtained for the observation horizon which coincides with the longest feasible interval $[0, 1]$. As for Problem 1, after setting the duration of the observation horizon as $T = 0.5$, the optimal moment for starting measurements was obtained at $t_0 = 0.5$, i.e., observations at the end of the evolution of the DPS provide best information about the parameters to be estimated. For Problem 2, we set the guaranteed efficiency at the level of $\eta = 0.8$ and obtained the optimal values $t_0^\star = 0.4692$ and $T^\star = 0.5308$.

Let us note that the diffusion coefficient values in the upper left of Ω are greater than those in the lower right. This means that the state changes during the system evolution are quicker when we move up and to the left (on the other hand, the system would have reached the steady state there earlier). This fact explains the form of the obtained trajectories—the sensors tend to measure the state in the regions where the distributed system is the most sensitive with respect to the unknown parameter κ, i.e. in the lower right.

VII. CONCLUSION

We have indicated possible ways of optimizing the observation horizon while designing moving sensor trajectories in experiments related to parameter estimation of DPSs. Two formulations have been proposed: the first for a specified duration and an open initial time, and the other for the case in which the elapsed observation time constitutes a decision variable. We have shown that they can be transcribed into equivalent optimal control problems with fixed initial and final times in Mayer form, and that they can be then efficiently solved by the MATLAB toolbox RIOTS_95, a high-performance tool for solving optimal control problems, combined with the Partial Differential Equation Toolbox.

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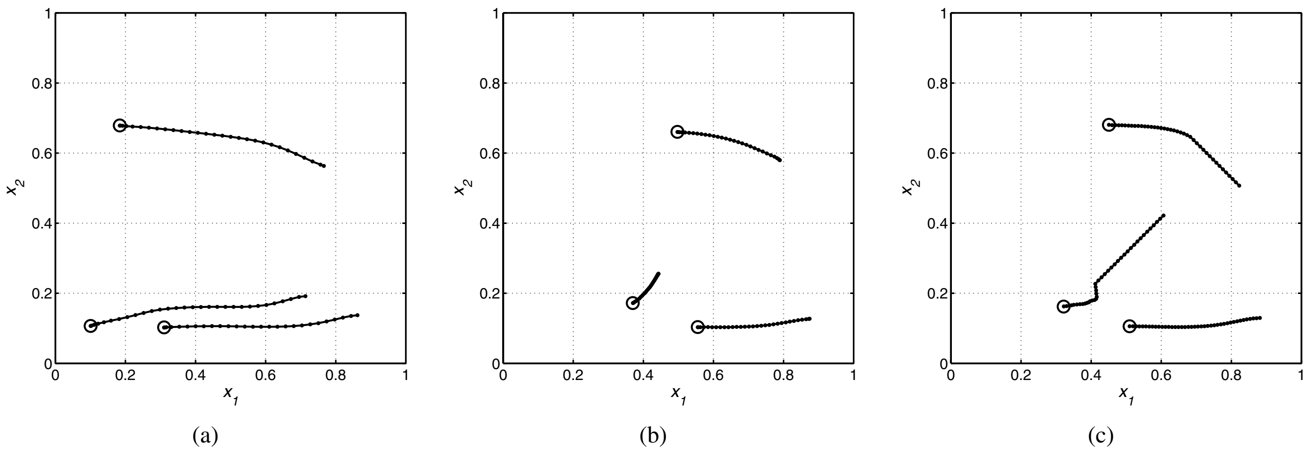

Fig. 1. Optimal sensor trajectories: D-optimum criterion for the time limits fixed at $t_0 = 0$ and $T = 1$ (a), Problem 1 for $T = 0.5$ (b), and Problem 2 for the guaranteed D-efficiency value set as 0.8. The initial sensor positions are marked with open circles, and the sensors positions at the consecutive points of the time grid are denoted by discs.

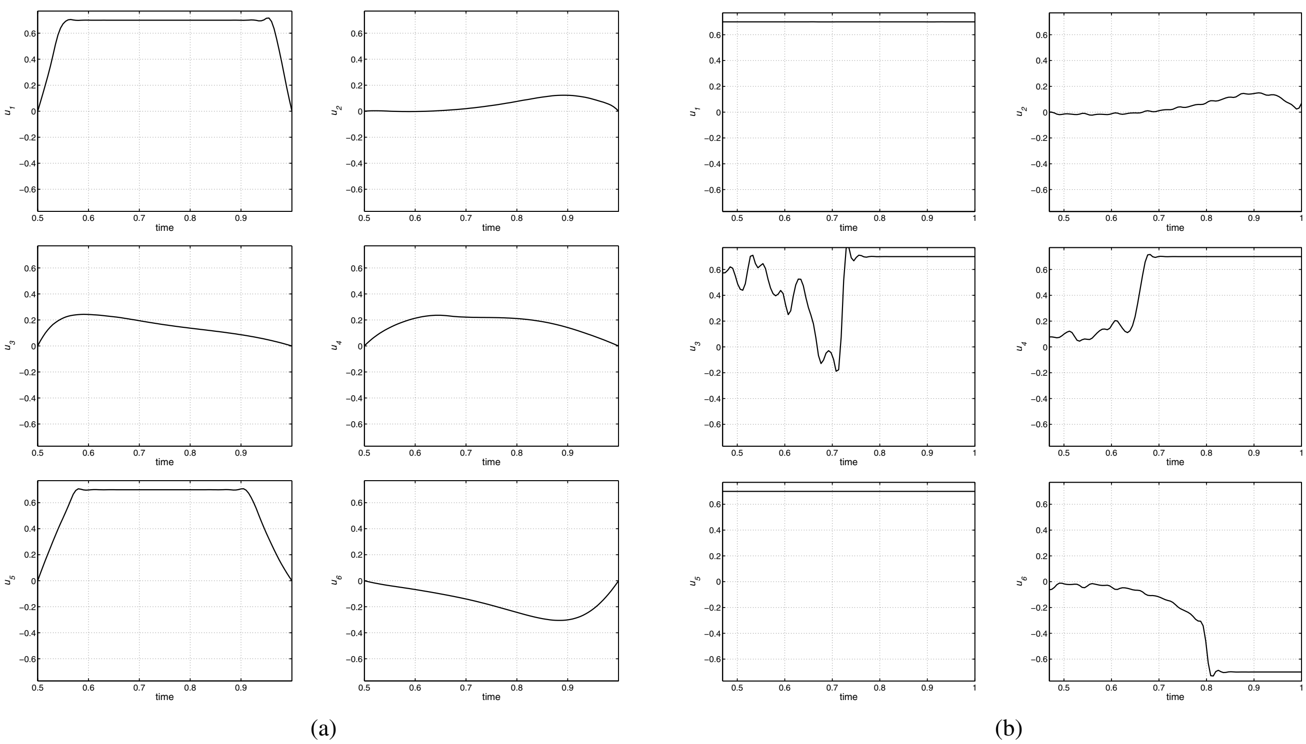

Fig. 2. Optimal controls: Problem 1 (a) and Problem 2 (b).

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