Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Delay-dependent Robust Stabilization for Uncertain Singular Time-delay Systems: Dynamic Output Feedback Case

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Abstract— **In this paper, the problem of delay-dependent robust dynamic output feedback stabilization for a class of uncertain singular time-delay system is investigated. Based on the delay-dependent stability criterion for nominal singular time-delay systems and the concept of generalized quadratic stabilization, the sufficient conditions of the existence of the feedback controller are obtained, which ensure that the closed-loop systems are regular, impulse free and robustly asymptotically stable. Moreover, the expression for the desired robustly stabilizing controller is given by using the LMIs and the cone complementarity linearization iterative algorithm.**

Index Terms — **singular time-delay systems; robust stabilization; dynamic output feedback; delay-dependent stability criteria; linear matrix inequality(LMI).**

I. INTRODUCTION

Singular time-delay systems are derived from electrical machinery, input-output model, econometric model, environmental pollution, spaceship attitude, to name a few. Recently, increasing attention has been devoted to the study of singular time-delay systems [1]-[4]. Above results are all delay-independent, so they are quite conservative, especially when the delay is comparatively small.

In this paper, the problem of delay-dependent robust dynamic output feedback stabilization for a class of singular time-delay system with norm-bounded uncertainties is investigated. All the coefficient matrices except the matrix E include uncertainties. We consider the case of single constant time-delay, the value of which is not required to be precisely known. Based on the delay-dependent stability criterion for nominal singular time-delay systems obtained in [5], and by using the idea of generalized quadratic stabilization, the sufficient conditions for the existence of the robust dynamic output feedback controller are given. And then a cone complementarity linearization iterative algorithm is proposed to design the desired robustly stabilizing controller, which ensures that the resulting closed-loop system is regular, impulse free and asymptotically stable for all admissible uncertainties. By using the delay-independent stability criterion, the problem of robust output feedback stabilization for singular time-delay systems was considered in [4]. However, the controller can not be designed by solving the matrix inequalities once. We have to solve the partial LMI first, then substitute the obtained partial variables into the whole set of inequalities to continue solving the LMIs. Since the LMI Toolbox in MATLAB can only give one feasible set, this method is quite conservative. While the cone complementarity linearization iterative algorithm proposed in this chapter converges [6], so it improves the results in [4] to a certain extent.

Notations: The symbol $*$ will be used in some matrix expressions to induce a symmetric structure, for example, $\begin{bmatrix} X & Y \\ * & Z \end{bmatrix} = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the uncertain singular time-delay system

$$\begin{cases} E\dot{x}(t) &= (A+\triangle A)x(t)+(A_\tau+\triangle A_\tau)x(t-\tau) \\ & \quad +(B+\triangle B)u(t), \\ y(t) &= (C+\triangle C)x(t), \\ x(t) &= \phi(t), \quad t\in[-\tau,0] \end{cases} \tag{1}$$

where $x(t)\in R^n, u(t)\in R^q, y(t)\in R^m$ are the state, control input and measurement output, respectively. E, A, A_τ, B, C are known real constant matrices with appropriate dimensions and $0 <$rank $E = p < n$. τ is an unknown constant delay and satisfies $0<\tau\le\tau_m$. $\phi(t)\in C_{n,\tau}$ is a compatible vector valued initial function. $\triangle A, \triangle A_\tau, \triangle B$ and $\triangle C$ are unknown time-invariant matrices representing norm-bounded uncertainties which are assumed to be of the following forms:

$$\begin{bmatrix} \triangle A & \triangle A_\tau & \triangle B \\ \triangle C & \star & \star \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} F \begin{bmatrix} E_1 & E_\tau & E_2 \end{bmatrix}$$
$$F^TF\le I_j, \quad F\in R^{i\times j} \tag{2}$$

where $D_1\in R^{n\times i}, D_2\in R^{m\times i}, E_1\in R^{j\times n}, E_\tau\in R^{j\times n}, E_2\in R^{j\times q}$ are known real constant matrices and F is an uncertain real constant matrix. $\triangle A, \triangle A_\tau, \triangle B$ and $\triangle C$ are said to be admissible if (2) is satisfied.

The objective of this paper is to design a dynamic output feedback controller

$$\begin{cases} E\dot{\eta}(t) &= A_k\eta(t)+B_ky(t), \\ u(t) &= C_k\eta(t), \\ E\eta(0) &= \eta_0 \end{cases} \tag{3}$$

such that the closed-loop system constructed by (1) and (3) is regular, impulse free and robustly asymptotically stable. Where $\eta(t)\in R^n$ is the state of the dynamic output feedback controller, η_0 can be chosen arbitrarily; (E, A_k) is regular and impulse free, that is, (3) is a p-dimensional proper controller, and then can be realized in practice.

This work was supported by the NSF of China under Grant 30271564

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To get the main result of this paper, the following two lemmas are needed.

Lemma 1 [5]: The singular time-delay system

$$\left\{\begin{array}{rcl} E\dot{x}(t) & = & Ax(t)+A_\tau x(t-\tau), \\ x(t) & = & \phi(t), \quad t\in[-\tau,0] \end{array}\right. \tag{4}$$

is regular, impulse free and asymptotically stable for any constant delay τ satisfying $0<\tau\leq\tau_m$, if there exist matrices $Q>0, X\geq 0, Z>0$ and P, Y satisfying

$$PE = E^TP^T \geq 0,$$

$$\left[\begin{array}{cc} \Gamma & PA_\tau - Y + \tau_m A^TZA_\tau \\ * & -Q+\tau_m A_\tau^T Z A_\tau \end{array}\right] < 0,$$

$$\left[\begin{array}{cc} X & Y \\ * & E^TZE \end{array}\right] \geq 0,$$

where $\Gamma = A^TP^T + PA + Q + \tau_m X + Y + Y^T + \tau_m A^TZA$.

Lemma 2: For any real matrices G, H with appropriate dimensions and scalar $\varepsilon>0$, the following inequality holds:

$$GH + H^TG^T \leq \varepsilon^{-1}GG^T + \varepsilon H^TH.$$

Remark 1: About the definition that the singular time-delay system is regular, impulse free and asymptotically stable, it can be referred to [5].

III. DYNAMIC OUTPUT FEEDBACK CONTROLLER DESIGN

Consider the closed-loop system constructed by (1) and (3):

$$\left\{\begin{array}{rcl} \tilde{E}\dot{\tilde{x}}(t) & = & (\tilde{A}+\triangle\tilde{A})\tilde{x}(t) + (\tilde{A}_\tau + \triangle\tilde{A}_\tau)\tilde{x}(t-\tau) \\ x(t) & = & \phi(t), \quad t\in[-\tau,0], \\ E\eta(0) & = & \eta_0 \end{array}\right. \tag{5}$$

where

$$\tilde{x}(t) = \left[\begin{array}{c} x(t) \\ \eta(t) \end{array}\right], \tilde{E} = \left[\begin{array}{cc} E & 0 \\ 0 & E \end{array}\right], \tag{6a}$$

$$\tilde{A} = \left[\begin{array}{cc} A & BC_k \\ B_kC & A_k \end{array}\right],\ \tilde{A}_\tau = \left[\begin{array}{cc} A_\tau & 0 \\ 0 & 0 \end{array}\right], \tag{6b}$$

$$\triangle\tilde{A} = \left[\begin{array}{cc} \triangle A & \triangle BC_k \\ B_k\triangle C & 0 \end{array}\right],\ \triangle\tilde{A}_\tau = \left[\begin{array}{cc} \triangle A_\tau & 0 \\ 0 & 0 \end{array}\right]. \tag{6c}$$

Using Schur complement argument and Lemma 1, the sufficient conditions are obtained guaranteeing that the system (5) is regular, impulse free and robustly asymptotically stable as: there exist matrices $\tilde{Q}>0, \tilde{X}\geq 0, \tilde{Z}>0$ and $\tilde{P}, \tilde{Y}$ such that

$$\tilde{E}^T\tilde{P}^T = \tilde{P}\tilde{E} \geq 0, \tag{7a}$$

$$\left[\begin{array}{ccc} \Theta & \tilde{P}(\tilde{A}_\tau+\triangle\tilde{A}_\tau)-\tilde{Y} & \tau_m(\tilde{A}+\triangle\tilde{A})^T\tilde{Z} \\ * & -\tilde{Q} & \tau_m(\tilde{A}_\tau+\triangle\tilde{A}_\tau)^T\tilde{Z} \\ * & * & -\tau_m\tilde{Z} \end{array}\right] < 0, \tag{7b}$$

$$\left[\begin{array}{cc} \tilde{X} & \tilde{Y} \\ * & \tilde{E}^T\tilde{Z}\tilde{E} \end{array}\right] \geq 0 \tag{7c}$$

hold for all admissible uncertainties, where

$$\Theta = (\tilde{A}+\triangle\tilde{A})^T\tilde{P}^T + \tilde{P}(\tilde{A}+\triangle\tilde{A}) + \tilde{Q} + \tau_m\tilde{X} + \tilde{Y} + \tilde{Y}^T.$$

Let

$$\tilde{P} = \left[\begin{array}{cc} P & P_2 \\ P_3 & P_4 \end{array}\right], P\in R^{n\times n}, P_i \in R^{n\times n}, i=2,3,4 \tag{8}$$

It is easy to know from (7b) that $\tilde{P}$ is nonsingular. Without loss of generality, we can assume that $P, P_i, i=2,3,4$, are all nonsingular. Then from (7a), we have

$$E^TP^T = PE,\ \ E^TP_3^T = P_2E,\ \ E^TP_4^T = P_4E. \tag{9}$$

Take

$$T_1 = \left[\begin{array}{cc} I & 0 \\ 0 & PP_3^{-1} \end{array}\right], T_2 = \left[\begin{array}{cc} I & 0 \\ 0 & P_2^{-1}P \end{array}\right], \tag{10a}$$

$$T_3 = diag\{T_1, T_1, T_2^T\}. \tag{10b}$$

Combining with (6), (8) and (9), we obtain

$$\begin{aligned} \bar{E} = T_2^{-1}\tilde{E}T_1^T &= \left[\begin{array}{cc} E & 0 \\ 0 & P^{-1}P_2EP_3^{-T}P^T \end{array}\right] \\ &= \left[\begin{array}{cc} E & 0 \\ 0 & P^{-1}E^TP_3^TP_3^{-T}P^T \end{array}\right] = \left[\begin{array}{cc} E & 0 \\ 0 & E \end{array}\right], \end{aligned} \tag{11a}$$

$$\bar{P} = T_1\tilde{P}T_2 = \left[\begin{array}{cc} P & P \\ P & PP_3^{-1}P_4P_2^{-1}P \end{array}\right], \tag{11b}$$

$$\begin{aligned} \bar{A} = T_2^{-1}\tilde{A}T_1^T &= \left[\begin{array}{cc} A & BC_kP_3^{-T}P^T \\ P^{-1}P_2B_kC & P^{-1}P_2A_kP_3^{-T}P^T \end{array}\right] \\ &= \left[\begin{array}{cc} A & B\bar{C}_k \\ \bar{B}_kC & \bar{A}_k \end{array}\right], \end{aligned} \tag{11c}$$

$$\begin{aligned} \triangle\bar{A} = T_2^{-1}\triangle\tilde{A}T_1^T &= \left[\begin{array}{cc} \triangle A & \triangle BC_kP_3^{-T}P^T \\ P^{-1}P_2B_k\triangle C & 0 \end{array}\right] \\ &= \left[\begin{array}{cc} \triangle A & \triangle B\bar{C}_k \\ \bar{B}_k\triangle C & 0 \end{array}\right], \end{aligned} \tag{11d}$$

$$\bar{A}_\tau = T_2^{-1}\tilde{A}_\tau T_1^T = \left[\begin{array}{cc} A_\tau & 0 \\ 0 & 0 \end{array}\right], \tag{11e}$$

$$\triangle\bar{A}_\tau = T_2^{-1}\triangle\tilde{A}_\tau T_1^T = \left[\begin{array}{cc} \triangle A_\tau & 0 \\ 0 & 0 \end{array}\right], \tag{11f}$$

where

$$\bar{A}_k = P^{-1}P_2A_kP_3^{-T}P^T, \bar{B}_k = P^{-1}P_2B_k, \bar{C}_k = C_kP_3^{-T}P^T, \tag{11g}$$

and denote

$$\bar{Q} = T_1\tilde{Q}T_1^T, \bar{X} = T_1\tilde{X}T_1^T, \bar{Y} = T_1\tilde{Y}T_1^T, \bar{Z} = T_2^T\tilde{Z}T_2. \tag{11i}$$

Pre-multiplying by T_1 and post-multiplying by T_1^T on both sides of (7a) results in

$$\bar{E}^T\bar{P}^T = \bar{P}\bar{E} \geq 0. \tag{12a}$$

Pre-multiplying by T_3 and post-multiplying by T_3^T on both sides of (7b) yields

$$\left[\begin{array}{ccc} \Lambda & \bar{P}(\bar{A}_\tau+\triangle\bar{A}_\tau)-\bar{Y} & \tau_m(\bar{A}+\triangle\bar{A})^T\bar{Z} \\ * & -\bar{Q} & \tau_m(\bar{A}_\tau+\triangle\bar{A}_\tau)^T\bar{Z} \\ * & * & -\tau_m\bar{Z} \end{array}\right] < 0 \tag{12b}$$

with

$$\Lambda = (\bar{A}+\triangle\bar{A})^T\bar{P}^T + \bar{P}(\bar{A}+\triangle\bar{A}) + \bar{Q} + \tau_m\bar{X} + \bar{Y} + \bar{Y}^T.$$

And Pre-multiplying by $\mathrm{diag}\{T_1,T_1\}$ and post-multiplying by $\mathrm{diag}\{T_1^T,T_1^T\}$on both sides of (7c) we get

$$\begin{bmatrix} \bar{X} & \bar{Y} \\ * & \bar{E}^T\bar{Z}\bar{E} \end{bmatrix} \geq 0. \tag{12c}$$

It can be seen that the closed-loop systems $(\tilde{E},\tilde{A}+\triangle\tilde{A},\tilde{A}_\tau+\triangle\tilde{A}_\tau)$ and $(\bar{E},\bar{A}+\triangle\bar{A},\bar{A}_\tau+\triangle\bar{A}_\tau)$ are algebraically equivalent under the r. s. e. transformation where T_2^{-1} and T_1^T are taken as the row full rank transformation matrix and the coordinate full rank transformation matrix respectively. And comparing the coefficient matrices of the two closed-loop systems, we can see that the difference between them is just the controller parameters A_k,B_k,C_k and $\bar{A}_k,\bar{B}_k,\bar{C}_k$. So, without loss of generality, in the next of this paper, we directly see $\tilde{E},\tilde{A}+\triangle\tilde{A},\tilde{A}_\tau+\triangle\tilde{A}_\tau$ and $\tilde{P}$ in (5), (6) and (8) as $\bar{E},\bar{A}+\triangle\bar{A},\bar{A}_\tau+\triangle\bar{A}_\tau$ and $\bar{P}$. It is easy to prove that $P_4-P_3P^{-1}P_2$ is nonsingular. Let

$$S^{-1}=PP_3^{-1}P_4P_2^{-1}P-P=PP_3^{-1}(P_4-P_3P^{-1}P_2)P_2^{-1}P, \tag{13}$$

then $\bar{P}$ can be written as

$$\bar{P}=\begin{bmatrix} P & P \\ P & P+S^{-1} \end{bmatrix}. \tag{14}$$

Denote

$$J^T=S+P^{-1},\quad T_4=\begin{bmatrix} J^T & -S \\ I & 0 \end{bmatrix}, \tag{15a}$$

$$T_5=\begin{bmatrix} J^{-T} & 0 \\ 0 & I \end{bmatrix}T_4=\begin{bmatrix} I & -J^{-T}S \\ I & 0 \end{bmatrix},\quad T_6=T_4\bar{P}\bar{Z}^{-1}, \tag{15b}$$

since $P(S+P^{-1})=(P+S^{-1})S=PP_3^{-1}P_4P_2^{-1}PS$ is nonsingular, J is also a nonsingular matrix.

When the uncertainty satisfies (2), we have

$$\triangle\bar{A}=\bar{D}_1\bar{F}\bar{E}_1,\quad \triangle\bar{A}_\tau=\bar{D}_1\bar{F}\bar{E}_\tau,\bar{D}_1=\begin{bmatrix} D_1 & 0 \\ 0 & \bar{B}_kD_2 \end{bmatrix}, \tag{16a}$$

$$\bar{E}_1=\begin{bmatrix} E_1 & E_2\bar{C}_k \\ E_1 & 0 \end{bmatrix},\quad \bar{E}_\tau=\begin{bmatrix} E_\tau & 0 \\ 0 & 0 \end{bmatrix},\quad \bar{F}=\begin{bmatrix} F & 0 \\ 0 & F \end{bmatrix}. \tag{16b}$$

Denote

$$\Sigma:=\begin{bmatrix} \begin{matrix}\bar{A}^T\bar{P}^T+\bar{P}\bar{A}+\bar{Q}\\ +\tau_m\bar{X}+\bar{Y}+\bar{Y}^T\end{matrix} & \bar{P}\bar{A}_\tau-\bar{Y} & \tau_m\bar{A}^T\bar{Z} \\ * & -\bar{Q} & \tau_m\bar{A}_\tau^T\bar{Z} \\ * & * & -\tau_m\bar{Z} \end{bmatrix}, \tag{17}$$

then (12b) can be written as

$$\Sigma+\begin{bmatrix} \bar{P}\bar{D}_1 \\ 0 \\ \tau_m\bar{Z}\bar{D}_1 \end{bmatrix}\bar{F}\begin{bmatrix} \bar{E}_1^T \\ \bar{E}_\tau^T \\ 0 \end{bmatrix}^T +(\begin{bmatrix} \bar{P}\bar{D}_1 \\ 0 \\ \tau_m\bar{Z}\bar{D}_1 \end{bmatrix}\bar{F}\begin{bmatrix} \bar{E}_1^T \\ \bar{E}_\tau^T \\ 0 \end{bmatrix}^T)^T<0. \tag{18}$$

From $F^TF\leq I_j$, it gets that $\bar{F}^T\bar{F}\leq I_{2j}$. So from Lemma 2, one obtains that if there exists $\varepsilon>0$ satisfying

$$\Sigma+\varepsilon^{-1}\begin{bmatrix} \bar{P}\bar{D}_1 \\ 0 \\ \tau_m\bar{Z}\bar{D}_1 \end{bmatrix}\begin{bmatrix} \bar{P}\bar{D}_1 \\ 0 \\ \tau_m\bar{Z}\bar{D}_1 \end{bmatrix}^T+\varepsilon\begin{bmatrix} \bar{E}_1^T \\ \bar{E}_\tau^T \\ 0 \end{bmatrix}\begin{bmatrix} \bar{E}_1^T \\ \bar{E}_\tau^T \\ 0 \end{bmatrix}^T<0, \tag{19}$$

then (18) holds. By using again a Schur complement argument, we have that (19) is equivalent to

$$\beta(\varepsilon,\bar{A},\bar{A}_\tau,\bar{D}_1,\bar{E}_1,\bar{E}_\tau,\bar{Q},\bar{P},\bar{Z},\bar{X},\bar{Y})$$
$$=\begin{bmatrix} \bar{A}^T\bar{P}^T+\bar{P}\bar{A}+\bar{Q}+\tau_m\bar{X}+\bar{Y}+\bar{Y}^T & \bar{P}\bar{A}_\tau-\bar{Y} & \tau_m\bar{A}^T\bar{Z} & \bar{P}\bar{D}_1 & \varepsilon\bar{E}_1^T \\ * & -\bar{Q} & \tau_m\bar{A}_\tau^T\bar{Z} & 0 & \varepsilon\bar{E}_\tau^T \\ * & * & -\tau_m\bar{Z} & \tau_m\bar{Z}\bar{D}_1 & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix}<0. \tag{20}$$

From (20), we have

$$diag\{T_4,T_5,T_6,I,I\}\beta(\varepsilon,\bar{A},\bar{A}_\tau,\bar{D}_1,\bar{E}_1,\bar{E}_\tau,\bar{Q},\bar{P},\bar{Z},\bar{X},\bar{Y})$$
$$\times diag\{T_4^T,T_5^T,T_6^T,I,I\}<0,$$

that is,

$$\begin{bmatrix} \begin{matrix}T_4\bar{A}^T\bar{P}^TT_4^T+T_4\bar{P}\bar{A}T_4^T+T_4\bar{Q}T_4^T\\ +\tau_mT_4\bar{X}T_4^T+T_4\bar{Y}T_4^T+T_4\bar{Y}^TT_4^T\end{matrix} & T_4\bar{P}\bar{A}_\tau T_5^T-T_4\bar{Y}T_5^T & \tau_mT_4\bar{A}^T\bar{Z}T_6^T & T_4\bar{P}\bar{D}_1 & \varepsilon T_4\bar{E}_1^T \\ * & -T_5\bar{Q}T_5^T & \tau_mT_5\bar{A}_\tau^T\bar{Z}T_6^T & 0 & \varepsilon T_5\bar{E}_\tau^T \\ * & * & -\tau_mT_6\bar{Z}T_6^T & \tau_mT_6\bar{Z}\bar{D}_1 & 0 \\ * & * & * & -\varepsilon I & 0 \\ * & * & * & * & -\varepsilon I \end{bmatrix}<0. \tag{21}$$

Noticing

$$T_4\bar{P}\bar{A}T_4^T=\begin{bmatrix} AJ-B\bar{C}_kS^T & A \\ PAJ+P\bar{B}_kCJ-PBC_kS^T-P\bar{A}_kS^T & PA+P\bar{B}_kC \end{bmatrix}, \tag{22a}$$

$$T_4\bar{P}\bar{A}_\tau T_5^T=\begin{bmatrix} A_\tau & A_\tau \\ PA_\tau & PA_\tau \end{bmatrix},T_4\bar{P}\bar{D}_1=\begin{bmatrix} D_1 & 0 \\ PD_1 & P\bar{B}_kD_2 \end{bmatrix}, \tag{22b}$$

$$T_4\bar{E}_1^T=\begin{bmatrix} J^TE_1^T-S\bar{C}_k^TE_2^T & J^TE_1^T \\ E_1^T & E_1^T \end{bmatrix},T_5\bar{E}_\tau^T=\begin{bmatrix} E_\tau^T & 0 \\ E_\tau^T & 0 \end{bmatrix}, \tag{22c}$$

$$T_4\bar{A}^T\bar{Z}T_6^T=T_4\bar{A}^T\bar{Z}\bar{Z}^{-1}\bar{P}^TT_4^T=T_4\bar{A}^T\bar{P}^TT_4^T, \tag{22d}$$

$$T_5\bar{A}_\tau^T\bar{Z}T_6^T=T_5\bar{A}_\tau^T\bar{Z}\bar{Z}^{-1}\bar{P}^TT_4^T=T_5\bar{A}_\tau^T\bar{P}^TT_4^T, \tag{22e}$$

$$T_6\bar{Z}T_6^T=T_4\bar{P}\bar{Z}^{-1}\bar{Z}\bar{Z}^{-1}\bar{P}^TT_4^T=T_4\bar{P}\bar{Z}^{-1}\bar{P}^TT_4^T, \tag{22f}$$

$$T_6\bar{Z}\bar{D}_1=T_4\bar{P}\bar{Z}^{-1}\bar{Z}\bar{D}_1=T_4\bar{P}\bar{D}_1, \tag{22g}$$

and denoting

$$Q=T_4\bar{Q}T_4^T=\begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix},X=T_4\bar{X}T_4^T=\begin{bmatrix} X_1 & X_2 \\ X_2^T & X_3 \end{bmatrix}, \tag{22h}$$

$$Z = T_4 \bar{P} \bar{Z}^{-1} \bar{P}^T T_4^T = \begin{bmatrix} Z_1 & Z_2 \\ Z_2^T & Z_3 \end{bmatrix}, Y = T_4 \bar{Y} T_4^T = \begin{bmatrix} Y_1 & Y_2 \\ Y_3 & Y_4 \end{bmatrix}, \tag{22i}$$

$$W_B = P\bar{B}_k, W_C = \bar{C}_k S^T, L = PAJ + P\bar{B}_k CJ - PB\bar{C}_k S^T - P\bar{A}_k S^T, \tag{22j}$$

one gets

$$T_5 \bar{Q} T_5^T = \begin{bmatrix} J^{-T} Q_1 J^{-1} & J^{-T} Q_2 \\ Q_2^T J^{-1} & Q_3 \end{bmatrix}, T_4 \bar{Y} T_5^T = \begin{bmatrix} Y_1 J^{-1} & Y_2 \\ Y_3 J^{-1} & Y_4 \end{bmatrix}. \tag{22k}$$

Introduce matrices $Q_4 > 0, Q_6 > 0$ and Q_5. Obviously, if there exist matrices $Q_1 > 0, Q_3 > 0, Q_4 > 0, Q_6 > 0, X_1 \geq 0, X_3 \geq 0, Z_1 > 0, Z_3 > 0, P, J, W_B, W_C, L, Q_2, Q_5, X_2, Z_2, Y_1, Y_2, Y_3, Y_4$ and a scalar $\varepsilon > 0$ with P, J nonsingular, satisfying

$$\left[\begin{array}{cccccccccc} \Xi_{11} & \Xi_{12} & A_\tau - Y_1 J^{-1} & A_\tau - Y_2 & \Xi_{15} & \tau_m L^T & D_1 & 0 & \varepsilon(J^T E_1^T - W_C^T E_2^T) & \varepsilon J^T E_1^T \\ * & \Xi_{22} & PA_\tau - Y_3 J^{-1} & PA_\tau - Y_4 & \tau_m A^T & \Xi_{26} & PD_1 & W_B D_2 & \varepsilon E_1^T & \varepsilon E_1^T \\ * & * & -Q_4 & -Q_5 & \tau_m A_\tau^T & \tau_m A_\tau^T P^T & 0 & 0 & \varepsilon E_\tau^T & 0 \\ * & * & * & -Q_6 & \tau_m A_\tau^T & \tau_m A_\tau^T P^T & 0 & 0 & \varepsilon E_\tau^T & 0 \\ * & * & * & * & -\tau_m Z_1 & -\tau_m Z_2 & \tau_m D_1 & 0 & 0 & 0 \\ * & * & * & * & * & -\tau_m Z_3 & \tau_m P D_1 & \tau_m W_B D_2 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon I \end{array}\right] < 0 \tag{23}$$

with

$$\Xi_{11} = AJ + J^T A^T - BW_C - W_C^T B^T + Q_1 + \tau_m X_1 + Y_1 + Y_1^T,$$

$$\Xi_{12} = A + L^T + Q_2 + \tau_m X_2 + Y_2 + Y_3^T,$$

$$\Xi_{22} = PA + A^T P^T + W_B C + C^T W_B^T + Q_3 + \tau_m X_3 + Y_4 + Y_4^T,$$

$$\Xi_{15} = \tau_m (J^T A^T - W_C^T B^T), \quad \Xi_{26} = \tau_m (A^T P^T + C^T W_B^T),$$

and

$$\begin{bmatrix} Q_4 & Q_5 \\ Q_5^T & Q_6 \end{bmatrix} \leq \begin{bmatrix} J^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} \begin{bmatrix} J^{-1} & 0 \\ 0 & I \end{bmatrix}, \tag{24}$$

then taking

$$\begin{aligned} &S = J^T - P^{-1}, \bar{B}_k = P^{-1} W_B, \bar{C}_k = W_C S^{-T}, \\ &\bar{A}_k = P^{-1}(PAJ + W_B CJ - PBW_C - L) S^{-T}, \end{aligned} \tag{25}$$

one obtains that there are solutions $\bar{Q} > 0, \bar{X} \geq 0, \bar{Z} > 0, \bar{P}, \bar{Y}$ and $\varepsilon > 0$ to (20).

From (12a) we have

$$T_4 \bar{E}^T \bar{P}^T T_4^T = T_4 \bar{P} \bar{E} T_4^T \geq 0. \tag{26}$$

Noticing

$$\begin{aligned} T_4 \bar{P} \bar{E} T_4^T &= \begin{bmatrix} EJ & E \\ PEJ - PES^T & PE \end{bmatrix} \\ &= \begin{bmatrix} EJ & E \\ PEP^{-T} & PE \end{bmatrix} = \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix}, \end{aligned} \tag{27a}$$

$$T_4 \bar{E}^T \bar{P}^T T_4^T = \begin{bmatrix} J^T E^T & E \\ E^T & E^T P^T \end{bmatrix}, \tag{27b}$$

then (26) is just

$$EJ = J^T E^T, PE = E^T P^T, \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} \geq 0. \tag{28}$$

It is obtained from (12c) that

$$\begin{bmatrix} T_4 \bar{X} T_4^T & T_4 \bar{Y} T_4^T \\ * & T_4 \bar{E}^T \bar{Z} \bar{E} T_4^T \end{bmatrix} \geq 0. \tag{29}$$

(22i) implies that $\bar{Z} = \bar{P}^T T_4^T Z^{-1} T_4 \bar{P}$, then

$$\begin{aligned} T_4 \bar{E}^T \bar{Z} \bar{E} T_4^T &= T_4 \bar{E}^T \bar{P}^T T_4^T Z^{-1} T_4 \bar{P} \bar{E} T_4^T \\ &= \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} Z^{-1} \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} \end{aligned} \tag{30}$$

Introduce matrix $W \geq 0$. Obviously, if

$$\begin{bmatrix} X & Y \\ * & W \end{bmatrix} \geq 0, \tag{31}$$

and

$$W \leq \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} Z^{-1} \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix}, \tag{32}$$

then (29) holds.

Now, we state the following theorem.

Theorem 1: If there are solutions $Q_1 > 0, Q_3 > 0, Q_4 > 0, Q_6 > 0, X_1 \geq 0, X_3 \geq 0, Z_1 > 0, Z_3 > 0, W \geq 0, P, J, W_B, W_C, L, Q_2, Q_5, X_2, Z_2, Y_1, Y_2, Y_3, Y_4$ and $\varepsilon > 0$ with P, J nonsingular, to (22h), (22i), (23), (24), (28), (31) and (32), then there exists dynamic output feedback controller (3) such that the closed-loop system constructed by (1) with respect to uncertainty (2) and (3) is regular, impulse free and robustly asymptotically stable for any $\tau : 0 < \tau \leq \tau_m$, and the controller parameters are given in (25).

Remark 2: It is obvious that (23), (24) and (32) are not LMIs. In order to use the LMI Toolbox in MATLAB to get the solutions, we can do as follows. In (23), let $Y_1 = 0, Y_3 = 0$, then for given $\varepsilon > 0$, (23) is a LMI about the variables $P, J, W_B, W_C, L, Q_i, i = 1,2,3,4,5,6, X_i, i = 1,2,3, Z_i, i = 1,2,3, Y_i, i = 2,4$. Without loss of generality, it is assumed that $E = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}$, then the matrices P, J satisfying (28) are of the forms:

$$P = \begin{bmatrix} P_{11} & P_{12} \\ 0 & P_{22} \end{bmatrix}, J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix}, P_{11} \in R^{p \times p}, J_{11} \in R^{p \times p} \tag{33a}$$

with

$$\begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix} \geq 0. \tag{33b}$$

It is obtained from Schur complement argument that (24) is equivalent to

$$\begin{bmatrix} \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} & \begin{bmatrix} J^T & 0 \\ 0 & I \end{bmatrix} \\ \begin{bmatrix} J & 0 \\ 0 & I \end{bmatrix} & \begin{bmatrix} Q_4 & Q_5 \\ Q_5^T & Q_6 \end{bmatrix}^{-1} \end{bmatrix} \geq 0. \tag{34}$$

Introduce $V = \begin{bmatrix} V_1 & V_2 \\ V_2^T & V_3 \end{bmatrix} > 0$, then (34) can be replaced by

$$\begin{bmatrix} Q_1 & Q_2 & J^T & 0 \\ Q_2^T & Q_3 & 0 & I \\ J & 0 & V_1 & V_2 \\ 0 & I & V_2^T & V_3 \end{bmatrix} \geq 0 \tag{35}$$

and

$$\begin{bmatrix} V_1 & V_2 \\ V_2^T & V_3 \end{bmatrix} \begin{bmatrix} Q_4 & Q_5 \\ Q_5^T & Q_6 \end{bmatrix} = I. \tag{36}$$

Introduce another variable $U > 0$, then (32) can be replaced by

$$W \leq \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} U \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} \tag{37}$$

and

$$UZ = I. \tag{38}$$

Write U as

$$U = \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ U_{12}^T & U_{22} & U_{23} & U_{24} \\ U_{13}^T & U_{23}^T & U_{33} & U_{34} \\ U_{14}^T & U_{24}^T & U_{34}^T & U_{44} \end{bmatrix} > 0, \tag{39}$$

where

$$U_{11} \in R^{p\times p}, U_{22} \in R^{(n-p)\times(n-p)},$$
$$U_{33} \in R^{p\times p}, U_{44} \in R^{(n-p)\times(n-p)}.$$

Noticing

$$\begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} U \begin{bmatrix} EJ & E \\ E^T & PE \end{bmatrix} = \begin{bmatrix} \Pi_{11} & 0 & \Pi_{13} & 0 \\ * & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 \\ * & * & * & 0 \end{bmatrix}, \tag{40}$$

where

$$\Pi_{11} = J_{11}U_{11}J_{11} + U_{13}^T J_{11} + J_{11}U_{13} + U_{33},$$
$$\Pi_{13} = J_{11}U_{11} + U_{13}^T + J_{11}U_{13}P_{11} + U_{33}P_{11},$$
$$\Pi_{33} = U_{11} + P_{11}U_{13}^T + U_{13}P_{11} + P_{11}U_{33}P_{11},$$

so we can assume

$$W = \begin{bmatrix} W_1 & 0 & W_2 & 0 \\ 0 & 0 & 0 & 0 \\ W_2^T & 0 & W_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \geq 0. \tag{41}$$

Then (37) is just

$$\begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \leq \begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix} \begin{bmatrix} U_{11} & U_{13} \\ U_{13}^T & U_{33} \end{bmatrix} \begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}. \tag{42}$$

Invoking again a Schur complement argument, we have that (42) is equivalent to

$$\begin{bmatrix} \begin{bmatrix} U_{11} & U_{13} \\ U_{13}^T & U_{33} \end{bmatrix}^{-1} & \begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}^{-1} \\ \begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix}^{-1} & \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix}^{-1} \end{bmatrix} \geq 0. \tag{43}$$

Introduce

$$\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^T & \alpha_3 \end{bmatrix} > 0, \ \theta = \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_2^T & \theta_3 \end{bmatrix} > 0, \tag{44}$$

then (44) can be replaced by

$$\begin{bmatrix} U_{11} & U_{13} & \alpha_1 & \alpha_2 \\ U_{13}^T & U_{33} & \alpha_2^T & \alpha_3 \\ \alpha_1 & \alpha_2 & \theta_1 & \theta_2 \\ \alpha_2^T & \alpha_3 & \theta_2^T & \theta_3 \end{bmatrix} \geq 0 \tag{45}$$

and

$$\begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^T & \alpha_3 \end{bmatrix} = I, \tag{46a}$$

$$\begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_2^T & \theta_3 \end{bmatrix} = I. \tag{46b}$$

Therefore, for given $\varepsilon > 0$, letting $Y_1 = 0, Y_3 = 0$, one can consider the dynamic output feedback stabilization problem as the the following cone complementary problems:

$$\text{Minimize } \{\mathrm{tr}(UZ + \begin{bmatrix} V_1 & V_2 \\ V_2^T & V_3 \end{bmatrix} \begin{bmatrix} Q_4 & Q_5 \\ Q_5^T & Q_6 \end{bmatrix}) + \mathrm{tr}(\begin{bmatrix} J_{11} & I \\ I & P_{11} \end{bmatrix} \begin{bmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^T & \alpha_3 \end{bmatrix} + \begin{bmatrix} W_1 & W_2 \\ W_2^T & W_3 \end{bmatrix} \begin{bmatrix} \theta_1 & \theta_2 \\ \theta_2^T & \theta_3 \end{bmatrix})\} \tag{47}$$

subject to LMIs: (22h), (22i), (23), (31), (33), (35), (39), (41), (44), (45) and

$$\begin{cases} Q_1 > 0, Q_3 > 0, Q_4 > 0, Q_6 > 0, & (48a) \\ X_1 \geq 0, X_3 \geq 0, Z > 0, V > 0, & (48b) \\ \begin{bmatrix} U & I \\ I & Z \end{bmatrix} \geq 0, \begin{bmatrix} V_1 & V_2 & I & 0 \\ V_2^T & V_3 & 0 & I \\ I & 0 & Q_4 & Q_5 \\ 0 & I & Q_5^T & Q_6 \end{bmatrix} \geq 0, & (48c) \\ \begin{bmatrix} \alpha_1 & \alpha_2 & I & 0 \\ \alpha_2^T & \alpha_3 & 0 & I \\ I & 0 & J_{11} & I \\ 0 & I & I & P_{11} \end{bmatrix} \geq 0, & (48d) \\ \begin{bmatrix} \theta_1 & \theta_2 & I & 0 \\ \theta_2^T & \theta_3 & 0 & I \\ I & 0 & W_1 & W_2 \\ 0 & I & W_2^T & W_3 \end{bmatrix} \geq 0. & (48e) \end{cases}$$

Then the controller (3) can be solved by using the iterative linearization algorithm as follows:

Algorithm 1:

(1) Make singular value decomposition to matrix E:

$\bar{U}E\bar{V} = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix}$, where $\bar{U},\bar{V}$ are orthogonal matrices and $\Sigma \in R^{p\times p}$ is a nonsingular diagonal matrix. Let $M = \begin{bmatrix} \Sigma^{-\frac{1}{2}} & 0 \\ 0 & I \end{bmatrix}\bar{U}$ and $N = \bar{V}\begin{bmatrix} \Sigma^{-\frac{1}{2}} & 0 \\ 0 & I \end{bmatrix}$, then $\bar{E} = MEN = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}$. Take M as the row full rank transformation matrix and N as the coordinate full rank transformation matrix, then system (1) is r. s. e. (restricted system equivalence) [7] to:

$$\begin{cases} \bar{E}\dot{\bar{x}}(t) = (\bar{A}+\triangle\bar{A})\bar{x}(t)+(\bar{A}_\tau+\triangle\bar{A}_\tau)\bar{x}(t-\tau) \\ \qquad\qquad +(\bar{B}+\triangle\bar{B})u(t), \\ \bar{x}(t) = N^{-1}\phi(t), \quad t\in[-\tau,0] \end{cases} \tag{49}$$

where, $\bar{E} = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}$, $\bar{A} = MAN, \bar{A}_\tau = MA_\tau N, \bar{B} = MB$, $\triangle\bar{A} = \bar{D}F\bar{E}_1, \triangle\bar{A}_\tau = \bar{D}F\bar{E}_\tau$, $\triangle\bar{B} = \bar{D}F\bar{E}_2$, $\bar{D} = MD, \bar{E}_1 = E_1N, \bar{E}_\tau = E_\tau N, \bar{E}_2 = E_2N, \bar{x} = N^{-1}x$. For convenience, we still denote $\bar{E},\bar{A},\bar{A}_\tau,\bar{B},\bar{D},\bar{E}_1,\bar{E}_\tau,\bar{E}_2$ as $E,A,A_\tau,B,D,E_1,E_\tau,E_2$.

(2) For given $\tau_m > 0$ and $\varepsilon > 0$, find a feasible set $Q_1,Q_2,Q_3,Q_4,Q_5,Q_6,X_1,X_2,X_3,Z_1,Z_2,Z_3,W,P,J,W_B,W_C,L,$ Y_1,Y_2,Y_3,Y_4 satisfying (22h), (22i), (23), (31), (33), (35), (39), (41), (44), (45) and (48). If there are none, exit. Otherwise set $U^{(0)} = U, Z^{(0)} = Z, V_1^{(0)} = V_1, V_2^{(0)} = V_2, V_3^{(0)} = V_3, Q_4^{(0)} = Q_4, Q_5^{(0)} = Q_5, Q_6^{(0)} = Q_6, J_{11}^{(0)} = J_{11}, P_{11}^{(0)} = P_{11}, \alpha_1^{(0)} = \alpha_1, \alpha_2^{(0)} = \alpha_2, \alpha_3^{(0)} = \alpha_3, W_1^{(0)} = W_1, W_2^{(0)} = W_2, W_3^{(0)} = W_3, \theta_1^{(0)} = \theta_1, \theta_2^{(0)} = \theta_2, \theta_3^{(0)} = \theta_3$, and verify the condition (24) and (32). If (24) and (32) are satisfied, then the parameters of the dynamic output feedback controller (3) are given in (25). Otherwise, set the index of the objective function in the next step as $k = 0$ and go to step (3).

(3) Solve the following convex optimization problem for the variables $Q_1,Q_2,Q_3,Q_4,Q_5, Q_6,X_1,X_2,X_3,Z_1,Z_2,Z_3,W,$ $P,J,W_B,W_C,L, Y_1,Y_2,Y_3,Y_4$:

$$\begin{aligned} &\text{Minimize } \{ \operatorname{tr}(U^{(k)}Z+Z^{(k)}U \\ &+\begin{bmatrix} V_1^{(k)} & V_2^{(k)} \\ * & V_3^{(k)} \end{bmatrix}\begin{bmatrix} Q_4 & Q_5 \\ * & Q_6 \end{bmatrix}+\begin{bmatrix} V_1 & V_2 \\ * & V_3 \end{bmatrix}\begin{bmatrix} Q_4^{(k)} & Q_5^{(k)} \\ * & Q_6^{(k)} \end{bmatrix}) \\ &+\operatorname{tr}(\begin{bmatrix} J_{11}^{(k)} & I \\ * & P_{11}^{(k)} \end{bmatrix}\begin{bmatrix} \alpha_1 & \alpha_2 \\ * & \alpha_3 \end{bmatrix}+\begin{bmatrix} J_{11} & I \\ * & P_{11} \end{bmatrix}\begin{bmatrix} \alpha_1^{(k)} & \alpha_2^{(k)} \\ * & \alpha_3^{(k)} \end{bmatrix} \\ &+\begin{bmatrix} W_1^{(k)} & W_2^{(k)} \\ * & W_3^{(k)} \end{bmatrix}\begin{bmatrix} \theta_1 & \theta_2^T \\ * & \theta_3 \end{bmatrix}+\begin{bmatrix} W_1 & W_2 \\ * & W_3 \end{bmatrix}\begin{bmatrix} \theta_1^{(k)} & \theta_2^{(k)} \\ * & \theta_3^{(k)} \end{bmatrix}) \end{aligned}$$

subject to LMIs: (22h), (22i), (23), (31), (33), (35), (39), (41), (44), (45) and (48).

Set $U^{(k+1)} = U, Z^{(k+1)} = Z, V_1^{(k+1)} = V_1, V_2^{(k+1)} = V_2, V_3^{(k+1)} = V_3, Q_4^{(k+1)} = Q_4, Q_5^{(k+1)} = Q_5, Q_6^{(k+1)} = Q_6, J_{11}^{(k+1)} = J_{11}, P_{11}^{(k+1)} = P_{11}, \alpha_1^{(k+1)} = \alpha_1, \alpha_2^{(k+1)} = \alpha_2, \alpha_3^{(k+1)} = \alpha_3, W_1^{(k+1)} = W_1, W_2^{(k+1)} = W_2, W_3^{(k+1)} = W_3, \theta_1^{(k+1)} = \theta_1, \theta_2^{(k+1)} = \theta_2, \theta_3^{(k+1)} = \theta_3$.

(4) Verify the condition (24) and (32). If (24) and (32) are satisfied, then the parameters of the dynamic output feedback controller (3) are given in (25). If condition (24) and (32) are not satisfied within a specified number of steps of iterations, then exit. Otherwise, set the index k of the objective function in Step (3) as $k+1$ and go to Step (3).

Remark 3: It should be noticed that the controller (3) obtained using the above method is not sure be proper. In fact, when $E = \begin{bmatrix} I_p & 0 \\ 0 & 0 \end{bmatrix}$, let $\bar{A}_k = \begin{bmatrix} A_{k11} & A_{k12} \\ A_{k21} & A_{k22} \end{bmatrix}$. If A_{k22} is nonsingular, the controller (3) is a p-dimensional proper controller. If A_{k22} is singular, we redefine $\bar{A}_k$ as $\bar{A}_k^* = \begin{bmatrix} A_{k11} & A_{k12} \\ A_{k21} & A_{k22}+\mu I \end{bmatrix}$, where scalar $\mu > 0$ is sufficient small such that $A_{k22}+\mu I$ is nonsingular and $\bar{A}^* = \begin{bmatrix} A & B\bar{C}_k \\ \bar{B}_kC & \bar{A}_k^* \end{bmatrix}$ still satisfies $\beta(\varepsilon,\bar{A}^*,\bar{A}_\tau,\bar{D}_1,\bar{E}_1,\bar{E}_\tau,\bar{Q},\bar{P},\bar{Z},\bar{X},\bar{Y}) < 0$. Thus $(\bar{A}_k^*,\bar{B}_k,\bar{C}_k)$ forms a proper dynamic output feedback controller for the system (1).

IV. CONCLUSIONS

In this paper, the problem of delay-dependent robust stabilization for a class of uncertain singular time-delay system with norm-bounded uncertainties is investigated. The sufficient conditions of the existence of the dynamic output feedback controller are proposed. The presented control law guarantees the resultant closed-loop system is regular, impulse free as well as robustly stable for all admissible uncertainties. Since the conditions given in this paper are delay-dependent and the controllers can be designed effectively by using the LMIs and the cone complementarity linearization algorithm, this paper improves the results in [4] to a certain extent.

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