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Maxwell Slip Model Based Identification and Control of Systems with Friction

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Abstract— **The problem of identifying models suitable for friction compensation is addressed. A Maxwell Slip model identification method, capable of accounting for presliding friction hysteresis, is postulated and assessed. Identification is achieved via a single pair of displacement – applied force signals, without requiring a priori knowledge of physical parameters. The method is successfully assessed via Monte Carlo experiments, as well as via signals from a detailed physics based simulation model. Based upon these results, a simple feedforward control scheme is subsequently postulated and assessed. The results indicate that the scheme yields very effective friction compensation.**

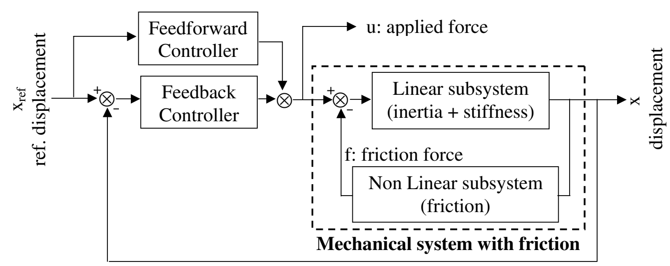

Fig. 1. Schematic diagram of a feedforward / feedback friction compensation scheme.

I. INTRODUCTION

Friction may be generally described as the resistance to motion when two surfaces are slid against each other. In many systems, friction is beneficial (for instance brakes), while in others it may seriously degrade performance (for instance motion systems). In both cases, successful friction characterization and control improves the system's functionality. However, this task is quite challenging due to the friction's complicated nature, which includes several nonlinear phenomena such as presliding (or micro - slip) displacement, rising static friction, the Stribeck effect, frictional lag, and so on [1].

Regarding the control of systems with friction the available methods may be generally classified into *non-model based* and *model based*. In the former case, friction may be treated as an unknown state element, and estimated as part of the augmented state [2]. Other efforts add a dither signal to the control signal for eliminating certain of the friction effects [1].

The premise on which many *model based* methods are founded is simple. The friction force (f) is estimated using some model, and a corresponding force $(\hat{f})$ that compensates for the actual friction is added to the control signal [1], [2], [3]. This concept is illustrated in Fig. 1 (note that friction is shown as a function of displacement), where a feedforward / feedback control scheme is utilised; the interested reader is referred to [1]. Nevertheless, since accurate modelling of friction based upon first principles and material / surface properties is not possible to date, identification methods based upon experimentally obtained data are typically used.

Several different friction models (Karnopp [4], LuGre [5], elastoplastic, [6], Leuven [7], [8], Maxwell – Slip [9], [10], [11], [12] and so on) and different identification techniques (both time – domain [7], [13], [14] and frequency – domain [15], [16], [17]), have been proposed for this purpose.

A recent study [12] suggests that an extended version of the Maxwell–Slip model is capable of accurately capturing the underlying friction dynamics, even when several transitions between the presliding and sliding regimes occur. Nevertheless, identification is based upon measured displacement and *friction force* signals. The current study aims at extending the above method by allowing for identification based upon the most commonly measured signals, which are the displacement and *applied force* (Fig. 1), and thus eliminating the need for friction force measurement.

Being based upon the Maxwell-Slip model [8], [11], the method is capable of accounting for the experimentally observed presliding friction hysteresis with nonlocal memory [7]. Besides, it is simple, it has good physical interpretation, and does not require a priori knowledge of the system's physical parameters (such as the mass). Furthermore, estimation is based upon a single pair of applied force and resulting displacement signals, without the need for obtaining velocity and acceleration (via numerical differentiation or otherwise). The experimental procedure is thus also simplified, as the usual need for a series of dedicated identification experiments [2], [7], [13] is overcome and a minimum number of sensors are utilized.

II. A MECHANICAL SYSTEM WITH FRICTION

Consider a simple mechanical system consisting of a mass m, a viscous damper c and a linear spring k. The system is excited by a force u, while the (unmeasurable) friction force f resists the motion (Fig. 2). The model for this system is:

$$m \cdot a + c \cdot v + k \cdot x = u - f \tag{1}$$

with x, v and a designating displacement, velocity and acceleration, respectively.

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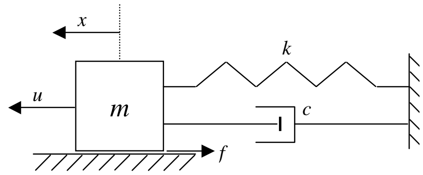

Fig. 2. A simple mechanical system with friction.

In addition to the displacement (x) and applied force (u), the identification of the system of (1) requires velocity (v), acceleration (a), and friction force (f) measurements. Alternatively, v, a and f may be expressed as suitable functions of x.

A. Velocity and Acceleration Approximations

Typically, the velocity $v(t)$ and acceleration $a(t)$ signals may be obtained via numerical differentiation of the displacement. This procedure is, nevertheless, non-unique and subject to numerical errors. The idea presently postulated is to adopt Moving Average representations of orders n_v, n_a and coefficients p_j, q_j, respectively, for approximating these signals as follows:

$$v(t) \approx \sum_{j=0}^{n_v} p_j \cdot x(t-j) \tag{2}$$

$$a(t) \approx \sum_{j=0}^{n_a} q_j \cdot x(t-j) \tag{3}$$

The filter orders and coefficients are allowed to be directly obtained from the data via the identification procedure. Note that t henceforth designates normalized discrete time ($t = 1, 2, \ldots$).

B. Friction Force Approximation

The next step is the parametrization of the friction force. A large number of such models are available [1], [2], [3]. In our case an extended version of the Maxwell – Slip model is selected [12]. This choice is guided by the fact that this model is simple, accounts for the presliding hysteresis with nonlocal memory characteristics [8], [11], and has been found capable of providing almost excellent friction force prediction [9], [10], [11], [12]. Moreover, it expresses friction as a function of displacement; this is in contrast to other popular models (such as the LuGre [5]) that adopt a velocity dependency.

The model is a suitable extension of the basic Maxwell – Slip model structure [12]. It consists of M elasto-slide operators in parallel configuration, subjected to common displacement $x(t)$ [Fig. 3]. Each operator has negligible inertia, and maximum spring deformation Δ_i (threshold). For spring deformation smaller, in magnitude, than Δ_i ($|\delta_i(t)| < \Delta_i$) the corresponding operator *sticks*; otherwise it *slips* ($|\delta_i(t)| = \Delta_i$). The whole system sticks (presliding regime) iff at least one operator sticks, and slides (sliding regime) iff all operators slip.

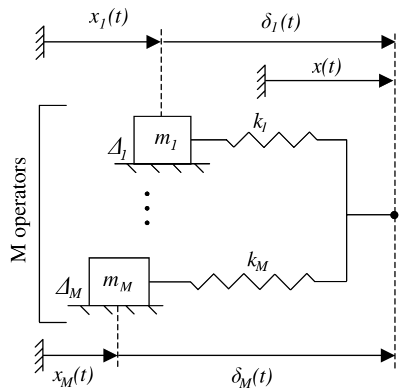

Fig. 3. The basic Maxwell Slip model structure.

In mathematical terms, the model is described by a set of nonlinear state equations [11]:

$$\begin{aligned}\delta_i(t+1) &= \operatorname{sgn}[x(t+1) - x(t) + \delta_i(t)] \cdot \\ &\quad \min\{|x(t+1) - x(t) + \delta_i(t)|, \Delta_i\}\end{aligned} \tag{4}$$

with $i = 1, \ldots, M$, while (unlike the basic model of Fig. 3 in which friction is approximated as the sum of the operators' spring forces [8], [11]) friction is presently expressed as:

$$f(t) = \sum_{j=0}^{n'} r_j \cdot x(t-j) + \sum_{j=0}^{n} \boldsymbol{\theta}_j^T \cdot \boldsymbol{\delta}(t-j) \tag{5}$$

subject to (4). The friction force thus depends upon the spring deformations [$\delta_i(t)$'s] and the displacement history. The above equation suggests that the friction force is generated by having the displacement driven through a Finite Impulse Response (FIR) filter of order n' (with coefficients r_j for $j = 0, \ldots, n'$), and the spring deformation vector $\boldsymbol{\delta}(t) \triangleq [\delta_1(t) \ldots \delta_M(t)]^T$ driven through an M-dimensional FIR filter of order n (with vector coefficients $\boldsymbol{\theta}_j$ for $j = 0, \ldots, n$).

C. Discrete–Time System Dynamics

Equations (2), (3) and (5) give a displacement – based approximation of the velocity, acceleration and friction force. Substituting them into (1) yields:

$$m \cdot \left(\sum_{j=0}^{n_a} q_j \cdot x(t-j) \right) + c \cdot \left(\sum_{j=0}^{n_v} p_j \cdot x(t-j) \right) + $$

$$k \cdot x(t) = u(t) - \sum_{j=0}^{n'} r_j \cdot x(t-j) - \sum_{j=0}^{n} \boldsymbol{\theta}_j^T \cdot \boldsymbol{\delta}(t-j) \tag{6}$$

which may be re-written as:

$$u(t) = \sum_{j=0}^{n_x} g_j \cdot x(t-j) + \sum_{j=0}^{n} \boldsymbol{\theta}_j^T \cdot \boldsymbol{\delta}(t-j) \tag{7}$$

and is subject to (4). In this expression $n_x \triangleq \max\{n_a, n_v, n'\}$ and g_j are the coefficients obtained by combining the common factors (for instance $g_0 \triangleq m \cdot q_0 + c \cdot p_0 + k + r_0$). This representation retains the two Finite Impulse Response (FIR) filters mentioned earlier (their orders now being n_x and n) and is of the form referred to as the *Dynamic*

NonLinear Regression with direct application of eXcitation (DNLRX) representation [12].

This representation provides the current value of the applied force given the displacement history (provided that the incorporated parameters are known). Thus, it may be considered as a discrete–time equivalent of the *inverse dynamics* of the system (the *direct dynamics* corresponding to the force–displacement causal relationship).

III. INVERSE DYNAMICS IDENTIFICATION

In this section the DNLRX identification method is presented (also see the joint paper [12] for the application of the method to pure friction identification – that is neglecting the inertial, elastic and dissipative dynamics of the system of Fig. 2).

A. The DNLRX Method

The DNLRX method performs identification using a pair of displacement – applied force signals (single experiment) via minimization of a quadratic cost function of the form:

$$\mathcal{J} \triangleq \sum_{t=\lambda}^{N} e^2(t) \tag{8}$$

with N designating the number of available signal samples (signal length) and $\lambda \triangleq \max\{n, n_x\} + 1$. $e(t)$ stands for the error, defined as the difference between the applied force $u(t)$ and the model provided force $\hat{u}(t)$ [see (7)]:

$$e(t) \triangleq u(t) - \hat{u}(t) \tag{9}$$

As is conventionally done, this error is assumed to be a stationary *zero mean* and *uncorrelated (white)* sequence with variance σ_e^2 [18]; the relaxation of these assumptions is generally possible, but is beyond the scope of the present study.

The method aims at simultaneously estimating the parameters in the following *DNLRX model structure* [compare with (7)]:

$$\text{DNLRX}(M, n, n_x; \boldsymbol{d}, \boldsymbol{\theta}): \;\; u(t) = \boldsymbol{\theta}^T \cdot \Big[x(t) \ldots x(t-n_x) \vdots\, \boldsymbol{\delta}^T(t) \,\ldots\, \boldsymbol{\delta}^T(t-n) \Big]^T + e(t) \tag{10}$$

subject to (4). In this expression $e(t)$ designates the model error, $\boldsymbol{d} \triangleq [\Delta_1 \ldots \Delta_M]^T$ and $\boldsymbol{\theta} \triangleq [g_0 \ldots g_{n_x} \vdots \boldsymbol{\theta}_0^T \ldots \boldsymbol{\theta}_n^T]^T$ being the M and $[(n+1)\cdot M + n_x + 1]$ – dimensional threshold and composite parameter vector, respectively.

Obviously, the model is nonlinear with respect to $\boldsymbol{d}$ [see (4)] but remains linear with respect to $\boldsymbol{\theta}$ [see (10)]. Therefore the estimator may be realized via a succession of nonlinear and linear regression operations as follows:

$$\Big[\hat{\boldsymbol{d}}^T \, \hat{\boldsymbol{\theta}}^T\Big]^T = \arg\min_{\boldsymbol{d},\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{d}, \boldsymbol{\theta}) = \arg\min_{\boldsymbol{d}} \left\{ \min_{\boldsymbol{\theta}} \mathcal{J}(\boldsymbol{\theta}/\boldsymbol{d}) \right\} \tag{11}$$

The nonlinear regression operation is based upon a postulated two-phase, hybrid, optimization scheme. The first (*pre-optimization*) phase utilizes Genetic Algorithm (GA) based optimization [19] in order to explore large areas of the parameter space and locate regions where global or local minima may exist. The second (*fine-optimization*) phase utilizes the Nelder-Mead Downhill Simplex algorithm [19] for locating the exact global or local minima within the previously obtained regions.

This two-phase scheme has been shown (see [9], [11]) to be very effective in locating the true global minimum of the cost function and circumventing problems associated with local minima. Furthermore, the Nelder-Mead algorithm utilizes only cost function evaluations but *no* derivative evaluations. This is useful as the latter is not defined everywhere since the cost function is *nonsmooth* in areas of the parameter space.

B. Initialization (Initial Spring Deformations)

In order to get the cost function $\mathcal{J}$ calculated, the (unmeasurable) $\delta_i(t)$ $(\forall i)$ evolutions are required. Since the $\delta_i(1)$'s (initial values) cannot be uniquely identified [11], their estimation should be avoided. A simple approach would be based upon their arbitrary selection, but this may reduce estimation accuracy [11].

A more appropriate approach is to take advantage of the system characteristics [11]. Indeed, when the system is in the sliding regime, all Maxwell – Slip model operators slip in the same direction. Thus, selecting a time instant t_{cr} such that the system slides, set $\delta_i(1) = \text{sgn}[x(t_{cr})] \cdot \Delta_i$ $(\forall i)$ as initial spring deformations and identify the model using data that correspond to $t \geq t_{cr}$. For the selection of t_{cr} recall that the system switches from the sliding to the presliding regime when velocity reverses, or, equivalently, right after a "dominant" extreme of the displacement.

C. Order Selection

Since the main objective of identification is simulation and control, model selection is tailored to these needs and is primarily judged in terms of the identified model's simulation ability. This is measured via a normalized quadratic function of the model error, referred to as the *Normalized Output Error (NOE)*:

$$NOE = \frac{\sum_{t=\lambda}^{N}(u(t) - \hat{u}(t))^2}{\sum_{t=\lambda}^{N}(u(t) - m_u)^2} \times 100\% \tag{12}$$

where m_u is the sample mean of the actual applied force, N the signal length (in samples), and λ is defined in (8). In addition, the condition number of the information matrix [18] is monitored for avoiding model overfitting.

DNLRX(M, n, n_x) model order selection is then based upon the estimation of models corresponding to various values of n, n_x for any given M. The final model is selected following consideration of various values of M. Model validation is based upon the model's simulation performance as assessed within a subset of the data, referred to as the *validation set*, that has not been used in the estimation and order selection *(cross validation principle)*.

IV. IDENTIFICATION AND CONTROL RESULTS

Two distinct cases are considered. Case I corresponds to DNLRX model identification, with the objective being

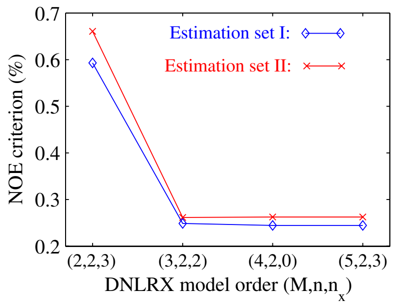

Fig. 4. The NOE criterion versus model orders (DNLRX model; 5% noise case; single experiment).

the assessment of the identification method's accuracy via Monte Carlo experiments. Case II corresponds to friction identification and control based upon DNLRX models. In this case the displacement – applied force signals are generated via a generic physics based friction model that works at the asperity level [21]. The objective in this case is to judge the ability of the DNLRX models to properly represent the "actual" friction dynamics and, also, to be used for friction compensation.

A. Case I: Monte Carlo Simulation Results

In this case 40 excitation – response $[x(t) - u(t)]$ realizations, obtained via a DNLRX model characterized by $M = 3$ operators and FIR orders of $n = 2$ and $n_x = 2$, respectively [see (7)] are used. The actual model parameters are indicated in Table I (left part). Each excitation (displacement) realization is a zero mean low pass filtered random signal consisting of $N = 2,000$ samples. The responses are either noise free or corrupted by zero mean uncorrelated noise at the 5% standard deviation level. The available data are split into two disjoint data sets of equal size. The former, designated as *estimation set I*, is employed for parameter estimation, while the latter (*estimation set II*) for evaluating model performance within a data segment that was not used in estimation. The selection of the final orders is based upon model performance within both data sets.

The design parameters (encoding, crossover, mutation, and so on [20]) of the Genetic Algorithm employed within the nonlinear regression operation of the identification method are similar to those in [12]. Moreover, the population size and number of generations are selected according to model complexity (not exceeding 100).

The first issue investigated is whether the proposed order selection procedure yields the correct orders. The investigation is performed for the 5% noise case, since it is closer to a real situation in which model overdetermination would be more likely to occur. Model order selection results are presented in Fig. 4. The identification leads to generally decreasing NOE as the FIR orders n and n_x increase ($n, n_x \in [0, 6]$). The FIR orders n and n_x (for each M) beyond which the reduction in the NOE is practically insignificant (below 1%) are selected and presented in Fig. 4.

As indicated in Fig. 4, a plateau in the NOE sequence (within estimation set I) is achieved for $M \geq 3$, hence the

TABLE I

DNLRX IDENTIFICATION RESULTS FOR 0% NOISE AND 5% NOISE (40 MONTE CARLO RUNS PER CASE; POINT ESTIMATES PLUS/MINUS TWO STANDARD DEVIATIONS).

	Actual Parameter	Estimated Parameter (0% noise)	Estimated Parameter (5% noise)
$\boldsymbol{d}$	0.1	$0.1000 \pm 4.7 \times 10^{-10}$	0.1007 ± 0.0075
	0.7	$0.7000 \pm 6.6 \times 10^{-10}$	0.7007 ± 0.0064
	1.3	$1.3000 \pm 4.7 \times 10^{-10}$	1.3000 ± 0.0046
$\boldsymbol{\theta}_0$	0.9	$0.9000 \pm 7.4 \times 10^{-10}$	0.9015 ± 0.0524
	-0.7	$-0.7000 \pm 9.8 \times 10^{-10}$	-0.7054 ± 0.0341
	0.8	$0.8000 \pm 7.4 \times 10^{-10}$	0.8039 ± 0.0235
$\boldsymbol{\theta}_1$	-0.3	$-0.3000 \pm 4.7 \times 10^{-9}$	-0.3032 ± 0.1234
	0.5	$0.5000 \pm 2.8 \times 10^{-9}$	0.5100 ± 0.0592
	-0.2	$-0.2000 \pm 1.7 \times 10^{-9}$	-0.2046 ± 0.0438
$\boldsymbol{\theta}_2$	-0.01	$-0.0100 \pm 2.3 \times 10^{-9}$	-0.0129 ± 0.0813
	0.05	$0.0500 \pm 1.7 \times 10^{-9}$	0.0448 ± 0.0348
	-0.04	$-0.0400 \pm 5.1 \times 10^{-10}$	-0.0383 ± 0.0245
g_0	-0.07	$-0.0700 \pm 5.6 \times 10^{-10}$	-0.0733 ± 0.0654
g_1	0.05	$0.0500 \pm 1.0 \times 10^{-9}$	0.0542 ± 0.1276
g_2	0.01	$0.0100 \pm 5.1 \times 10^{-10}$	0.0087 ± 0.0638

DNLRX$(3, 2, 2)$ model is selected (notice that this model is characterized by the correct orders). This result is also confirmed from the obtained NOE within the estimation set II, in which case the minimum NOE is achieved for the DNLRX$(3, 2, 2)$ model [with small differences from the DNLRX$(4, 2, 0)$ and DNLRX$(5, 2, 3)$ models].

The ability of the DNLRX method to provide accurate parameter estimates is investigated via Monte Carlo experiments (40 runs per case) using the correct model order ($M = 3$, $n = 2$ and $n_x = 2$). The estimated model parameters are contrasted to their actual counterparts in Table I. Examination of the results for the no noise case indicates that the estimates coincide with their actual counterparts, while the standard deviations are (owing to the absence of added noise) insignificant.

Similar behavior is reported for the 5% noise case, where the estimates are in very good agreement with the actual parameters (right part of Table I). These results indicate the effectiveness of the identification procedure.

B. Case II: A Mechanical System with Friction

In this case the capabilities of DNLRX modelling for accurately representing and controlling a mechanical system with friction are investigated. Toward this end a simple mechanical system (Fig. 5) which is subject to inertial and frictional forces (no elastic or dissipative forces) is considered. Within this system friction is generated by the generic physics based friction model working at the asperity level [21]. This simulates the interaction of a large number of idealized asperities subject to adhesion, creep and deformation. This friction model, which has been provided by the Katholieke Universiteit Leuven (Belgium – research team led by Professors H. Van Brussel and F. Al-Bender), is capable of approximating actual friction behavior (also see [10]).

1) Simulation Setup: The $x(t)$ and $u(t)$ signals are obtained from a closed-loop system simulation using only proportional ($P = 1,000$) feedback control (Fig. 5 – without the feedforward controller). The mass is selected as $m = 5$

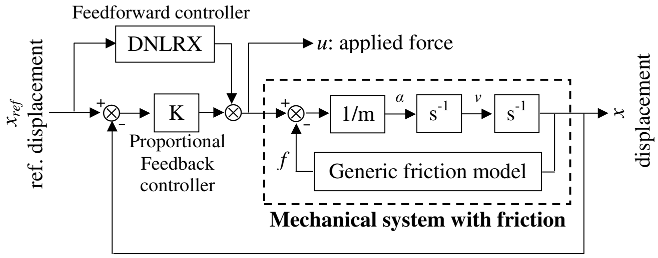

Fig. 5. Block diagram of the closed–loop system with friction.

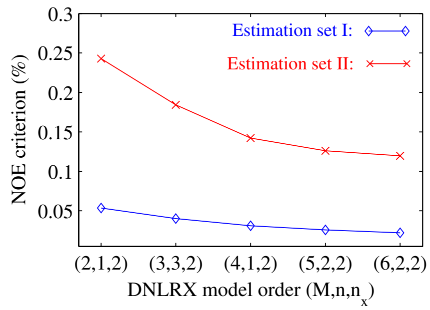

Fig. 6. Order selection for the mechanical system with friction: NOE versus model orders.

kg, while the sampling period is set at $f_s = 2,000\ Hz$. The numerical integration is implemented via the Dormand-Prince fixed step solver (ODE5 - Simulink / Matlab). The reference displacement x_{ref} is a lowpass filtered [with cutoff $f_c = 3\ Hz$] random signal. $100,000$ sample-long displacement – applied force signals are thus obtained.

2) Identification Results: Initial spring deformations are set by locating a "dominant" extremum (maximum) of the excitation (see subsection III-B). An extreme at $t_{cr} = 2,525$ is specifically selected. Thus, the remaining $97,476$ samples are divided into three disjoint sets: An $8,100$ sample-long *estimation set I* used for estimating the parameters and selecting the orders, a $38,900$ sample-long *estimation set II* also used for order selection (but not for parameter estimation), and a $50,476$ sample-long *validation set* used for independent evaluation and assessment of the estimated model performance.

The order selection procedure is depicted in Fig. 6, which presents the FIR orders n and n_x (for each M) beyond which the NOE decrease is judged as practically insignificant. Notice that the FIR order $n_x = 2$ is selected for all examined DNLRX models. The NOE criterion within the *estimation set I* is a decreasing function of the order [Fig. 6]. However, the NOE remains almost unchanged for $M \geq 5$, hence for purposes of model economy (principle of parsimony) the DNDLRX$(5, 2, 2)$ model is selected. Additionally, the NOE criterion remains practically the same for $M \geq 5$ within the estimation set II [Fig. 6], indicating that $M = 5$ is adequate for representing the system dynamics.

As the results of Fig. 7 indicate, the DNLRX$(5, 2, 2)$ model manages to capture the underlying system dynamics. Indeed, the (simulation) error between the actual force and model-based force is presented in Fig. 7(b) and is quite small

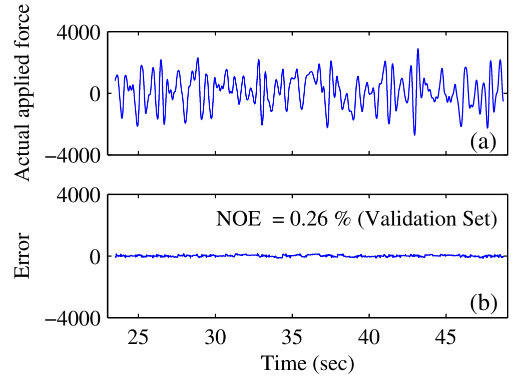

Fig. 7. Mechanical system with friction: (a) Actual applied force $u(t)$; (b) DNLRX$(5, 2, 2)$ model based error (validation set).

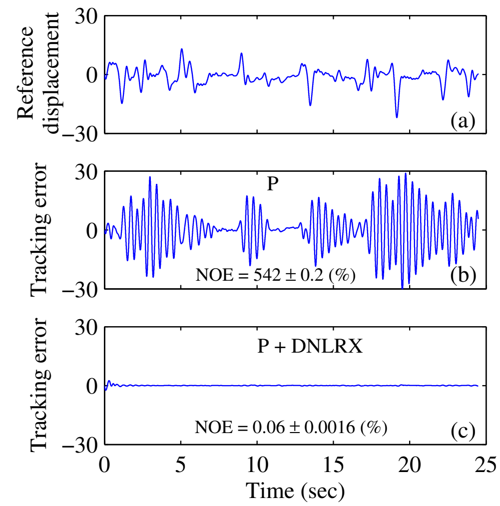

Fig. 8. Control results: (a) Reference trajectory; (b) P control tracking error; (c) P + DNLRX control tracking error (for each setup the NOE mean plus/minus one standard deviation is indicated based upon 30 realizations).

(NOE=0.26%), thus indicating very good agreement between the actual [shown in Fig. 7(a)] and model-based (simulated) force within the validation set (notice that the agreement is generally better within the estimation set I).

3) Model-Based Control Results: In order to examine the possibility for DNLRX model-based control (friction compensation), two simple control setups are formulated. The first includes a typical Proportional (P) controller in the feedback loop, while the second a Proportional controller combined with a DNLRX$(5, 2, 2)$ based feedforward controller (P+DNLRX) [Fig. 5]. In each setup the proportional gain is equal to $1,000$. The effectiveness of each setup is assessed using a "tricky" reference displacement signal [Fig. 8(a)], which is generated using a lowpass filtered (cutoff frequency of $f_c = 3\ Hz$) random signal.

30 experiments are implemented for each setup, and the sample mean and standard deviation of the NOE (normalized tracking error) for each setup, are computed. This is done because the "generic" friction model employed is based upon stochastic events, a feature that resembles real frictional behavior [21]. A typical result, indicating the reference displacement and the corresponding tracking errors for both the P and the P+DNLRX controller cases, is presented in Fig.

8 (mean and standard deviation for all 30 experiments are also noted). Evidently, the system based upon the P controller alone is unable to follow the reference displacement, whereas excellent tracking is achieved when the P+DNLRX controller is used. These results demonstrate the DNLRX method's potential for use in controlling systems with friction.

V. CONCLUSIONS

The problem of identification and control of systems with friction was addressed and a model structure, designated as the Dynamic NonLinear Regression with direct application of eXcitation (DNLRX), was postulated. This model structure is an extended version of the basic Maxwell – Slip model, and is capable of accounting for the presliding friction hysteresis with nonlocal memory. An identification method was subsequently developed and its effectiveness was assessed via Monte Carlo experiments, as well as via signals obtained from a mechanical system with friction in the presence of several transitions between the presliding and sliding friction regimes. Finally, a friction control scheme based upon the DNLRX model structure was discussed. The obtained results have indicated that:

(i) The postulated two-phase identification method achieves accurate estimation of the DNLRX model structure.

(ii) The DNLRX based identification method, using a single pair of displacement – applied force signals and no prior knowledge, captures, with excellent accuracy, the inverse dynamics of a mechanical system with friction.

(iii) The DNLRX model based friction compensation scheme appears quite effective. This signifies the potential of the method for feedforward control of systems with friction.

Future research is, among other things, to be devoted to the comparison of the friction compensation method with other existing schemes, as well as to its application to a laboratory setup.

VI. ACKNOWLEDGMENTS

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