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Evaluating PID Control for Supply Chain Management:
A Freshman Design Project

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Abstract— **Supply chain management is concerned with the efficient movement of goods through a network of suppliers and retailers. As delayed dynamical systems, supply chains represent an excellent opportunity for illustrating the benefits of engineering control principles to what may normally be perceived as a "business" process. This paper describes an Excel modeling project developed by the authors for a first-year engineering course at Arizona State University that 1) introduces students to a meaningful application of control engineering principles, and 2) enables them to develop their computer-based problem-solving skills. The project contrasts standard inventory management policies based on traditional Economic Order Quantity (EOQ) approaches with a judiciously-designed Proportional-Integral-Derivative (PID) controller in the case of a single inventory in a supply chain. A well-tuned PID-based policy is able to generate effective decisions on orders that ultimately reduce the need for safety stock, eliminate backorders, and mitigate the "bullwhip effect"; such behavior in an actively managed supply chain represents desirable outcomes for the enterprise.**

I. INTRODUCTION

All engineering freshmen students at Arizona State University are required to take ECE 100: Introduction to Engineering Design. The course plays an important role in meeting accreditation requirements for the Fulton School under the ABET EC2000 criteria [1]. The course objectives are: 1) students will learn to apply the engineering design methodology, 2) students will acquire and use basic engineering skills and tools, and 3) students will understand and value engineering as a career and a profession. In support of objective 2), students are introduced to the use of Excel for engineering-oriented modeling assignments and projects. Control engineering problems are naturally suited to illustrate basic modeling principles in engineering. However, course prerequisites, which are limited to high-school computing, physics, and algebra, place constraints on the ability to develop meaningful problems at the freshman level involving dynamics and controls. The design project described in this article, nonetheless, takes the approach that this dilemma is not an obstacle if one keeps in mind the philosophy expressed by Lennart Ljung in the preface of his well-known system identification text [2]: "...it is a redeeming feature of life that we are able to use many things without understanding every detail of them".

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Supply chains, also known as demand networks or value webs, consist of interconnected entities, such as factories, distribution centers, retailers, and transportation links, that transform ideas and materials into value-added goods and services. An effective supply chain is managed with the goal of maintaining a high level of customer satisfaction while minimizing costs and maximizing profits. Supply chain management (SCM) has become an increasingly important aspect of our national economy; reports of savings achieved by best-in-class companies as a result of improving their supply chain operations amount to 5 - 6% of sales [3]. As sophisticated consumers themselves, we find that many of our students are conceptually aware of the importance of supply chains, and are motivated to examine how an engineering-based approach can be used to improve what they may normally consider as a business process.

The paper is organized as follows. Section II provides a general problem description of the exercise given to the ECE 100 students, and summarizes the Economic Order Quantity (EOQ) policies applied to the problem. Section III presents the theoretical basis for the design of the PID controller implemented by students in this exercise. Section IV describes how the project is conducted, while Section V provides a summary and conclusions.

II. PROBLEM DESCRIPTION

The problem considered in the ECE 100 student project is this: Sparky Computer (SC) Inc., a hypothetical diversified global computer manufacturer based in Tempe, Arizona, has realized that it can significantly increase its profit margins and customer satisfaction levels by improving its inventory managment system. To this end, upper management at SC has hired a consulting firm (the student team) to study its current inventory management practices, which are based on the use of traditional Economic Order Quantity (EOQ)-style approaches [4], and advise on improvements and modifications. In particular, Sparky Computer is opting to license technology from two PID control system vendors; these are also hypothetical firms named IMC, Inc. and ACME®. Specifically, management wishes to determine the following:

- whether replacing the existing EOQ policies with PID ones has merit,
- which one of the two PID control vendors offers the most suitable technology, and
- which specific product within that vendor's suite of offerings represents the best buy.

To this end, the student team must provide *demonstrated* answers to the following questions:

1) *How do standard EOQ policies, tested under conditions that involve both stochastic and deterministic demand variations, compare with PID-type decision policies?*

2) *How can the design and adjustable parameters in the PID controller be specified to minimize total costs over 60 days while avoiding undesirable amplification of demand variations in the orders determined by the decision policy? This phenomenon is commonly known as the "bullwhip effect" [5] in the business literature.*

The problem chosen for analysis is that of a retailer with a single item inventory and a 3-day order fulfillment time. A fluid analogy for the problem is shown in Figure 1. The translation to a control problem follows the analysis of Braun *et al.* [6]. Inventory at the retailer is the controlled variable, which must be maintained at a desired safety stock/setpoint to cushion the retailer against unpredictable demand variations. Demand at the retailer is treated as a disturbance variable. Students must use Excel to implement and evaluate control policies that will automatically reorder product to maintain inventory at a predetermined set point. Four classical EOQ policies, ((s, Q), (s, S), (R, S), and (R, s, S)) which are simple feedback policies relying on IF-THEN rules for reordering inventory [4], have to be evaluated and contrasted with a number of policies relying on PID control. In the EOQ policies, a decision to place an order is made when either the inventory position (IP), defined as

$$\text{Inventory Position} = \text{Stock on Hand} + \text{Stock on Order}$$
$$- \text{Backorders} - \text{Committed}$$

is lower than or equal to a threshold value s, or a review period R is reached. The magnitude of the order then corresponds to either a fixed quantity Q or to the amount that will take the inventory position to an "order-up-to value" S. In the (R, s, S) policy both conditions R and s must be satisfied before an order can be made. Rules for the four standard EOQ strategies can be summarized as follows:

(s,Q): If $IP < s$, then order quantity Q.

(s,S): If $IP < s$, then order quantity $S - IP$.

(R,S): If the review period R is reached, then order quantity $S - IP$.

(R,s,S): If the review period R is reached and $IP < s$, then order quantity $S - IP$.

The underlying theory behind the open-loop model describing this system is briefly explained. The dynamics of inventory in the system are represented by a simple difference equation reflecting the conservation of mass:

$$I(k+1) = I(k) + O(k-\theta) - D(k) \quad (1)$$

where $I(k)$ is the net stock inventory (the difference between on-hand inventory and backorders), θ represents the order fulfillment time, $O(k-\theta)$ represents the orders

made θ days prior, and $D(k)$ is the demand signal. k is the index for the day in question. The open-loop model can be conceptually understood by students, since it basically states that $I(k+1)$, the net stock inventory at the end of the day, equals $I(k)$, the net stock inventory at the start of the day, plus the orders that were received that day (which were placed θ days prior) less $D(k)$, the sales/demand for that day. For simplicity the order fulfillment time is assumed fixed, but the demand signal $D(k)$ can have both deterministic and stochastic components.

III. PID-BASED DECISION POLICY

There are multiple ways in which engineering control policies can be developed for systems with dynamics per (1). One approach towards obtaining a model-based, engineering feedback control law for this system is recognize that the model per (1) corresponds to a delayed, integrating plant. The use of a first-order Padé approximation on the delay leads to an integrating system with lag and a Right-Half Plane zero,

$$I(s) = \frac{e^{-\theta s}}{s} O(s) - \frac{1}{s} D(s) \quad (2)$$

$$\approx \frac{\frac{-\theta}{2}s + 1}{s(\frac{\theta}{2}s + 1)} O(s) - \frac{1}{s} D(s) \quad (3)$$

The resulting model per (3) is thus amenable to the Internal Model Control (IMC) rules for Proportional-Integral-Derivative (PID) type controllers as shown in [7] and [8]. One such rule, which results in no offset for ramp-like disturbances, is presented to the students as the IMC "Platinum" rule, and corresponds to a PID with filter structure per

$$c(s) = K_c(1 + \frac{1}{\tau_I s} + \tau_D s)\frac{1}{(\tau_F s + 1)} \quad (4)$$

$$\beta = \tau = \frac{\theta}{2} \quad (5)$$

$$K_c = \frac{2(\beta + \lambda) + \tau}{2\beta^2 + 4\beta\lambda + \lambda^2} \quad (6)$$

$$\tau_I = 2(\beta + \lambda) + \tau \quad (7)$$

$$\tau_D = \frac{2\tau(\beta + \lambda)}{2(\beta + \lambda) + \tau} \quad (8)$$

$$\tau_F = \frac{\beta\lambda 2}{2\beta^2 + 4\beta\lambda + \lambda^2} \quad (9)$$

This tuning rule includes an adjustable parameter λ which can be used to determine the speed-of-response and to influence the peformance-robustness tradeoff inherent in all feedback control systems. Similar rules with product names "Bronze", "Silver", "Gold", and "Platinum 2" are generated by using different types of delay approximations and factorizations for the zero dynamics in the model. The corresponding PID tuning rules for the competitor's products (ACME®) do not include any adjustable parameters, and arise from the use of alternate design procedures, such

as the open-loop Ziegler-Nichols tuning rules for delayed integrating systems described in [9].

Since students are neither control experts nor required to know calculus, the engineering control law must be presented to them in a manner that is conceptually simple as well as readily implementable in Excel. By using first-order backward-difference approximations on the integral and derivative modes, the continuous-time PID controller per (4) can be expressed as a difference equation in velocity form, written as follows:

$$
\begin{aligned}
O(k) \;=\;& O(k-T) + K_1\, e(k) + K_2\, e(k-T) \quad (10)\\
& + K_3\, e(k-T) + K_4\, \Delta O(k-T)
\end{aligned}
$$

$$
\begin{aligned}
K_1 \;&=\; \frac{TK_c}{\tau_F + T}\left(1 + \frac{T}{\tau_I} + \frac{\tau_D}{T}\right)\\
K_2 \;&=\; -\frac{TK_c}{\tau_F + T}\left(1 + \frac{2\tau_D}{T}\right)\\
K_3 \;&=\; \frac{K_c \tau_D}{\tau_F + T}\\
K_4 \;&=\; \frac{\tau_F}{\tau_F + T}
\end{aligned}
$$

$\Delta O(k-T) = O(k-T) - O(k-2T)$ is the previous controller move, while $e(k) = I(k) - R(k)$ is a control error, which represents the discrepancy between inventory and the reference setpoint $R(k)$ corresponding to a desired safety stock level. T, the sampling time, is the review period for the PID policy. (11) indicates that the current order made by the PID policy is determined by the previous order plus some scaled corrections based on the current and previous control errors and prior control action, the scaling determined by the controller coefficients K_1, K_2, K_3, and K_4. These coefficients are not arbitrarily specified, but determined on the basis of a model-based tuning rule, where the model is defined by a fundamental conservation principle, the conservation of total mass. This is in contrast to the EOQ policies, which are somewhat *ad hoc* policies whose performance can vary drastically based on the choices for the inventory position threshold s, the review period R, the order quantity Q and the order-up-to value S.

IV. CONDUCTING THE SCM-PID CONTROL PROJECT

In the initial part of the course, students use Excel to model the four basic EOQ policies ((s, Q), (s, S), (R, S), and (R, s, S)) and the IMC Platinum policy over a 60-day time period; this helps them build some basic proficiency with spreadsheeting. To improve the students' intuitive understanding of open and closed-loop system behavior, we then make available two advanced spreadsheets with animation features that enable them to better visualize the phenomena under study. The first spreadsheet, the Interactive Order Simulator (Figure 3) serves to introduce students to the challenges associated with manually controlling a

delayed system. Through a series of scenarios the students attempt to minimize inventory holding and order costs without incurring backorder expenses by deciding how much product to order each day. Students can immediately see the results of their actions in a set of graphs arranged to represent the system as a series of tanks. When orders are placed, students can see them represented as fluid in the topmost tank draining into another tank representing the delay involved in order fulfillment, eventually reaching the tank symbolizing the on-hand inventory, which is concurrently depleted due to demand. As students go through the scenarios they can observe the current state of the supply chain, and try to avoid excessive surpluses or stockouts as best they can. The scenarios introduce conditions with different levels of difficulty, such as variable initial safety stock levels, varying deterministic and stochastic demand changes, and the presence or absence of potentially inaccurate demand forecasts. Throughout the simulation students can observe the real time cost information broken down into order, backorder, and holding costs.

Having become acquainted with the basics of delayed systems and inventory management from the Interactive Order Simulator, students are ready to progress and try implementing automatic PID control on the problem using the single node Supply Chain Inventory Dynamics Simulator (Figure 4). This spreadsheet includes a master control page similar in appearance to the previous spreadsheet, but unlike its manually controlled predecessor the amount to be ordered each day and, consequently, the inventory is now decided by various adjustable EOQ and PID based policies. Some of these policies come preprogrammed and only require the students to adjust the various settings and interpret the results. A few worksheets, however, are set aside for the students to implement their assigned IMC and ACME® PID tuning rules. All of the policies can be viewed graphically as they would occur in a real-time environment over a 60-day period using the same visual interface as in the Interactive Order spreadsheet. A representative set of responses is shown in Figure 2. The IMC Platinum decision policy for two values of λ (1.5 and 0.1) is contrasted against an (s, Q) EOQ policy where $s = Q = 20$. A lower λ implies a faster speed of response; however, if made too low, the result is aggressive control behavior, resulting in underdamped amplification in the order signal in spite of small changes of demand (the bullwhip effect per [5]). The (s, Q) EOQ policy, which reorders Q amounts whenever the inventory position goes at or below s, naturally introduces significant oscillations in the inventory dynamics and is clearly less desirable than the PID response. One exception is when order costs are significant relative to inventory holding or backorder costs; however, this scenario is becoming a less frequent occurrence in practice.

Both spreadsheets are designed to run from macros written in Microsoft's Visual Basic for Applications.

The purpose of the macros is to apply the settings chosen on the control page throughout the workbook and either take input from the user or re-output the data generated by the control policies sequentially as opposed to simultaneously, as is normal in Excel. The sequential output of data is especially important for the Dynamics Simulator because it allows the graphs to appear to change dynamically over the course of the simulation. Both the Interactive Order Simulator and the Single Node Supply Chain Inventory Dynamics Simulators, as well as the project description distributed to students and some supporting presentations, can be downloaded from the link `http://www.eas.asu.edu/~csel/ECE100SCM.htm`.

V. SUMMARY AND CONCLUSIONS

The use of a PID-controller based decision policy to manage a single inventory in a supply chain is a relatively simple yet challenging exercise that can be used to illustrate to first-year students the usefulness of an engineering-based approach in a non-traditional application such as supply chain management. By representing both the model and controller in terms of difference equations, the exercise can be presented to students without the need for calculus, and is amenable to implementation using readily-available spreadsheeting software such as Excel. A comparison with EOQ-based approaches shows that a well-tuned PID policy can reduce variability in the supply chain, leading to reduced need for safety stock, lower incidence of backorders, and mitigation of the "bullwhip effect"; accomplishing these goals in an actively managed supply chain is ultimately beneficial for the financial well-being of the enterprise.

Many extensions of the project can be considered, particularly those that involve using demand forecasts as feedforward signals in the control policy. The problem is also useful for upper division courses; for example, a Matlab-based version of the problem has been developed for the senior-level ChE 461 Introduction to Process Dynamics and Control course to demonstrate the effectiveness of combined feedback-feedforward control strategies over feedback-only control.

VI. ACKNOWLEDGEMENTS

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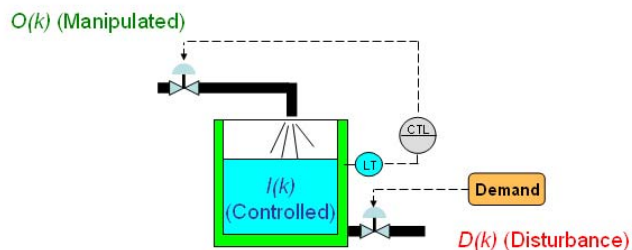[9] K. Erickson and J. Hedrick, *Plantwide Process Control*. New York: Wiley, 1999.

Fig. 1. Fluid Analogy for single node, single inventory supply chain problem under feedback-only control. The inlet flow to the pipe represents orders made by the controller/decision policy, while the length of the pipe is proportional to the order fulfillment time. The material in the tank represents stockpiled inventory that is used to meet demand, which dictates the outflow from the tank.
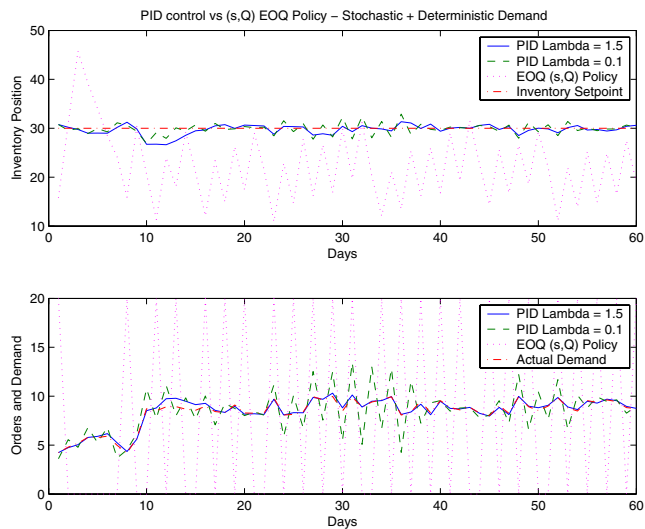

Fig. 2. Response comparison between representative PID and EOQ policies. The IMC Platinum decision policy for two values of λ (1.5 and 0.1) is contrasted against an (s, Q) EOQ policy for $s = Q = 20$ for stochastic demand changes plus a deterministic step change at day 10. The system is well-tuned for $\lambda = 1.5$, but displays aggressive control behavior and amplification in the order signal in spite of small changes of demand (the bullwhip effect per [5]) when $\lambda = 0.1$. The (s, Q) EOQ policy naturally introduces significant oscillations in the inventory dynamics.
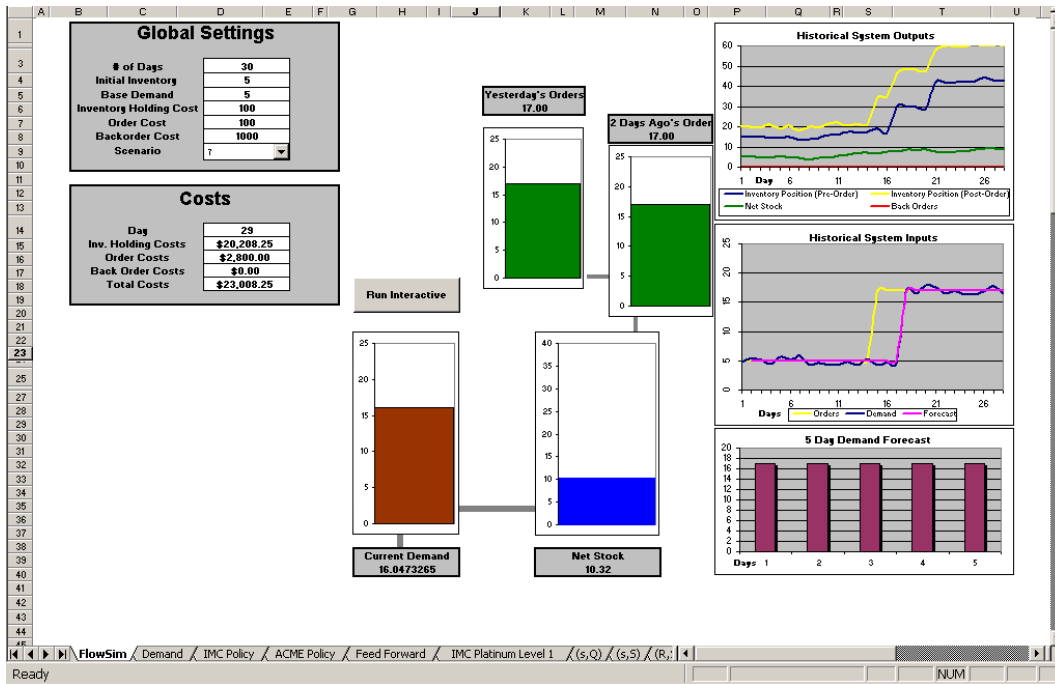
Fig. 3. Screen shot of the animated portion of the Interactive Order simulator spreadsheet. Students manually control the inventory dynamics of the system by deciding how much to order each day.
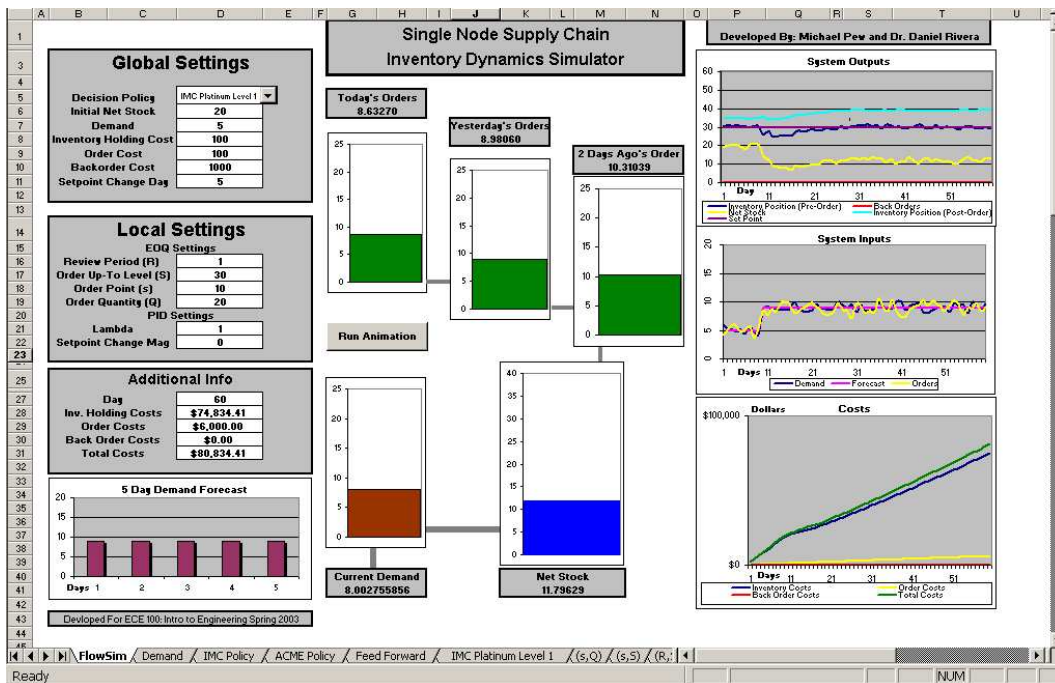

Fig. 4. Screen shot of the animated portion of the single node Supply Chain Inventory Dynamics simulator. This spreadsheet enables students to tune and evaluate various closed-loop PID and EOQ policies on the system.

3419