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On Passivity and Impulsive Control of Complex Dynamical Networks with Coupling Delays

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Abstract— **This paper presents a set of sufficient conditions for a class of nonlinear complex dynamical networks with coupling delays in the state to be passive. Based on the passivity property, impulsive control of the dynamical networks is addressed. An illustrative example is included.**

I. INTRODUCTION

In the past few years, there has been increasing interest in studying complex networks as relevant to many areas of science [20]. A complex network is a set of interconnected nodes, where a node is defined by a basic unit of the network. There are many different types of nodes, such as routers in the Internet [4], document files in the World Wide Web [1], individuals, organizations or countries in the social network, etc., while the edges represent the interactions among the individual elements. The ubiquity of complex networks in science and technology naturally stimulates the study of the structure and dynamics of complex networks.

For nearly 40 years, complex networks have been studied extensively as a branch of mathematics, namely random graphs. In order to describe the transition from a regular network to a random network, Watts and Strogatz (WS), studied the so-called small-world networks. However, many large-scale complex networks such as the World Wide Web, the Internet, social networks, etc., belong to a class of inhomogeneous networks called scale-free networks-see [20], [23] for references.

From a nonlinear dynamics point of view, one can introduce dynamical element models to be the network nodes. For the resulting dynamical network, there have been many studies of behaviour, particularly synchronization and bifurcations [3], [8], [12], [13], [14], [15], [20], [21], [23], [25], [27]. A particular point of interest in the problem of synchronization of dynamical systems was highlighted in the paper [17] where it was reported that two chaotic systems being interconnected may synchronize. After the pioneering work of Ott et al. [16], several control strategies for stabilizing chaos have been proposed. In this paper, we are interested in control issues for such networks. Control techniques based on impulses have been applied extensively in recent years [5], [6], [26], due to their theoretical and practical significance. Compared to continuous methods, an impulsive method can increase the efficiency of bandwidth usage. Recently networks with coupling delays have received a great deal of attention. In [3], [27], the effects of time delays in a specific coupled oscillator network were discussed. The work in [12], [13], [25] focuses on dynamical behaviors in delayed networks.

The concepts of dissipativity and passivity, motivated by the dissipation of energy across resistors in an electrical circuit, has been widely used in order to analyze stability of a general class of interconnected nonlinear systems [19], [22], [24]. The work [2] investigated conditions under which a nonlinear system can be rendered passive via smooth state feedback. In [18], the conditions of feedback passivity allow one to design an adaptive synchronizing control law which ensure global synchronization.

In this paper, we address the passivity of a class of complex dynamical networks in the presence of communication time-delays. Our goal in this paper is to render the dynamical networks to be passive via impulsive and state feedback controls. Using the Lyapunov functional approach, stability and control results are derived in terms of solutions of linear matrix inequalities. The paper is organized as follows: a class of nonlinear dynamical networks is presented in Section 2. Passive control of the complex dynamical networks with delay interconnections is addressed in Section 3. Section 4 contains a numerical example. Concluding remarks are collected in Section 5.

II. PROBLEM FORMULATION

Let $J = [t_0, +\infty)\,(t_0 \geq 0)$, and R^n denote the n-dimensional Euclidean space. For $x = (x_1, \ldots, x_n)^\top \in R^n$, the norm of x is $\|x\| := \left(\sum_{i=1}^{n} x_i^2 \right)^{\frac{1}{2}}$. Correspondingly, for $A = (a_{ij})_{n\times n} \in R^{n\times n}$, $\|A\| := \lambda_{\max}^{\frac{1}{2}} \left(A^T A\right)$. The identity matrix of order n is denoted as I_n (or simply I if no confusion arises).

In general, the dynamics of nonlinear systems are described by the behaviour of state variables as solutions of a set of nonlinear differential equations. In many cases, the dynamical equations can be decomposed into two parts:

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linear dynamics with respect to state variables; and a continuous vector-value function. In this paper, we consider a class of complex dynamical networks with time-delay interconnections, due to finite speed of information processing and communication. A dynamical network of such kind is assumed to consist of N coupled nodes, with each node being an $n-$dimensional dynamical system with linear and nonlinear parts. Such a dynamical network is described by

$$\left\{\begin{array}{rcl} \dot{x}_i(t) & = & A_i x_i(t) + f_i(t, x_i(t)) + u_i(t) \\ & & + E_i w_i(t) + \sum\limits_{j=1}^{N} D_{ij} x_j(t-\tau), \\ z_i(t) & = & F_i x_i(t), \quad i = 1, 2, \cdots N \end{array}\right. \tag{1}$$

where $t \in J$ ($t_0 \geq 0$), A_i, E_i and F_i are known matrices with appropriate dimensions. $f : J \times R^n \rightarrow R^n$ is a nonlinear vector-valued function with $f(t, 0) \equiv 0, t \in J$, $x_i = (x_{i1}, x_{i2}, \cdots, x_{in})^\top \in R^n$ are the state variables of node i, and $u_i \in R^m$ is the control input. $w_i \in R^q$ and $z_i \in R^q$ denote the exogenous input vector and the output vector of each subsystem, respectively. $\tau > 0$ represents the delay. $D = (D_{ij})_{N \times N}$ is the coupling configuration matrix. If there is a connection between node i and node j ($i \neq j$), then $D_{ij} = D_{ji} > 0$; otherwise, $D_{ij} = D_{ji} = 0$ ($i \neq j$), and the diagonal elements of matrix D are defined by

$$D_{ii} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} D_{ij} = -\sum_{\substack{j=1 \\ j \neq i}}^{N} D_{ji}, \quad i = 1, 2, \cdots N. \tag{2}$$

Construct a hybrid impulsive and feedback controller $u_i = u_{i1} + u_{i2}$ for network system (1) as follows:

$$\left\{\begin{array}{l} u_{i1}(t) = \sum\limits_{k=1}^{\infty} B_i x_i(t) l_k(t), \\ u_{i2}(t) = \sum\limits_{k=1}^{\infty} C_{ik} x_i(t) \delta(t - t_k), \end{array}\right. \quad i = 1, 2, \cdots N \tag{3}$$

where B_i and C_{ik} are $n \times n$ constant matrices, $\delta(\cdot)$ is the Dirac impulse function, and $l_k(t)$ is the ladder function, i.e.,

$$l_k(t) = \left\{\begin{array}{ll} 1, & t_{k-1} < t \leq t_k, \\ 0, & \text{others}, \end{array}\right.$$

with discontinuity points

$$t_1 < t_2 < \cdots < t_k < \cdots, \quad \lim_{k \rightarrow \infty} t_k = \infty \tag{4}$$

where $t_1 > t_0$.

Clearly, from (3) we have

$$u_{i1}(t) = B_i x_i(t), \ t \in (t_{k-1}, t_k], \ k = 1, 2, \cdots$$

with $i = 1, 2, \cdots N$.

On the other hand, $u_{i2}(t) = 0$ as $t \neq t_k$, and (1) and (3) together imply that

$$x_i(t_k + h) - x_i(t_k) = \int_{t_k}^{t_k + h} \Big[A_i x_i(s) + f_i(s, x_i(s)) + u_i(s) + E_i w_i(s) + \sum_{j=1}^{N} D_{ij} x_j(s - \tau))\Big] ds, \quad i = 1, 2, \cdots N$$

where $h > 0$ is sufficiently small, as $h \rightarrow 0^+$, which reduces to

$$\triangle x_i(t)|_{t_k} = x_i(t_k^+) - x_i(t_k) = C_{ik} x_i(t_k), \tag{5}$$

where $x_i(t_k^+) = \lim\limits_{h \rightarrow 0^+} x_i(t_k + h)$, and, for simplicity, it is assumed that $x_i(t_k) = x_i(t_k^-) = \lim\limits_{h \rightarrow 0^+} x_i(t_k - h)$. This implies that the controller $u_i(t)$ has the effect of suddenly changing the state of system (1) at the points t_k.

Accordingly, under control (3), the closed-loop system of (1) becomes

$$\left\{\begin{array}{l} \dot{x}_i = \tilde{A}_i x_i(t) + f_i(t, x_i(t)) + E_i w_i(t) \\ \qquad + \sum\limits_{j=1}^{N} D_{ij} x_j(t - \tau), \qquad t \in (t_{k-1}, t_k] \\ \triangle x_i = C_{ik} x_i, \qquad t = t_k \\ z_i(t) = F_i x_i(t) \qquad k = 1, 2, \cdots \\ x_i(t_0^+) = x_{i0}, \qquad i = 1, 2, \cdots N \end{array}\right. \tag{6}$$

where $\tilde{A}_i = A_i + B_i$, $\{t_k\}$ satisfies (4), and $\triangle x_i$ is given by (5). For convenience, let $t_0 = 0$.

Our goal is to find a set of (sufficient) conditions on the hybrid control time sequence $\{t_k\}$ and the constant control gain matrices, $\{B_i\}$ and $\{C_{ik}\}$, guaranteeing that the closed-loop system (6) is exponentially stable and strictly passive. In many applications, e.g., power systems, control actions include line switching which can be modelled by $u_i(t) = -D_{ik} x_k$ and $u_k(t) = -D_{ki} x_i$.

III. PASSIVELY CONTROLLED NETWORK WITH COUPLING DELAYS

The following lemmas and definitions are need to facilitate the development of the main results of this paper.

Lemma 3.1: [10] For any $x \in R^n$, if $P \in R^{n \times n}$ is a positive definite matrix, $Q \in R^{n \times n}$ is an symmetric matrix, then

$$\lambda_{\min}(P^{-1}Q) x^\top P x \leq x^\top Q x \leq \lambda_{\max}(P^{-1}Q) x^\top P x.$$

Lemma 3.2: [7] Let $v(t)$ be a continuous function with $v(t) \geq 0$ for $t \geq t_0$. If

$$v'(t) \leq -a v(t) + b v(t - \tau), \qquad t \geq t_0 \tag{7}$$

with the initial condition $v(t) = \phi(t), t \in [t_0 - \tau, t_0]$, where $\phi(t)$ is piecewise continuous, a and b are positive constants with $a > b > 0$, then

$$v(t) \leq v(t_0) e^{-\lambda(t - t_0)}, \quad t \geq t_0, \tag{8}$$

where λ is a positive solution of the equation $-\lambda = -a + b e^{\lambda \tau}$.

Definition 3.1: ([9], [11]) A system with input w and output z where $w(t), z(t) \in R^q$ is said to be passive if there is a constant β such that

$$\int_0^T w^\top(s) z(s) ds \geq -\beta^2 \tag{9}$$

for all $T \geq 0$. If in addition, there are constants $\varepsilon_1 \geq 0$ and $\varepsilon_2 \geq 0$ such that

$$\int_0^T w^\top(s)z(s)ds \geq -\beta^2 + \varepsilon_1 \int_0^T w^\top(s)w(s)ds + \varepsilon_2 \int_0^T z^\top(s)z(s)ds \qquad (10)$$

for all $T \geq 0$, then the system is input strictly passive if $\varepsilon_1 > 0$, output strictly passive if $\varepsilon_2 > 0$.

This definition is for input-output models. Allowing for internal dynamics, the β is in general to be dependent on the initial state x_0 [9].

Definition 3.2: The closed-loop network system (6) is said to be passively controlled if each subsystem is internally stable and the input w and output z satisfy the inequality (10).

Assume that $t_k - t_{k-1} \geq \delta\tau$, $\delta > 1$. For convenience, define the following functions and parameters by inequalities and equalities:

$$\Delta(P_i,t) = \begin{bmatrix} \psi_i(t) & P_iD_{i1} & \cdots & P_iD_{iN} \\ D_{i1}P_i & 0 & \cdots & 0 \\ D_{i2}P_i & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ D_{iN}P_i & 0 & \cdots & 0 \end{bmatrix}, \qquad (11)$$

$$\Omega(P_i,t) = \begin{bmatrix} \psi_i(t)+aP_i & P_iD_{i1} & P_iD_{i2} & \cdots & P_iD_{iN} \\ D_{i1}P_i & -bP_1 & 0 & \cdots & 0 \\ D_{i2}P_i & 0 & -bP_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ D_{iN}P_i & 0 & 0 & \cdots & -bP_N \end{bmatrix}, \qquad (12)$$

$$\Gamma(P_i,t) = \begin{bmatrix} \psi_i(t)-\gamma F_i^\top F_i & P_iD_{i1} & \cdots & P_iD_{iN} \\ D_{i1}P_i & 0 & \cdots & 0 \\ D_{i2}P_i & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ D_{iN}P_i & 0 & \cdots & 0 \end{bmatrix}, \qquad (13)$$

$$\beta_k = \max_{1\leq i\leq N} \lambda_{\max}[P_i^{-1}(I+C_{ik})^\top P_i(I+C_{ik})], \qquad (14)$$

$$\lambda_m = \min_{1\leq i\leq N} \lambda_{\min}(P_i), \;\; \lambda_M = \max_{1\leq i\leq N} \lambda_{\max}(P_i) \qquad (15)$$

where a, b, and γ are constants to be determined,

$$\psi_i(t) = (\tilde{A}_i)^\top P_i + P_i\tilde{A}_i + 2\varphi_i(t)P_i,$$

$i = 1, 2, \cdots N$, $k = 1, 2, \cdots$, P_i are positive definite matrices, $\varphi_i(t)$ are continuous functions on J, which satisfy

$$f_i^\top(t,x_i)P_ix_i \leq \varphi_i(t)x_i^\top P_ix_i, \;\; i = 1, 2, \cdots, N. \qquad (16)$$

where $x_i \in R^n, t \in J$.

Theorem 3.1: For the closed-loop network system (6) with $w_i = 0$, assume that there exist positive-definite matrices P_i and scalars $a > bN > 0$ such that

$$\Omega(P_i,t) \leq 0, \qquad i = 1, 2, \cdots, N. \qquad (17)$$

If $\beta_k \leq \beta$ and ε is a positive solution of the equation $-\varepsilon = -a + bNe^{\varepsilon\tau}$, where β is a constant satisfying $\beta \geq 1$, β_k is given by (14), then $(\ln\beta/\delta\tau) - \varepsilon < 0$ implies that (6) is globally exponentially stable.

Proof. Construct a Lyapunov function in the form of

$$v(x) = \sum_{i=1}^N x_i^\top P_ix_i, \qquad (18)$$

where P_i are positive-definite matrices satisfying (17) and let $v(t){:=}v(x(t))$. For $t \in (t_{k-1}, t_k]$, the total derivative of $v(x(t))$ with respective to (6) is

$$\begin{aligned}
\dot{v}(x(t))\Big|_{(6)} &= \sum_{i=1}^N \Big\{\dot{x}_i^\top P_ix_i + x_i^\top P_i\dot{x}_i\Big\}\Big|_{(6)} \\
&= \sum_{i=1}^N \Big\{x_i^\top(\tilde{A}_i^\top P_i + P_i\tilde{A}_i)x_i \\
&\quad +2f_i^\top(t,x_i)P_ix_i \\
&\quad +\sum_{j=1}^N \Big[x_i^\top P_iD_{ij}x_j(t-\tau) \\
&\quad +x_j^\top(t-\tau)D_{ij}P_ix_i\Big] \\
&\quad +x_i^\top P_iE_iw_i + w_i^\top E_i^\top P_ix_i\Big\} \\
&\leq \sum_{i=1}^N \Big\{x_i^\top\Big[\tilde{A}_i^\top P_i + P_i\tilde{A}_i + 2\varphi_i(t)P_i\Big]x_i \\
&\quad +\sum_{j=1}^N \Big[x_i^\top P_iD_{ij}x_j(t-\tau) \\
&\quad +x_j^\top(t-\tau)D_{ij}P_ix_i\Big] \\
&\quad +x_i^\top P_iE_iw_i + w_i^\top E_i^\top P_ix_i\Big\} \\
&= \sum_{i=1}^N \Big\{y_i^\top\Delta(P_i,t)y_i \\
&\quad +x_i^\top P_iE_iw_i + w_i^\top E_i^\top P_ix_i\Big\}, \\
&\qquad t \in (t_{k-1}, t_k]
\end{aligned}$$

where $y_i(t) = \mathrm{col}(x_i(t), x_1(t-\tau), \cdots, x_N(t-\tau))$, $\Delta(P_i,t)$ is given by (11).

When $w_i(t)=0$, from (17), we obtain

$$\begin{aligned}\dot{v}(x(t)) &\le \sum_{i=1}^{N}\Big\{-ax_i^\top P_i x_i \\ &\quad +b\sum_{j=1}^{N}\Big[x_j^\top(t-\tau)P_j x_j(t-\tau)\Big]\Big\} \\ &= -av(t)+bNv(t-\tau), \\ &\qquad t\in(t_{k-1},t_k],\ k=1,2,\cdots, \quad (19)\end{aligned}$$

where a and b satisfy $0<bN<a$.

Accordingly, by Lemma 3.2, it follows from (19) that

$$v(t)\le v(t_{k-1}^+)e^{-\varepsilon(t-t_{k-1})},\ t\in(t_{k-1},t_k],\ k=1,2,\ldots, \quad (20)$$

where ε is the unique positive root of $-\varepsilon=-a+bNe^{\varepsilon\tau}$.

It follows from (6), (18) and Lemma 3.1 that

$$\begin{aligned}v(t_k^+) &= \sum_{i=1}^{N}x_i^\top(t_k^+)P_i x_i(t_k^+) \\ &= \sum_{i=1}^{N}[(I+C_{ik})x_i(t_k)]^\top P_i[(I+C_{ik})x_i(t_k)] \\ &\le \sum_{i=1}^{N}\lambda_{\max}[P_i^{-1}(I+C_{ik})^\top P_i(I+C_{ik})] \\ &\quad \times x_i^\top(t_k)P_i x_i(t_k) \\ &\le \beta_k v(t_k),\quad k=1,2,\ldots, \quad (21)\end{aligned}$$

where β_k are defined by (14).

For $t\in(t_0,t_1]$,

$$v(t)\le v(t_0)e^{-\varepsilon(t-t_0)},$$

which leads to

$$v(t_1)\le v(t_0)e^{-\varepsilon(t_1-t_0)},$$

and

$$v(t_1^+)\le\beta_1 v(t_1)\le\beta_1 v(t_0)e^{-\varepsilon(t_1-t_0)}.$$

Similarly, for $t\in(t_1,t_2]$,

$$\begin{aligned}v(t)&\le v(t_1^+)e^{-\varepsilon(t-t_1)}\le\beta_1 v(t_1)e^{-\varepsilon(t-t_1)} \\ &\le\beta_1 v(t_0)e^{-\varepsilon(t-t_0)}.\end{aligned}$$

In general, for $t\in(t_k,t_{k+1}]$,

$$v(t)\le v(t_0)\beta_1\cdots\beta_k e^{-\varepsilon(t-t_0)}. \quad (22)$$

Since $\beta_k\le\beta$, $t_k-t_{k-1}\ge\delta\tau$, $(\delta>1)$ and $\beta\ge1$

$$\begin{aligned}\beta_1\cdots\beta_k &\le\beta^k\le\exp\big[\frac{\ln\beta}{\delta\tau}(t_k-t_0)\big] \\ &\le\exp\big[\frac{\ln\beta}{\delta\tau}(t-t_0)\big],\ \ t\in(t_k,t_{k+1}] \quad (23)\end{aligned}$$

From (22) and (23), we have

$$v(t)\le v(t_0)\exp\big[(\frac{\ln\beta}{\delta\tau}-\varepsilon)(t-t_0)\big],\ \ t\ge t_0. \quad (24)$$

Then we get

$$\begin{aligned}\sum_{i=1}^{N}\lambda_{\min}(P_i)\|x_i(t)\|^2 &\le\sum_{i=1}^{N}\lambda_{\max}(P_i)\|x_i(t_0)\|^2 \\ &\quad\times\exp\big[(\frac{\ln\beta}{\delta\tau}-\varepsilon)(t-t_0)\big],\ \ t\ge t_0,\end{aligned}$$

namely,

$$\|x(t)\|\le\sqrt{\frac{\lambda_M}{\lambda_m}}\|x(t_0)\|\exp\big[\frac{1}{2}(\frac{\ln\beta}{\delta\tau}-\varepsilon)(t-t_0)\big],t\ge t_0. \quad (25)$$

where $x=\mathrm{col}(x_1,x_2,\cdots,x_N)$. So the closed-loop system (6) is globally exponentially stable. This immediately completes the proof. ♢

Theorem 3.2: For the closed-loop network system (6), assume that there exist positive-definite matrices P_i and scalars $a>bN>0$ and $\gamma\le0$ such that (17) is satisfied and

$$\Gamma(P_i,t)\le0,\quad i=1,2,\cdots,N. \quad (26)$$

If $\beta_k\le1$, ε is a positive solution of the equation $-\varepsilon=-a+bNe^{\varepsilon\tau}$, and $F_i=E_i^\top P_i$, then the controlled network (6) is passively controlled. Particularly, $\gamma<0$ implies that network (6) is strictly passively controlled.

Proof. Similar to the proof of Theorem 3.1, we obtain

$$v(t)\le v(t_0)\beta_1\cdots\beta_k e^{-\varepsilon(t-t_0)},\quad t\in(t_k,t_{k+1}]. \quad (27)$$

Since $\beta_k\le1$, for $t\ge t_0$

$$v(t)\le v(t_0)e^{-\varepsilon(t-t_0)}, \quad (28)$$

namely,

$$\|x(t)\|\le\sqrt{\frac{\lambda_M}{\lambda_m}}\|x(t_0)\|\exp\big[-\frac{1}{2}\varepsilon(t-t_0)\big],\ t\ge t_0. \quad (29)$$

So the closed-loop system (6) is internally stable.

On the other hand, for any given $T\ge0$, we have

$$\begin{aligned}&2\int_0^T w^\top(s)z(s)ds \\ =&\int_0^T\Big[w^\top(s)z(s)+z^\top(s)w(s)\Big]ds \\ =&\sum_{i=1}^{N}\int_0^T\Big[w_i^\top(s)z_i(s)+z_i^\top(s)w_i(s)\Big]ds \\ =&\sum_{i=1}^{N}\int_0^T\Big[w_i^\top(s)E_i^\top P_i x_i(s)+x_i^\top(s)P_iE_iw_i(s)\Big]ds\end{aligned}$$

$$
\begin{aligned}
= & \sum_{i=1}^{N} \int_{0}^{T} \Big\{ \Big[\frac{dx_i}{ds} - \tilde{A}_i x_i(s) - f_i(t, x_i(s)) \\
& - \sum_{j=1}^{N} D_{ij} x_j(s-\tau) \Big]^\top P_i x_i(s) \\
& + x_i^\top(s) P_i \Big[\frac{dx_i}{ds} - \tilde{A}_i x_i(s) - f_i(t, x_i(s)) \\
& - \sum_{j=1}^{N} D_{ij} x_j(s-\tau) \Big] \Big\} ds \\
\geq & \sum_{i=1}^{N} \int_{0}^{T} \Big\{ \frac{d(x_i^\top(s) P_i x_i(s))}{ds} \\
& - x_i^\top(s) \Big[\tilde{A}_i^\top P_i + P_i \tilde{A}_i + 2\varphi_i(s) P_i \Big] x_i(s) \\
& - \sum_{j=1}^{N} \Big[x_i^\top(s) P_i D_{ij} x_j(s-\tau) \\
& + x_j^\top(s-\tau) D_{ij} P_i x_i(s) \Big] \Big\} ds \\
= & \int_{0}^{T} \frac{dv(s)}{ds} ds - \int_{0}^{T} \sum_{i=1}^{N} y_i^\top(s) \Delta(P_i, s) y_i(s) ds
\end{aligned}
$$

where $y_i(t) = \mathrm{col}(x_i(t), x_1(t-\tau), \cdots, x_N(t-\tau))$. Since for any $T \in (t_k, t_{k+1}]$,

$$
\begin{aligned}
& \int_{0}^{T} \dot{v}(t) dt \\
= & \int_{0}^{t_1} \dot{v}(t) dt + \int_{t_1}^{t_2} \dot{v}(t) dt + \cdots \\
& + \int_{t_{k-1}}^{t_k} \dot{v}(t) dt + \int_{t_k}^{T} \dot{v}(t) dt \\
= & v(t_1) - v(0^+) + v(t_2) - v(t_1^+) + \cdots \\
& + v(t_k) - v(t_{k-1}^+) + v(T) - v(t_k^+) \\
= & v(t_1) - v(t_1^+) + \cdots + v(t_{k-1}) - v(t_{k-1}^+) \\
& + v(t_k) - v(t_k^+) + v(T) - v(0^+) \\
\geq & \sum_{i=1}^{k} (1-\beta_i) v(t_i) - v(0^+) + v(T) \\
\geq & v(T) - v(0^+),
\end{aligned} \tag{30}
$$

then from (26) we have

$$
\begin{aligned}
& 2 \int_{0}^{T} w^\top(s) z(s) ds \\
\geq & v(T) - v(0^+) - \int_{0}^{T} \sum_{i=1}^{N} \gamma x_i^\top(s) F_i^\top F_i x_i(s) ds \\
= & v(T) - v(0^+) - \gamma \int_{0}^{T} z^\top(s) z(s) ds \\
\geq & -v(0^+) - \gamma \int_{0}^{T} z^\top(s) z(s) ds.
\end{aligned}
$$

That is,

$$
\int_{0}^{T} w^\top(s) z(s) ds \geq -\frac{1}{2} v(0^+) - \frac{\gamma}{2} \int_{0}^{T} z^\top(s) z(s) ds.
$$

If $\gamma = 0$, then the system is passive. For $\gamma < 0$, it gives the output strict passivity property of the closed-loop system (6). This immediately completes the proof. $\diamondsuit$

IV. A NUMERICAL EXAMPLE

In this section, we give an example to demonstrate the effectiveness of the proposed methods.

We consider a network (1) with 5 coupled nodes in which each node is a unified chaotic system [14]. The state equation is

$$
\begin{cases}
\dot{x}_1 = (25\alpha + 10)(x_2 - x_1) \\
\dot{x}_2 = (28 - 35\alpha) x_1 - x_1 x_3 + (29\alpha - 1) x_2 \\
\dot{x}_3 = x_1 x_2 - \frac{\alpha+8}{3} x_3,
\end{cases} \tag{31}
$$

where $\alpha \in [0, 1]$.

In [14], it is observed: when $0 \leq \alpha < 0.8$, system (31) belongs to the generalized Lorenz system; when $\alpha = 0.8$, it belongs to the class of chaotic systems introduced in [15]; when $0.8 < \alpha \leq 1$, it belongs the generalized Chen system formulated in [21]. That is, system (31) is chaotic when $\alpha \in [0, 1]$.

Rewrite system (31) as

$$
\dot{x} = Ax + f(x), \tag{32}
$$

where $x = (x_1, x_2, x_3)^\top, f(x) = (0, -x_1 x_3, x_1 x_2)^\top$ and

$$
A = \begin{bmatrix} -(25\alpha + 10) & (25\alpha + 10) & 0 \\ 28 - 35\alpha & 29\alpha - 1 & 0 \\ 0 & 0 & -\frac{\alpha+8}{3} \end{bmatrix}.
$$

α has different value for each node of dynamical network (31). Here, $\alpha = 0, 0.2, 0.8, 0.96, 1$ for each node. The coupling strengths are chosen to be $D_{13} = D_{31} = 0.36$, $D_{23} = D_{32} = 6.4$, $D_{34} = D_{43} = 0.189$, $D_{45} = D_{54} = 3.45$ and

$$
E_1 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 2 & -1 & 0 \\ 0 & -3 & 1 \\ -1 & 0 & 2 \end{bmatrix},
$$

$$
E_3 = \begin{bmatrix} 6 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix},
$$

$$
E_4 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}, \qquad E_5 = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 1 & 4 \\ -2 & 0 & 1 \end{bmatrix}.
$$

Let $\tau = 0.5$, $\gamma = -0.001$, $a = 6.2$ and $b = 1$ satisfying $a > Nb$. Select the controller gain matrices as

$$
B_1 = \begin{bmatrix} -500 & -2 & 0 \\ 10 & -40 & 0 \\ 0 & 0 & -19 \end{bmatrix},
$$

$$B_2 = \begin{bmatrix} -68 & -1 & 0 \\ 1 & -5800 & 0 \\ -10 & 0 & -300 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} -98 & 3 & 0 \\ -8 & -160 & 1 \\ 0 & 0 & -5000 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} -290 & 0 & 0 \\ 0 & -560 & 0 \\ 0 & 0 & -270 \end{bmatrix},$$

$$B_5 = \begin{bmatrix} -230 & 2 & 0 \\ -3 & -580 & 0 \\ 0 & 0 & -270 \end{bmatrix}.$$

and

$$B_{2k} = B = \mathrm{diag}\{b_1, b_2, b_3\} = \mathrm{diag}\{-0.58, -0.65, -0.75\}.$$

Note that for system (32) if take $P_i \equiv I$, then $f^\top(x)x = 0$, that is, in (16), $\varphi_i(t) = 0$. Then it is easy to calculate and obtain

$$\Omega(P_i, t) \leq 0, \text{ and } \Gamma(P_i, t) \leq 0, \ i = 1, 2, \cdots, 5,$$

$$\beta_k = \beta = \max\left\{(1+b_1)^2, (1+b_2)^2, (1+b_3)^2\right\} = 0.1764,$$

which implies, from Theorem 3.2, that each subsystem is exponentially internal stable and the input w and output z satisfy the inequality (10), namely, the closed-loop network system (6) is passively controlled.

V. CONCLUSIONS

In this paper, the passivity property and impulsive control of complex dynamical networks with coupling time-delays are addressed. Sufficient conditions are obtained in terms of solutions of linear matrix inequalities. An example is provided to illustrate the effects of time delays on dynamical networks and to verify the effectiveness of the proposed stability analysis and control design methods.

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