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Diagnostic Symptom Generation Aggregating Model Validity and Residual Evaluation

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Abstract— FDI commonly relies on an analytical model of the process to be diagnosed. This model is supposed to represent the process behavior in a particular mode for a given operating set point. Residuals are computed with this model and some measures acquired online from the process. They are compared to thresholds in order to provide the diagnostic decision. However, the model that has been used to compute a residual can be not suitable. For instance, this may be due to the identification procedure itself that has been performed with data not fully relevant. Another reason may be that the operating set point has changed, leading to a model that is not valid at this new set point. Thus, the diagnostic decision may produce a false alarm. Therefore, it is highly desirable to be able to measure the validity of the model used to provide the residuals. The aim of this paper is to propose an evaluation of this model validity indicator based upon fuzzy aggregation of several partial criteria. The model validity is then aggregated with the diagnostic decision in order to provide a symptom sensitive both to the model validity and to the residual threshold exceeding. Examples will illustrate the proposed method.

Keywords: model-based diagnosis, Fault Detection, symptom generation, model validity, fuzzy aggregation.

I. INTRODUCTION

Fault Detection and Isolation (FDI) is now a major area of control science. This is firstly due to the increasing complexity of industrial processes. Safety is a major subject of interest not only for the process itself but also for its environment and for men living around. A second reason is purely financial. A shut-down induced by security constraints is very costly. Thus, the interest of the supervision system is clearly understandable: avoiding an emergency shutdown is often more relevant from an economical viewpoint than slightly improving the quality of finished product.

Many different diagnostic approaches have been developed among control scientists. The most common approach relies on an analytical model of the process to be diagnosed [4][7][11]. The *model* is supposed to represent the behavior of the process, either in the normal mode or in a particular faulty one. Actually, it constitutes a *prototype behavior* that is compared to the actual state evaluated with data acquired on-line from the process. Usually, analytical equations are used to represent the (normal) process behavior. From the measures and this analytical model, quantities called residuals are computed. From these numerical residuals, detection algorithms have to provide symptoms.

Ideally, when the residual is “null” and the model corresponds to “normal behavior”, the corresponding symptom must indicate that the process is safe. If this residual is different from zero, the symptom must indicate that the process is faulty. In practice, the residual is never exactly zero due to noises corrupting the measures and model imprecision. Data used to provide the prototype behavior must be very reliable, which cannot be always ensured when they are acquired in an industrial context. For instance, the digital signal must be obtained during a time window long enough to allow statistical hypotheses to be checked or parameter estimation to be enough precise. However, during data acquisition, disturbance signals may not affect the process and change the operating point. Unfortunately, long experimental recordings are difficult to program on a process in production. Moreover, input signals must be persistently excited when an identification procedure is performed. Theoretically, several data sets need to be studied; some of them are used for model identification while others are used for model validation. Alas, engineers in industry have seldom the time necessary to process a fully relevant identification procedure. Furthermore, they have no time to re-identify the system each time a component is changed or modified.

The main consequence is that the diagnostic decision is made with a model (or prototype behavior) that is not fully relevant. Hence, the evaluation of the *model validity* must be part of the diagnostic decision. The objective of this work is to evaluate *a priori* the model validity. This *a priori* model validity can be reevaluated when the diagnostic algorithm is running online in order for instance to take into account the actual operating conditions.

Several concepts used to establish diagnostic principles are vague notions, for instance, “the residuals are near zero” or “the model is precise”. These concepts have to be translated into mathematical formulations easily

A simplified version of the algorithms described in the paper has been implemented within the MAGIC project supported by the European Commission under the project EU-IST-2000-30009 MAGIC [8].

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implemented in an algorithm that can be processed online. Moreover, in an industrial context, diagnosis should not be a Boolean process with answers such as "the process is faulty/the process is safe". Using a gradual point of view allows focusing attention on a component before a fault is completely installed [3][10].

The symptom generation proposed in this paper uses both the model validity (section III) and the evaluation of the residuals (section IV), that are represented as fuzzy sets, to generate a relevant decision. An appendix reminds the theoretical basis of fuzzy decision making. Section V illustrates the proposed method.

II. Classical Fault Detection Approach

The FDI community is especially concerned with industrial process diagnosis based on quantitative dynamic models. Two basic representations can be used: state space models and input-output relations. In this work, input-output relations are used. (1) takes into consideration the way faults f and unknown disturbances d affect the measurable output y of the system, excited by an input u:

$$y = g(u, f, d, t) \tag{1}$$

y and u represent observations. Disturbances are un-controlled input signals whose presence is undesired but normal and must be distinguished from faults. Noise is a special kind of disturbance related to random uncertainty. Faults are deviations from normal behavior in the plant or its instrumentation. The *support* of (1) consists in the set of components modeled by this relation. Additive process faults are unknown inputs acting on the plant, which are normally zero. Multiplicative process faults lead to changes in model parameters. Sensor and actuator faults are other significant types of faults, represented as additive signals. Model (1) can take into account both additive faults (extra signals) and modifications to the model parameters (change in h).

The model is used to compute numerical fault indicators, known as residuals r_j, that are null when there is no fault affecting the system. Residual generation refers to the elaboration of relevant fault indicators. It is worth noting that a residual, by using appropriate filters, can represent a much more elaborate quantity than a simple comparison of a process measurement with its model prediction.

A residual r_j must have a computational form gc_j (2), known as an analytical redundancy relation, deduced from the model, depending only on observations, possibly at different times:

$$r_j = gc_j(u, y, t) \tag{2}$$

The residual r_j evaluation form ge_j is expressed by (3):

$$r_j = ge_j(u, y, f, d, t) \tag{3}$$

which shows how it is influenced by the faults and the un-known disturbances. Ideally, a residual should be decoupled from the unknown disturbances and dependent only on a single fault f_j:

$$r_j = ge_j(u, y, f_j, t) \tag{4}$$

In (4), when f_j is null, r_j should be zero. If r_j is not zero, this results from an inconsistency between the model and the observations. When new data come from the acquisition system, residuals are computed using (2) and are interpreted to obtain symptoms. This step is known as fault detection.

FDI relies implicitly on the exoneration assumption, as has been fully highlighted in a collective work published in [1] and is briefly reminded here 0. The exoneration assumption means that a faulty component necessarily shows its faulty behavior, i.e. causes any analytical redundancy relation in which its model is involved not to be satisfied by any given set of observations. Equivalently, given the set of observations, any set of components whose model is involved in a satisfied ARR is exonerated, i.e. each component of the ARR support is considered to behave correctly. In this general exoneration assumption, there is a single-fault exoneration assumption — each individual component shows its faulty behavior — and a non-compensation assumption — the individual effects of faulty components never compensate each other.

Let $\{ARR_1\} \subset \{ARR\}$ be the subset of relations potentially affected by a set of faults $F_1 \subset F$, and let f_p be the present fault. The exoneration assumption is expressed as follows:

$$\{ARR_1(OBS)\} = 0 \leftrightarrow f_p \in \overline{F_1} \tag{5}$$

where $\overline{F_1}$ is the complement of the set F_1 in F. (5) is equivalent to (6):

$$\{ARR_1(OBS)\} \neq 0 \leftrightarrow f_p \in F_1 \tag{6}$$

When the exoneration assumption is not made, then:

$$\{ARR_1(OBS)\} = 0 \leftarrow f_p \in \overline{F_1} \tag{7}$$

whose contrapositive is:

$$\{ARR_1(OBS)\} \neq 0 \rightarrow f_p \in F_1 \tag{8}$$

These principles will be used in section IV to justify the choice of fuzzy operators.

III. Model Validity

The *model validity* $MV \in [0,1]$ measures the confidence that can be set to the model used for the fault detection, which obviously influences the confidence in the symptom. $MV = 1$ means that the confidence attached to this model is complete while $MV = 0$ corresponds to an invalid model.

This section presents the evaluation of the *a priori* and *a posteriori* model validity when the model is the reference transfer function:

$$H^R(z) = \frac{y(z)}{u(z)} = z^{-k} \frac{b_1 + b_2 z^{-1} + \cdots + b_{nb} z^{-nb}}{1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}} \tag{9}$$

corresponding to (1) where f and d are null. H^R is computed during an identification step performed with the so-called reference signals x_j^R, $j = 1:K$, where x stands for y and u. Note that usually, several sets of the input/output data should be used for the identification of the model parameters and for its validation.

A. A priori model validity

The *a priori* model validity AMV computation takes into account the trust that can be put in the K reference signals x^R_j, $j = 1{:}K$, used to identify (and validate) the model, in the identified parameters and in the model structure itself. The AMV evaluation is based on a fuzzy aggregation of p partial criteria c_i, $i = 1 : p$.
Each transfer function $H^R(z)$ is obtained with an appropriate identification technique, for instance an output error method, an ARX method, etc. depending on the application and *a priori* knowledge of the process.

In order to compute AMV, a first criterion quantifies the adjustment of the identified model to the measures. Let:

$$e_k = y_k - y_k^* \tag{10}$$

be the output error of the model where y_k and y_k^* are respectively the output of the system and of the model respectively to the operating point OP. k is the sampling time. Define the relative error over the identification window of length N:

$$F = \sum_{k=1}^{N} e_k^2 \Big/ \sum_{k=1}^{N} y_k^2 \tag{11}$$

$c_1 \in [0,1]$ measures the fit of the simulated output to the real output of the system:

$$c_1 = 1 - \min\{1, F\} \tag{12}$$

$c_1 = 1$ corresponds to a perfect fit ($e_k = 0$ over the validation window). When $F \to 1$, which means that the error is important, $c_1 \to 0$. Identification software tools as Matlab implement an index computation to evaluate the fitness of the model identified respectively to the validation data. In [9], such an index is proposed for neural networks based models.
Let $\theta_j, j = 1 : na + nb$ be the identified parameters. The second criterion takes into account the standard deviation σ_j that is evaluated for each θ_j during the identification of $H^R(z)$. Define the parameter relative precision:

$$PRP^j = \left|\frac{\sigma_j}{\theta_j}\right| \quad if \quad \theta_j \neq 0 \tag{13}$$

Then, the partial criterion related to this parameter is:

$$c_2^j = 1 - \min\{1, PRJ^j\} \tag{14}$$

Note that the case $\theta j = 0$, $\sigma_j \neq 0$ is handled in a more intricate way that is not presented in this paper. $c_2^j \to 1$ corresponds to a very precise parameter (*i.e.* $PRP^j \to 0$) while $c_2^j \to 0$ stands for an imprecise parameter.
A third criterion c_3 is related to the input excitation. The condition number $cond(H)$ of the Hankel-like matrix built form u_k and y_k, $k = 1 : N$:

$$H = \left[\begin{array}{cccc|ccc} y_1 & y_2 & \cdots & y_{na} & u_{k+1} & \cdots & u_{k+nb} \\ y_2 & y_3 & & y_{na+1} & u_{k+2} & & \vdots \\ y_3 & \vdots & & \vdots & \vdots & & \\ \vdots & & & & & & \end{array}\right] \tag{15}$$

measures the process excitation. $cond(H) = 1$ means that the system is perfectly well excited while $cond(H) >> 1$ stands for a poorly excited process. c_3 must be in the interval $[0,1]$. Therefore, an elegant solution is to consider for this third indicator the inverse of $cond(H)$:

$$c_3 = 1/cond(H_u) \tag{16}$$

AMV is then given by a fuzzy aggregation:

$$AMV = h\left(c_1, c_2^1, \ldots, c_2^J, c_3\right) \tag{17}$$

This paper proposes for h a compromise operator such that:

$$AMV = \frac{1}{na + nb + 2}\left(c_1 + \sum_{j=1}^{na+nb} c_2^j + c_3\right) \tag{18}$$

Thus, all partial criteria are equally weighted. Therefore, the same confidence is attached to the output fit and to each parameter, which means that the structure of the model is supposed to have been properly chosen.
It is well known that the relative identification error F can be small with an over-parameterized model that will give bad simulation results when used with other type of inputs. An over-parameterized model can be detected when PRP^j is increasing. AMV as proposed in (18) reflects these two phenomena.

B. Online criteria for updating the model validity

During the online diagnostic procedure, AMV is modified into *a posteriori model validity* $MV \in [0,1]$. Actually, it is quite evident that when the model has been identified for particular conditions (*i.e.* operating point, functioning mode, etc.) and the actual conditions are slightly or completely different, AMV may be unsuitable or inappropriate. Therefore, the AMV numerical value must be updated.
For instance, consider that the signals used during the identification x^R_j, $j = 1{:}K$, have been acquired around an operating point $OP_{ref} \in \mathbb{R}^M$ (supposed to be defined as a

vector, for example OP_{ref} = [*speed, load, current*] at the reference operating point) in unfaulty functioning mode. If the process is non-linear, which is quite often verified in industry, the model computed with the signals around OP_{ref} is not suitable for other operating points even in unfaulty functioning mode. Suppose that the process is now functioning around a new set point $OP_{act} \in \mathbb{R}^M$ quite far from OP_{ref}. Therefore, the computed model is no more relevant for this new operating point and AMV must be updated. A normalized distance $D(OP_{act})$ between the new and the reference operating points can be used in order to decrease the *a priori* model validity:

$$D(OP_{act}) = f(OP_{ref}, OP_{act}) \tag{19}$$

The model has been identified around the reference set point (u_{ref}, y_{ref}) and it is supposed to be valid for $y \in [y_{\min}, y_{\max}]$ with, $\Delta = y_{\max} - y_{\min}$. In this paper, (19) depends on the distance between the actual output y_k and the bounds $y_{\max}$ and $y_{\min}$ (see Figure 1).

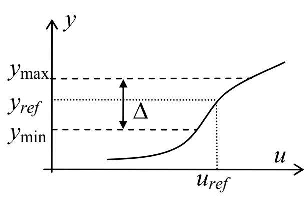

Figure 1: Definition of $D(OP_{act})$

A criterion based on this distance has been implemented:

$$c_4 = \begin{cases} 0 \ \text{if} \ \ y_k \in [y_{\min}, y_{\max}] \\ \min\left(1, \dfrac{|y_k - y_{\max}|}{\Delta/2}\right) \ \text{if} \ \ y_k > y_{\max} \\ \min\left(1, \dfrac{|y_k - y_{\min}|}{\Delta/2}\right) \ \text{if} \ \ y_k < y_{\min} \end{cases} \tag{20}$$

Thus, c_4 = 0 means that the system output is in the neighborhood of y_{ref} while c_4 = 1 stands for an operating set point far from the reference one, which means that the model currently used is not fully relevant.

Another partial criterion, $c_5 \in [0,1]$, could be related to the sensor used for the signal acquisition. Even during the unfaulty functioning mode, the sensor characteristics are modified. For instance, the bias introduced in the measures may change due to the ageing of the sensor. Its sensitivity may also be altered by external conditions. Another classical situation in an industrial context is that the sensor employed when the reference signals $x^R{}_j$, have been acquired has been replaced with another one, the characteristics of which being slightly different (for instance the linearity of the actual sensor is lower that the linearity of the sensor used during the acquisition of $x^R{}_j$). If these changes are quantifiable, the *a priori* model validity AMV must be modified thanks to c_5 associated with the presently recorded signal. Note that c_5 is application dependant and could be evaluated with a fuzzy aggregation of partial criteria (sensor accuracy, sensitivity, linearity, sensor ageing, etc.).

C. A posteriori model validity

The *model validity* MV is computed using a fuzzy aggregation of AMV and of the partial criteria evaluated online (such as c_4 and c_5):

$$MV = h(AMV, c_4, c_5) \tag{21}$$

where h is the aggregation function. Note that in (21) a conjunctive operator seems quite natural because it takes into account the worse partial criterion. It decreases the value of the *a priori* model validity, depending on the experimental conditions:

$$MV = \min(AMV, c_4, c_5) \tag{22}$$

These concepts will be exemplified in section V.

D. Association of Models

A residual can correspond to the agglomeration (product, summation) of M partial models H_m^R, $m = 1:M$. The global *a priori* model validity is then computed with the aggregation of the partial *a priori* model validity:

$$AMV = h(AMV_1, ..., AMV_M) \tag{23}$$

In equation (23), a conjunctive operator is to be expected, for instance:

$$AMV = \min(AMV_1, ..., AMV_M) \tag{24}$$

In that way, the global *a priori* model validity exhibits the worse partial model *a priori* validity. Therefore, the confidence in the global model cannot be better than the "worse" partial model. MV is then evaluated as in (22) where c_4 and c_5 are computed for the global model.

IV. Residual Evaluation and Diagnostic Decision

A. Residual Evaluation

The residual evaluation can simply be made by comparing it to a fixed threshold, obtained empirically. In this paper, it is based on fuzzy set theory [14], well suited to detection in systems with model uncertainty or disturbed by random perturbations. The most simple procedure consists in evaluating the residual with two fixed values t_0 and t_1 (see Figure 2) to obtain the fault detection indicator $FD(res)$.

Parameters t_0 and t_1 are specific to each residual and thus need to be considered as a parameterization of the algorithm. When the residual is greater than t_1, it is sure that there is a fault. Thus, $FD(res)$ is 1. When the residual has the same order of magnitude as the noise standard deviation σ, the component/support of the model used to compute it can be considered in its normal state if *exoneration* is used. Then, the fault detection indicator is 0.

Therefore, σ is used as the lower threshold t_0. If σ is unknown, t_0 can be fixed to zero, but it is worth noting that classical identification procedures allow estimating σ at the same time as the model parameters. If no value of t_1 can be *a priori* fixed, a reasonable value is to choose $t_1 = 4\sigma$.

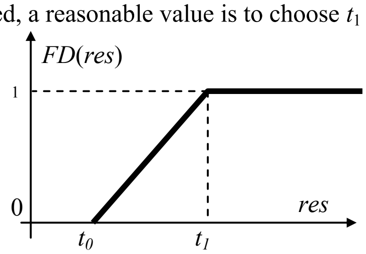

Figure 2: Fuzzy residual evaluation.

More intricate procedures can be used to compute $FD(res)$. For instance, [3] proposes to base the evaluation on the dynamics of the residual on a time window.

B. Diagnostic Decision

The diagnostic decision measures the membership of the symptom S to an alarm, which means that the component whose model is used to make the test is not in normal state. The variation of the decision is gradual and S satisfies $S \in [0,1]$. It combines both the *a posteriori* model validity MV and the residual evaluation $FD(res)$.

Several semantics can be used for S. The first one is as follows: "1 means that the actual component behavior is inconsistent with respect to the model (faulty) one" while "0 stands for the actual behavior is consistent with it (unfaulty in case of exoneration)". "0.5 means that the status is not decidable (neither faulty nor safe)". A polynomial can be defined to match these requirements:

$$S = \alpha FD(res) + \beta MV + \chi MV * FD(res) + \delta \qquad (25)$$

The value of these polynomial coefficients fulfill:

- for $MV = 1$, $S = FD(res)$;
- for $MV = 0$, $S = 0.5$.

Their identification results in:

$$\begin{cases} \alpha = 0 \\ \beta = -0.5 \\ \chi = 1 \\ \delta = 0.5 \end{cases} \qquad (26)$$

Finally, the diagnostic decision is given by:

$$S = \frac{1 + MV(2FD(res) - 1)}{2} \qquad (27)$$

which has been used in [12].

When S evaluation is interpreted in the context of fuzzy aggregation, a conjunction operator can be proposed:

$$S = h(FD(res), MV) \qquad (28)$$

The semantic of (28) is that a "high" residual ($FD(res) \to 1$) and a "high" *a posteriori* model validity ($MV \to 1$) must exist together to decide that there is a fault. If this is not the case, no decision is made. In particular, either a small residual or a small model validity results in S = 0. This situation corresponds to the non-exoneration case: a small residual does not allow deciding about the support state.

Note that h can also be chosen as a min function:

$$S = \min(FD(res), MV) \qquad (29)$$

or as a more intricate operator [13]:

$$S = \max(0, (FD(res) + MV - 1)) \qquad (30)$$

Some examples are given in TABLE 1 to illustrate the norm influence. For comparison purpose, results achieved with (27) are also exhibited. Remember that the semantics of (29)-(30) and (27) are a bit different.

Table 1: Symptom Evaluation.

Eq.	$FD(res)$	$MV = 1$	$MV = 0.5$	$MV = 0$
(29)	0	$\to S = 0$	$\to S = 0$	$\to S = 0$
	1	$\to S = 1$	$\to S = 0.5$	$\to S = 0$
(30)	0	$\to S = 0$	$\to S = 0$	$\to S = 0$
	1	$\to S = 1$	$\to S = 0.5$	$\to S = 0$
(27)	0	$\to S = 0$	$\to S = 0.25$	$\to S = 0.5$
	1	$\to S = 1$	$\to S = 0.75$	$\to S = 0.5$

V. APPLICATION

The proposed *a priori* model validity computation is now tested on a second order transfer function:

$$H(z) = z^{-1}\frac{1 - 0.7z^{-1}}{1 - 1.8z^{-1} + 0.89z^{-2}} = \frac{Y(z)}{U(z)} \qquad (31)$$

The simulated data are given in Figure 3 for two different inputs The first one is a PRBS sequence while the second one is a triangle signal that provides a poorly excited system. The sampling period is $T_s = 0.02s$. Note that a Gaussian white noise $\mathcal{N}(0, \sigma = 0.2)$ has been added to the output data y. AMV is computed with (18) and some results are summarized in TABLE 2. Note that (12) is computed with validation data (actually, another PRBS sequence).

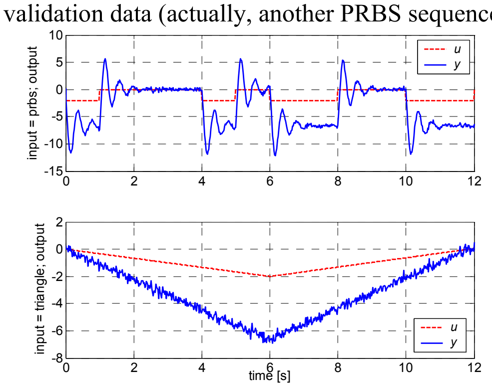

Figure 3: Data (top, u = PRBS; bottom, u = triangle)

As expected, AMV computed with the triangle input signal is smaller than AMV obtained with the PRBS input sequence. For the PRBS signal, the "best" model is of course obtained with the proper model structure. Remark that an inappropriate choice of the model structure leads to a small value of *AMV*. Lastly, (16) is computed with the model output y_k^* instead of the process one. Actually, the output noise artificially decreases the condition number of matrix H, leading to a wrong c_3 numerical value, and therefore to an inappropriate *AMV* evaluation.

Table 2: *AMV* for Different Model Structures.

Model structure [*nb*,*na*,*k*]	(u = PRBS) *AMV*	(u = triangle) *AMV*
[1,1,1]	0.56	0.53
[1,2,1]	0.73	0.56
[2,2,1]	0.79	0.43
[2,2,2]	0.78	0.39
[2,3,2]	0.60	0.25
[3,3,1]	0.37	0.24

VI. Conclusion and Further Work

This paper has proposed a diagnostic symptom S generation that aggregates the usual residual evaluation *FD*(*res*) together with an indicator of the model validity *MV*. *MV* is obtained by a fuzzy aggregation of the offline *a priori* model validity *AMV* with online criteria. *AMV* is itself obtained with fuzzy aggregation of partial criteria that measure the relevance of the identification step. A crude version of S has been implemented in [5].Note that an “optimal” computation of *AMV* is intricate because the weight attached to each partial criterion is context dependant. In fact, the *AMV* evaluation is strongly related with the model validation problem in identification context, and it is well known that no global answer exists. Different formula for (17) are under comparison (in the context of diagnosis) and results will be published later. Note that other partial criteria, such as the Akaike one, could also be implemented. An extension to nonlinear system is under study.

Appendix: Fuzzy Decision Making

Fuzzy decision making for diagnostic decision allows formal modeling of decision-making for imprecise and uncertain conditions in order to select a solution characterized by partial points of view, called partial criteria [2]. In a known environment, each decision $d \in D$ (where D is the set of possible decisions) is evaluated by a series of values [$c_1(d)$, $c_2(d)$,…, $c_p(d)$] where $c_i(d)$ measures the decision d in the sense of criterion i. d is defined as a fuzzy subset obtained by aggregation of p partial criteria. Thus, the membership function μ_d is such that:

$$\mu_d = h\left(c_1\left(d\right), c_2\left(d\right), ..., c_p\left(d\right)\right) \tag{32}$$

where h is a fuzzy set operator connective to be determined [1]. Necessary conditions on operator h are:

- h is a continuous function;
- $h\left(0,0,...,0\right)=0$ and $h\left(1,1,...,1\right)=1$;
- $\forall\left(u_i, v_i\right) \in \left[0,1\right]^2$, if $u_i \geq v_i$ then $h(u_1,\ldots, u_p) \geq h(v_1,\ldots, v_p)$.

Three main decision-making attitudes can be modeled.

1. For an operator h expressing that all the criteria are met simultaneously, a natural axiom is:

$$\forall\left(u_1,...,u_p\right),\ h\left(u_1,...,u_p\right) \leq \min_i\left(u_i\right),\ i=1:p \tag{33}$$

which means that the overall evaluation of a decision cannot be better than the smallest (*i.e.* “worst”) value of the partial evaluations. These operators correspond to conjunctions.

2. To express the redundancy of the objectives, the h operator must meet the following condition:

$$\forall\left(u_1,...,u_p\right),\ \max_i\left(u_i\right) \leq h\left(u_1,...,u_p\right),\ i=1:p \tag{34}$$

which means that the overall evaluation is determined by the highest (*i.e.* “best”) value of the partial evaluations. These operators are disjunctions.

3. Operator h becomes a compromise when the following axiom is verified:

$$\forall\left(u_1,...,u_p\right),\ \min_i\left(u_i\right) \leq h\left(u_1,...,u_p\right) \leq \max_i\left(u_i\right),\ i=1:p \tag{35}$$

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