Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Robust Uplink Resource Allocation in CDMA Cellular Radio Systems

Erik Geijer Lundin, Fredrik Gunnarsson and Fredrik Gustafsson
Division of Automatic Control
Department of Electrical Engineering, Linköping University
SE-581 83 LINKÖPING, SWEDEN
{geijer, fred, fredrik}@isy.liu.se*

Abstract— **Radio resource management (RRM) in cellular radio system is an example of automatic control. The system performance may be increased by introducing decentralization, shorter delays and increased adaptation to local demands. However, it is hard to guarantee system stability without being too conservative while using decentralized resource management.**

In this paper, two algorithms that both guarantee system stability and use local resource control are proposed for the uplink (mobile to base station). While one of the algorithms uses only local decisions, the other uses a central node to coordinate resources among different local nodes.

In the chosen design approach, a feasible solution to the optimization problems corresponds to a stable system. Therefore, the algorithms will never assign resources that lead to an unstable system. Simulations indicate that the proposed algorithms also provide high capacity at any given uplink load level.

I. Introduction

The control aspect of radio resource management may be stated as: provide as high capacity as possible while offering acceptable *quality of service* to the users without jeopardizing system stability, despite user movement and external disturbance. An overview of radio resource management is given in [1].

A system is feasible if there exist finite transmission powers to support the allocated services (typically related to the uplink signal to noise ratio) [2]. System feasibility is thus a necessary requirement for system stability.

To efficiently assign resources to the users, while guaranteeing system feasibility, it is our strong belief that the control algorithm should be a combination of centralized and decentralized control. Centralization enables coordination of resources over a wider area. This is useful for example when there is a relatively high concentration of users in a small area. Because of the limited bandwidth between local nodes and the central node, detailed information can not be made available in the central node. Decentralization, on the other hand, gives additional performance gains by using the detailed information on the immediate radio environment and current user demands. An addition to WCDMA (the radio interface of UMTS) called *Enhanced Uplink* is currently being standardized by 3GPP [3]. The objective is to enable high uplink data rates and short delays. This will be done by using more efficient retransmission protocols and decentralizing much of the RRM [4].

The resource allocation problem can, like many control problems, be formulated as an optimization problem. The utility function in this optimization problem can be *user centric*, as in maximizing each users transmission rate, or *network centric* as in maximizing the system throughput [5]. We will adopt a network centric approach.

Examples of where optimization is used and feasibility is guaranteed without the use of a central node but instead giving all local nodes a common rule are [6], [7], [8]. A drawback with the methods given in the above references is that the local nodes require knowledge of the current received interference power, which is generally considered hard to measure accurately.

Despite this, it is recommended to study the uplink interference power in the radio resource management since it provides better robustness to variations in the radio environment [9], [10].

In this contribution, two algorithms that combine guaranteed system feasibility with decentralized resource management are proposed. The algorithms control the interference power induced in neighboring cells as well as the own cell. Furthermore, the only measurements that the algorithms rely on are of the signal power gain between mobile and base, which are assumed reported by the mobiles. No measurements of the interference power are required.

Section II explains the notation and system model used. A criteria for system feasibility is provided in Section III. The radio resource management algorithms studied in this contribution are then explained in Section IV through Section VI. Simulations are used to investigate the performance of the algorithms. Results of those are discussed in Section VII before conclusions are made in Section VIII.

II. System Model

Consider a scenario consisting of B base stations, or local nodes[1], and M users. User i is solely connected to local node K_i and the set of users connected to local node j is denoted c_j. The power gain between user i and local node j is denoted $g_{i,j} < 1$. Each user i is transmitting with power p_i. The uplink channel is modeled by the power gain g_{i,K_i}.

*This work is supported by the Swedish Agency for Innovation Systems (VINNOVA), Information Systems for Industrial Control and Supervision (ISIS) and in cooperation with Ericsson Research, which are all acknowledged.

[1]The terms base station and local nodes will be used interchangeably in this contribution.

The bit rate and bit error probability perceived by user i is related to the *carrier-to-interference-ratio* (CIR),

$$\gamma_i \triangleq \frac{p_i g_{i,K_i}}{I_{K_i}^{tot} - (1-\alpha) p_i g_{i,K_i}},$$

where α is the self interference factor. For notational ease, the *carrier-to-total-interference-ratio* (CTIR) is introduced[2],

$$\beta_i \triangleq \frac{p_i g_{i,K_i}}{I_{K_i}^{tot}}.$$

The quantity I_j^{tot} is the total interference power in base station j. Power control adjusts the users' transmission powers, p_i, $i = 1, 2, \ldots, M$ to meet user individual target CTIR values, β_i^{tgt}, while mitigating time-varying disturbances.

Definition 1 (System Feasibility): Given all users' power gain to all local nodes, $g_{i,j}$, $i \in \{1, M\}$, $j \in \{1, B\}$, and user individual target CTIR values, β_i^{tgt}, a system is *feasible* if it exists user individual finite positive transmission powers such that

$$\beta_i \geq \beta_i^{tgt} \text{ for all } i.$$

Otherwise, the system is *infeasible*.

System feasibility is one out of many names used for this system property [2], [11]. In view of this definition, the resource management algorithms can be seen as mechanisms to assign CTIR levels to users, while ensuring system feasibility.

The primary uplink resource of a CDMA cellular radio system at a particular base station j is the total received interference power, I_j^{tot}. This quantity is modeled as the sum of a base station specific background noise power, N_j, and contributions from all users in the entire network, $p_i g_{i,j}$,

$$I_j^{tot} = N_j + \sum_{i=1}^{M} p_i g_{i,j}. \tag{1}$$

As system feasibility will be related to the maximum eigenvalue of a matrix, it is useful to introduce the notation $\lambda(A)$ for the eigenvalues of a general matrix A. Furthermore, the maximum eigenvalue is denoted by $\bar{\lambda}(A)$, i.e.,

$$\lambda(A) \triangleq \mathrm{eig}(A)$$
$$\bar{\lambda}(A) \triangleq \max \lambda(A).$$

III. System Feasibility

This section is devoted to finding a criteria for establishing whether or not the system is feasible, i.e., whether it exists finite transmission powers to support the users' target CTIR values.

If the resource control algorithm manages to maintain a reasonable load, it is fair to approximate the actual experienced CTIR, β_i, with β_i^{tgt}, i.e., it is assumed that power control manages to track β_i^{tgt} despite time-varying variations in g_{i,K_i} and interference from other connections. We thus concentrate on the equations as if the power control has reached a steady state. User i's transmission power can then be found as

$$\beta_i^{tgt} = \frac{p_i g_{i,K_i}}{I_{K_i}^{tot}} \Rightarrow p_i = \beta_i^{tgt} \frac{I_{K_i}^{tot}}{g_{i,K_i}}.$$

[2]The CTIR, β, is related to γ through $\beta = \frac{\gamma}{1+(1-\alpha)\gamma}$.

The interference power contribution in base station j from users connected to base station k is

$$I_{k,j} \triangleq \sum_{i \in c_k} p_i g_{i,j} = \sum_{i \in c_k} \beta_i^{tgt} \frac{g_{i,j}}{g_{i,K_i}} I_{K_i}^{tot} = \sum_{i \in c_k} \beta_i^{tgt} z_{i,j} I_k^{tot},$$

where the definition of relative power gain between user i and local node j, $z_{i,j}$, is

$$z_{i,j} \triangleq \frac{g_{i,j}}{g_{i,K_i}}.$$

By duality between K_i and c_k,

$$K_i = k \text{ for all } i \in c_k.$$

Let the element on row k and column j of the *system matrix* L be defined by

$$L_{k,j} \triangleq \frac{I_{k,j}}{I_k^{tot}} = \sum_{i \in c_k} \beta_i^{tgt} z_{i,j}. \tag{2}$$

Adding all cells' contributions to the background noise yields

$$I_j^{tot} = N_j + \sum_{k=1}^{B} I_{k,j} = N_j + \sum_{k=1}^{B} L_{k,j} I_k^{tot},\ j = 1, 2, \ldots, B.$$

A compact expression for the total received interference power in all base stations is thus

$$I^{tot} = N + L^T I^{tot}, \tag{3}$$

where $I^{tot} = [I_j^{tot}]$.

Lemma 1: All elements of the vector $I^{tot} = N + L^T I^{tot}$ are positive and finite if $\bar{\lambda}(L) < 1$, $L > 0$ and $N > 0$.

Proof: Let $B(\alpha)$ be the adjoint matrix of the characteristic matrix $\alpha E - A$, i.e., $B(\alpha) \triangleq (\alpha E - A)^{-1} \Delta(\alpha)$, where $\Delta(\alpha) = \det(\alpha E - A)$ is the characteristic polynomial of A and E is the identity matrix. If all elements of the matrix L are positive ($L > 0$), Perron-Frobenius theory states that all elements of the matrix $B(\alpha)$ is non-negative and finite if $\bar{\lambda}(A) < \alpha$ [12]. By choosing α to 1 and A to L^T, we get that if $\bar{\lambda}(L^T) < 1$ then

$$0 < I^{tot} = (E - L^T)^{-1} N < \infty,$$

since all elements in the inverse as well as in N are positive and finite. Using $\lambda(L) = \lambda(L^T)$ completes the proof. ■

Theorem 1: A wireless network with user's uplink CTIR targets, β_i^{tgt}, and uplink relative power gain $z_{i,j}$ is uplink feasible if $\bar{\lambda}(L) < 1$. The resulting interference vector I^{tot} is given by

$$I^{tot} = (E - L^T)^{-1} N.$$

Proof: There are finite transmission powers to support all users' target CTIR values, i.e., the system is feasible, if and only if all elements of a solution to Equation (3), I^{tot}, are positive. According to Lemma 1, all elements are positive if $\bar{\lambda}(L) < 1$. The corresponding interference vector is the solution to Equation (3),

$$I^{tot} = N + L^T I^{tot} \text{ implies } I^{tot} = (E - L^T)^{-1} N.$$

■

IV. A Centralized Algorithm

Provided that the central node has knowledge of the entire power gain matrix $[g_{i,j}]$, it can make a user target CTIR assignment while considering the situation in the entire network. According to Theorem 1, the system is feasible if $\bar{\lambda}(L) < 1$. An example of an optimization problem is thus

$$\begin{aligned} &\underset{\beta_i^{tgt}}{\text{maximize}}\ U(\beta_i^{tgt}) \\ &\text{s.t.}\ \begin{cases} \bar{\lambda}(L) \leq L_f^{tgt}, \\ \beta_{min} \leq \beta_i^{tgt} \leq \beta_{max},\ i = 1, 2, \ldots, M. \end{cases} \end{aligned} \tag{4}$$

where $U(\beta^{tgt})$ is some utilization function representing the chosen resource management policy and $L_f^{tgt} < 1$ is a tuning parameter. A higher L_f^{tgt} yields higher capacity, but it also makes it harder for the power control algorithm to maintain a satisfactory low variance in the users' received CIR. The first constraint guarantees system feasibility while the second yields that the assigned target CTIR values are in $[\beta_{min}, \beta_{max}]$.

Inspired by Shannon capacity [13], the following definition of capacity is made.

Definition 2: The *system capacity* is

$$\sum_{i=1}^{M} \log_2(1 + \gamma_i^{tgt}).$$

The system capacity can be also be put in terms of target CTIR,

$$\sum_{i=1}^{M} \log_2(1 + \gamma_i^{tgt}) = \sum_{i=1}^{M} \log_2 \frac{1 + \alpha\beta_i^{tgt}}{1 - (1 - \alpha)\beta_i^{tgt}}.$$

The algorithm that assigns resources to the users by solving problem (4) with

$$U(\beta_i^{tgt}) = -\sum_{i=1}^{M} \log_2 \left(\frac{1 + \alpha\beta_i^{tgt}}{1 - (1 - \alpha)\beta_i^{tgt}} \right).$$

will be referred to as the *centralized algorithm.*

$$\begin{aligned} &\underset{\beta_i^{tgt}}{\text{maximize}}\ -\sum_{i=1}^{M} \log_2 \left(\frac{1 + \alpha\beta_i^{tgt}}{1 - (1 - \alpha)\beta_i^{tgt}} \right) \\ &\text{s.t.}\ \begin{cases} \bar{\lambda}(L) \leq L_f^{tgt}, \\ \beta_{min} \leq \beta_i^{tgt} \leq \beta_{max},\ i = 1, 2, \ldots, M. \end{cases} \end{aligned} \tag{5}$$

Solving the optimization problem in (5) requires knowledge of all users' relative power gain values to all base stations. A centralized solution like this therefore comes with the price of heavy signaling and/or slow adaptation to changes in the radio environment users experience. In practice this leads to a demand for back-off in the optimization, which yields decreased utilization. Here, the algorithm is merely used to provide an upper bound on the system capacity.

V. A Semi-Centralized Algorithm

A. Guaranteeing System Feasibility

The fundamental idea here is to limit the intercell interference, $I_{k,j}$, $k \neq j$. The variable $L_{k,j}$ is a measure of how much load cell k introduces in cell j. More specifically, $L_{k,j} I_k^{tot}$ is the intercell interference power users in cell k introduce in cell j.

One way of limiting the off diagonal elements of L, i.e., to limit the intercell interference, is to limit each term in (2). A natural strategy is then that users with small relative power gain $z_{i,j}$ can be given a higher β_i^{tgt}. Feeding back complete knowledge on the relative power gains from the local nodes to a central node requires heavy signaling, and is thus avoided in practice if possible. The amount of information sent from each local node to the central node can be reduced if, instead of sending all users' relative power gain measurements, a weighted version of the relative power gain values is sent. The information fed back from local node k to the central node is

$$Y_{k,j} \triangleq \frac{1}{\sum_{i \in c_k} \beta_i^{tgt}} \sum_{i \in c_k} z_{i,j} \beta_i^{tgt},\ j = 1, 2, \ldots, B. \tag{6}$$

This way, the amount of information sent to the central node depends only on the number of local nodes and not on the number of users. Introduce the matrix $\bar{L} = [\bar{L}_{k,j}]$ as

$$\bar{L}_{k,j} \triangleq s_k Y_{k,j},$$

where s_k is a scalar.

Lemma 2: If $L_{k,j} \leq \bar{L}_{k,j}$ for all $k, j \in \{1, B\}$ and $\bar{\lambda}(\bar{L}) \leq L_f^{tgt}$ then $\bar{\lambda}(L) \leq L_f^{tgt}$.

Proof: Follows from the fact that increasing an element of a positive matrix can not decrease the maximum eigenvalue. ■

If the resource assignment algorithm chooses target CTIR values such that

$$L_{k,j} = \sum_{i \in c_k} \beta_i^{tgt} z_{i,j} \leq \bar{L}_{k,j} \text{ for all } k, j \in \{1, B\}, \tag{7}$$

then, according to lemma 2, $\bar{\lambda}(\bar{L}) \leq L_f^{tgt}$ yields $\bar{\lambda}(L) \leq L_f^{tgt}$. An interpretation of s_k as a resource pool is given by studying Equation (7) in the case when $k = j$, i.e., on the diagonal of L (note that $z_{i,K_i} = 1$ for all i),

$$L_{k,k} = \sum_{i \in c_k} \beta_i^{tgt} z_{i,k} = \sum_{i \in c_k} \beta_i^{tgt} \leq \bar{L}_{k,k} = s_k,$$

i.e., s_k is an upper bound on the sum of the target CTIR values of users connected to local node k.

Lemma 3: If

$$\begin{pmatrix} E & \bar{L} \\ \bar{L}^T & L_f^{tgt\,2} E \end{pmatrix} \succeq 0 \text{ then } \bar{\lambda}(\bar{L}) \leq L_f^{tgt}. \tag{8}$$

Proof: See the appendix. ■

Theorem 2 (System Feasibility): A system using a RRM algorithm that meets the inequalities in Equations (7) and (8) is feasible if $L_f^{tgt} < 1$.

Proof: As the requirement in Equation (8) is met, $\bar{\lambda}(\bar{L}) \leq L_f^{tgt}$. According to Lemma 2, $\bar{\lambda}(L)$ is then also less than L_f^{tgt} if the inequality in (7) is met. Finally, according to Theorem 1 this yields feasibility if L_f^{tgt} is less than 1. ■

B. Optimization Formulation

We propose an algorithm choosing the users' target CTIR values as the solution to optimization problems in each local node. In order to meet the requirements of Theorem 2, however, a central node is required.

The resource pools, s_k, given to the local nodes are produced by solving an optimization problem in the central node as well. System feasibility is obtained by meeting the inequality in (8). There may also be constraints on the users' possible β_i^{tgt} assignments given by β_{min} and β_{max}. The optimization problem in the central node is thus

$$\begin{aligned} &\underset{s_k\ k\in\{1,B\}}{\text{maximize}} \sum_{k=1}^{B} s_k \\ &\text{s.t.} \begin{cases} \begin{pmatrix} E & \bar{L} \\ \bar{L}^T & L_f^{tgt^2} E \end{pmatrix} \succeq 0, \\ \bar{L}_{k,j} = s_k Y_{k,j},\ k,j = 1,2,\dots,B. \\ s_k = \sum_{i\in c_k} \beta_i^{tgt} \\ \beta_{min} \le \beta_i^{tgt} \le \beta_{max},\ i = 1,2,\dots,M. \end{cases} \end{aligned} \tag{9}$$

Our network centric approach has led to the above choice of utility function, however, any concave function would lead to a convex problem [14].

In order to meet the requirements of Lemma 2, the inequalities in Equation (7) are considered when choosing the target CTIR values, β_i^{tgt}, in the local nodes. This is done by solving the following optimization problem in each local node k.

$$\begin{aligned} &\underset{\beta_i^{tgt}\in c_k}{\text{maximize}} \ -\sum_{i\in c_k} \log_2 \left(\frac{1+\alpha\beta_i^{tgt}}{1-(1-\alpha)\beta_i^{tgt}} \right) \\ &\text{s.t.} \begin{cases} \sum_{i\in c_k} \beta_i^{tgt} z_{i,j} \le Y_{k,j} s_k \text{ for all } j \\ \beta_{min} \le \beta_i^{tgt} \le \beta_{max}, \text{ for all } i \in c_k. \end{cases} \end{aligned} \tag{10}$$

If the first constraint is respected in all local nodes, each element of L is indeed upper bounded by the corresponding element in $\bar{L}$, i.e., Equation (7) is met.

The relation between a matrix's eigenvalues and its singular values yields that the maximum eigenvalue $\bar{\lambda}(\bar{L})$ is less than or equal to L_f^{tgt} if the matrix constraint is met with equality [15]. This implies a possible loss in utilization[3]. By scaling all users' β^{tgt} values with $L_f^{tgt}/\bar{\lambda}(\bar{L})$, while respecting the upper bound on allowed β^{tgt} values in (9), this loss can be partially regained.

The information flow in this algorithm can be visualized as in Figure 1, where the central node controls the resources using one control signal for each base station j, s_j. These signals are chosen such that system feasibility is guaranteed while the local nodes perform the actual resource (re-)assignment to the separate users. In this context, resource assignment means that each local node j allocates resources γ_i to all served users $i \in c_j$, where c_j is the set of users served by local node j.

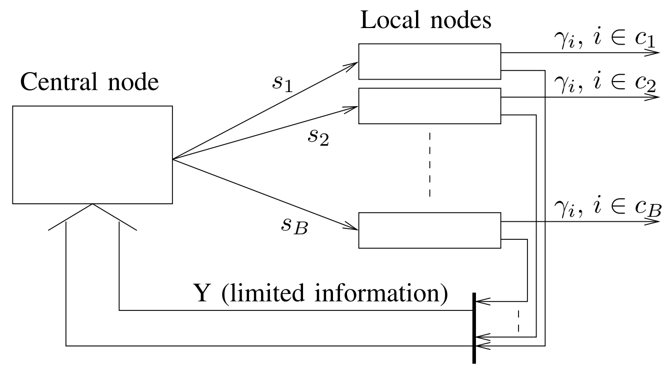

Fig. 1. Information flow in a RRM algorithm using local nodes.

C. The Semi-Centralized Algorithm

Combining the optimization problems in (9) and (10) with an iterative procedure yields the following algorithm which will be referred to as the *semi-centralized algorithm*.

---

1) Given an initial choice of target CTIR values, calculate Y according to Equation (6)
2) The central node assigns each base station a resource pool, s_k, $k = 1,2,\dots,B$, according to the solution to problem (9).
3) Each base station, k, chooses target CTIR values, $\beta_i^{tgt} \in c_k$, according to a solution to problem (10).
4) Each base station k calculates a new vector $Y_{k,j}, j = 1,2\dots,B$ according to Equation (6) and feed it back to the central node.
5) If convergence
   assign the current target CTIR values to the users
   else
   continue from point 2 above.

D. Properties of the Algorithm

Since system feasibility is guaranteed by the decisions made in the central node, the local nodes can focus on improving system performance by using information locally available. By feeding back $Y_{k,j}$ the central node has an idea of how much the different cells interfere with each other. If, for example, none of the users connected to base station k are from the base station, $Y_{k,j}$ will be relatively small and the central node can then for example choose s_k higher without jeopardizing system feasibility.

As the users move around in the cell or even enter other cells, the algorithms adapt while always maintaining system feasibility. The central node also adapts to a new radio environment via the iterations in the algorithm.

VI. Decentralized Algorithms

Two decentralized algorithms, i.e., algorithms not using a central node at all, will be introduced. Out of these, one will always make a robust resource assignment leading to a

[3]The optimization results in a maximum singular value less than $\bar{L}$.

feasible system, and the other considers only the situation in the own cell and can therefore not guarantee system feasibility.

Since no central node is used, the local nodes will have constant resource pools, $s_k = s_0$, $k = 1, 2, \ldots, B$, where s_0 is a fixed parameter. This parameter can be seen as a tuning parameter and will be chosen in advance. While a higher s_0 enables better utilization, it also increases the probability of too much uplink interference, or even infeasibility if the intercell interference is not constrained.

A. Local Robust Algorithm

A property of non-negative matrices, i.e., matrices with only non-negative elements in them, is

$$r_{min} \leq \bar{\lambda}(A) \leq r_{max},$$

where r_{min} and r_{max} is the minimum and maximum row sum, respectively [12].

The fact that each local node controls a row in the system matrix L, makes it possible for the local nodes to solve the following optimization problem on their own.

$$\begin{aligned} &\underset{\beta_i^{tgt} \in c_k}{\text{maximize}} \sum_{i \in c_k} - \log_2 \left(\frac{1 + \alpha \beta_i^{tgt}}{1 - (1 - \alpha)\beta_i^{tgt}} \right) \\ &\text{s.t.} \begin{cases} \sum_{j=1}^{B} \sum_{i \in c_k} \beta_i^{tgt} z_{i,j} \leq s_0 \\ \beta_{min} \leq \beta_i^{tgt} \leq \beta_{max}, \text{ for all } i \in c_k. \end{cases} \end{aligned} \tag{11}$$

Since the above optimization problem is solved in each local node, the first constraint guarantees that the maximum row sum of the system matrix L does not exceed L_f^{tgt}. The algorithm solving (11) in each local node will be referred to as the *local robust algorithm.*

B. Fair Algorithm

As comparison, we will use a local algorithm simply looking at the number of users in the own cell.

For each local node k, assign

$$\beta_i^{tgt} = \frac{s_0}{M_k},$$

where M_k is the number of users in cell k.

Since this algorithm uses neither feedback to a central node nor takes any consideration to the surrounding cells, system feasibility will not be guaranteed. This RRM algorithm will be referred to as the *fair algorithm*. Using this algorithm corresponds to an RRM policy with a more user-centric focus. This algorithm is meant to somewhat resemble what is done today.

VII. Simulations

The algorithms have been compared using simulations. The focus has been on system capacity as well as on fairness between users. The simulation scenario is a set of $B = 9$ cells distributed over 3 sites. A wrap around technique is used to eliminate boarder effects. Monte Carlo simulations are used for increased accuracy. The simulations are static, so the number of users and their positions are fixed for each simulation. The users are randomly spread such that 30% of the users are concentrated to a small area between two base stations and the remaining 70% are uniformly distributed over the entire simulation area. Both distance attenuation and shadow fading are modeled. The shadow fading has a log-normal standard deviation and a correlation distance according to the table below.

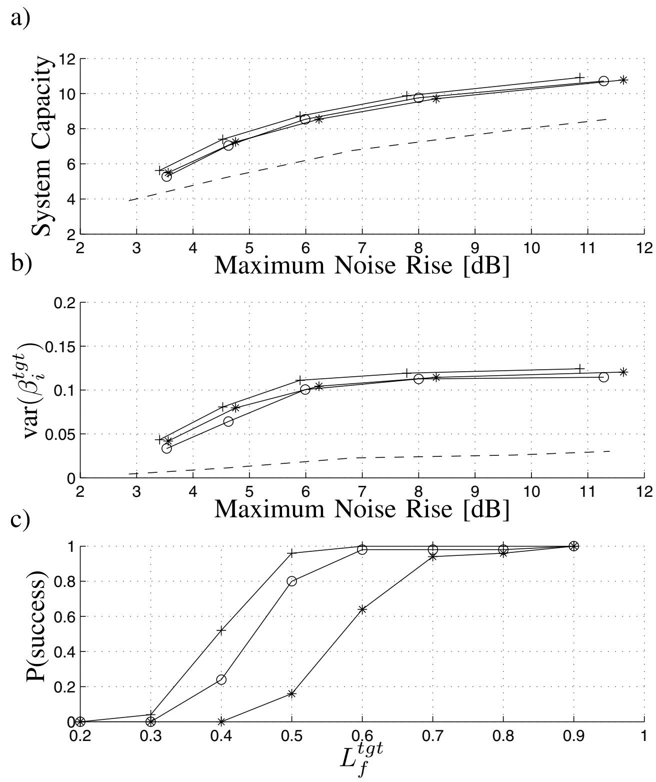

Fig. 2. Result of system simulations. +: Centralized, o: semi-centralized, *:local robust, –: fair. a) System Capacity, b) Fairness, c) The robust algorithms' success rate versus maximum allowed load.

When averaging the system capacity over the Monte Carlo simulations, only those simulations where all robust algorithms, i.e., those considering the users' relative power gain values, succeeded in find a resource assignment meeting the requirement on users target CTIR values were considered.

Table I shows some of the parameters used in the simulations. Plot a) shows that the semi-centralized and the local robust algorithms provide almost equal capacity as the completely, more theoretical, centralized algorithm for all

TABLE I

SIMULATION PARAMETERS

B	9
M	40
γ_{min}	-17 dB
γ_{max}	1
α	0.1

Max. Transmission Power	21 dBm
Average Cell Radius	500 m
No. of MC Simulations	50
Attenuation Exponent	-3.52
Log-normal Standard dev.	8
Log-normal Corr. Dist.	100

maximum noise rise levels. This capacity is also much higher than that of the fair algorithm. The maximum noise rise in each simulation is calculated as

$$\text{maximum noise rise} \triangleq \max_j \frac{I_j^{tot}}{N_j}.$$

As higher fairness means lower variance in the users' target CTIR values, plot b) indicates that the higher capacity is achieved at the expense of less fairness to individual users. This, however, is in line with today's trend of providing packet switched services by using fast scheduling.

Plot c) shows the only significant difference between the semi-centralized and the local robust algorithms. The plot indicates the different robust algorithms' ability to find a feasible solution as a function of allowed load, L_f^{tgt}. The fact that the semi-centralized algorithm seams more likely to find a feasible solution is explained by the use of a central node which coordinates resources between the different local nodes. Compared to the local robust algorithm, which does not use a central node, this property of the semi-centralized algorithm makes it more likely to find a feasible solution as the number of users grows and the target load is kept constant.

VIII. Conclusions

Two practically tractable resource assignment algorithms are proposed, both based on local decisions and information expected to be known in a commercial system. The proposed algorithms are robust in the sense that they will never make a resource assignment corresponding to an infeasible system provided that the measurements are accurate.

By using a central node, one of the proposed algorithms can coordinate resources between different base stations. This is useful in, for example, a situation where many users are concentrated to a small area. Since the actual resource assignments are done in local nodes, the decisions can be based on local information and be made with a high update rate.

Simulations indicate that the proposed algorithms provide practically the same capacity as a more idealized, completely centralized, algorithm using complete knowledge of the radio environment in the entire system.

Appendix

The *Schur Complement* of a matrix is a useful tool for establishing whether a matrix is positive definite or not [16]. Consider the matrix

$$X = \begin{pmatrix} A & B \\ B^T & C \end{pmatrix},$$

where A is a symmetric matrix. Assume that $\det(A) \neq 0$ and define the *Schur complement* of X as the matrix

$$S = C - B^T A^{-1} B.$$

By the construction of S, it follows that[4] $X \succeq 0 \Rightarrow S \succeq 0$ if $A \succ 0$. Comparing X with the extended matrix in lemma 3 yields

$$A = E,\ C = {L_f^{tgt}}^2 E \text{ and } B = \bar{L}.$$

Now, since $A = E \succ 0$, the requirement in lemma 3 gives

$${L_f^{tgt}}^2 E - \bar{L}^T \bar{L} \succeq 0 \text{ implies } \bar{L}^T \bar{L} \preceq {L_f^{tgt}}^2 E.$$

This means that

$$\bar{\lambda}(\bar{L}^T \bar{L}) \leq {L_f^{tgt}}^2,\ k = 1, 2, \dots B.$$

As the maximum eigenvalue of a matrix is less than or equal to its maximum singular value [15] this gives

$$\bar{\lambda}(\bar{L}) \triangleq \max_k \lambda_k(\bar{L}) \leq \sqrt{\bar{\lambda}(\bar{L}^T \bar{L})} \leq L_f^{tgt}.$$

References

[1] J. Zander, "Radio resource management - an overview," in *Proceedings of the IEEE Vehicular Technology Conference*, Atlanta, GA, USA, April 1996.

[2] J. D. Herdtner and E. K. Chong, "Analysis of a class of distributed asynchronous power control algorithms for cellular wireless systems," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 3, Mar. 2000.

[3] 3GPP, "FDD Enhanced Uplink; Overall description," 3GPP, Technical Specification 3GPP TS 25.309, 2005.

[4] S. Parkvall, J. Peisa, J. Torsner, and P. Malm, "WCDMA enhanced uplink - principles and basic operation," in *Proceedings of the IEEE Vehicular Technology Conference*, Stockholm, Sweden, May 2005.

[5] X. Duan, Z. Niu, D. Huang, and D. Lee, "A dynamic power and rate joint allocation algorithm for mobile multimedia DS-CDMA networks based on utiliity functions," in *Proceedings of the IEEE Personal,Indoor and Mobile Radio Communications Conference*, Sep 2002.

[6] T. Javidi, "Decentralized rate assignments in a multi-sector CDMA network," in *Proceedings of the IEEE Global Telecommunications Conference*, Dec 2003.

[7] R. Rezaiifar and J. Holtzman, "Proof of convergence for the distributed optimal rate assignment algorithm," in *Proceedings of the IEEE Vehicular Technology Conference*, Houston, TX, USA, May 1999.

[8] N. Feng, S.-C. Mau, and N. B. Mandayam, "Pricing and power control for joint network-centric and user-centric radio resource management," *IEEE Transactions on Communications*, vol. 52, no. 9, 2004.

[9] Y. Ishikawa and N. Umeda, "Capacity design and performance of call admission control in cellular CDMA systems," *IEEE Journal on Selected Areas in Communications*, vol. 15, no. 8, Oct. 1997.

[10] F. Gunnarsson, E. Geijer Lundin, G. Bark, and N. Wiberg, "Uplink admission control in WCDMA based on relative load estimates," in *IEEE International Conference on Communications*, New York, NY, Apr. 2002.

[11] J. Zander, "Optimum global transmitter power control in cellular radio systems," in *Personal, Indoor and Mobile Radio Communications., IEEE International Symposium on*, September 1991.

[12] F. R. Gantmacher, *The Theory of Matrices, Vol. II, Ch. XIII*. New York, NY, USA: Chelsea Publishing Company, 1974.

[13] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*. John Wiley & Sons, LTD, 1965.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*. The Edinburgh Building, Cambridge, CB2 2RU, UK: Cambridge University Press, 2004.

[15] G. H. Golub and C. F. van Loan, *Matrix Computations*, 3rd ed. Baltimore, MD, USA: The Johns Hopkins University Press, 1996.

[16] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Upper Saddle River, NJ, USA: Prentice Hall, 1995.

[4] The notation $A \prec B$, where A and B are matrices, means that the maximum eigenvalue of $A - B$ is less than 0.