Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Adaptive Neural Network Output Feedback Control of Nonlinear Systems with Actuator Saturation

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***Abstract*— An indirect adaptive neural network (NN) saturation compensator is presented for a class of nonlinear systems. Output feedback control is considered where only the system output is assumed to be measurable. The imposed actuator saturation is assumed to be unknown and treated as the system input disturbance. A NN-based state observer estimates derivatives of the output and another NN-based feedback controller is inserted into a feedforward path to capture the nonlinearities of the observed system and to cancel the effects of the unknown disturbances and the unknown saturation nonlinearity. The unknown system states identified by the NN observer are inputs of the NN-based controller. Two NNs interact together to achieve the desired performance. Both adaptive, neural network control laws and on line neural net weights tuning rules are rigorously derived based on feedback linearization and Lyapunov approach. The overall robust adaptive scheme guarantees that the states estimation errors, NN weights estimation errors, and output tracking errors are uniformly ultimately bounded. The simulation conducted indicates the proposed scheme can effectively estimate the unknown nonlinear system states and accommodate the unknown actuator constraints.**

I. Introduction

Adaptive control techniques adjust the controller parameters using system response to achieve the desired performance. Adaptive control can accommodate the system modeling, and parametric and environmental uncertainties [19]. Due to the non-analytic nature of the actuator nonlinear dynamics and the fact that the exact actuator nonlinear functions, namely operation uncertainty, are unknown, such systems provide an application field for adaptive control, sliding control and neural network-based control. Proportional-derivative (PD) controller has observed limit cycles if the actuator nonlinearity exists. To overcome the unavoidable actuator nonlinearities such as saturation, deadzone, backlash, hysteresis and the actuator failures, a number of researches [6], [8], [9], [11], [16], [18], [19] have developed techniques for their compensation.

Saturation is a common nonlinearity in most actuators. Categories of saturation nonlinearities include constraints of the magnitude and the rate of actuator inputs. These constraints may be deliberately placed on the actuator to avoid the damage on the system. When an actuator has reached such an input limit, it is said to be “saturated”, since efforts to further increase the actuator output would not result in any variation in the output, which is a challenge to the control design engineer. Particularly when integrator is present in the controller, an integrator wind up occurs and may generate additional problems.

To tackle the problem, many saturation compensation schemes have been developed based on a full state measurement [1], [2], [3], [4], [6], [8], [9], [11], [12], [14]. However, this assumption is not always realistic in practical control systems. Sometimes, only the output or some of system states are known and available.

For adaptive control design, usually, the linearity in the system parameters condition is required. Our NN-based observer that estimates the unknown nonlinear function as well as the unknown actuator saturation nonlinearity does not require this condition. NNs can successfully deal with nonlinear systems that are linearly unparameterizable.

Observers for linear systems were proposed by Luenberger. Recently, many researchers [5], [6], [7], [8], [11], [13], [15] utilized NNs to synthesize the feedback linearization based controls and incorporate the Lyapunov theory to ensure the overall system stabilization, command following, and disturbance rejection.

This paper presents a robust neural net, observer-based saturation compensator for a class of nonlinear systems in Brunovsky canonical form. The paper is organized as follows. Section 2 provides the preliminary remarks. Section 3 presents the nonlinear systems with saturation nonlinearity. Section 4 addresses the NN-based observer. Section 5 shows the design process of outer loop tracking controller. In Section 6, a simulation example on “pendulum type” nonlinear system with saturation nonlinearity is given. The conclusion is drawn in Section 7.

II. Preliminary Remarks

Assuming unknown saturation operation limits $\tau_{\min}$ and $\tau_{\max}$, the output of the actuator $\tau(t)$ is given by

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$$\tau(t)=\begin{cases}\tau_{\max}: & u(t)\ge \tau_{\max}/m\\ mu(t): & \tau_{\min}/m<u(t)<\tau_{\max}/m\\ \tau_{\min}: & u(t)\le \tau_{\min}/m\end{cases}, \quad (2.1)$$

where $\tau_{\max}$ is chosen positive and $\tau_{\min}$ is negative saturation limits. The control that cannot be implemented by the actuator $\delta(t)$, is given by

$$\delta(t)=\begin{cases}\tau_{\max}-u(t): & u(t)\ge \tau_{\max}/m\\ (m-1)u(t): & \tau_{\min}/m<u(t)<\tau_{\max}/m\\ \tau_{\min}-u(t): & u(t)\le \tau_{\min}/m\end{cases}. \quad (2.2)$$

The nonlinear actuator saturation can equivalently be described using $\delta(t)$ [6] and NN can be used for its approximation.

III. Nonlinear Systems With Actuator Saturation

Consider the nonlinear systems with state space representation in Brunovsky canonical form

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= x_3\\ &\vdots\\ \dot{x}_n &= f(x)+d(t)+\tau(t)\\ y &= x_1\end{aligned} \quad (3.1)$$

where $x=[x_1,x_2,\cdots,x_n]^T$, $f:\Re^n\rightarrow\Re$ is an unknown smooth function, d is the unknown bounded disturbance with a known upper bound D, and τ is the control input.

Define matrices A, b, and h as

$$A=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\ & &\vdots& & \\0&0&\cdots&1&0\\0&0&0&\cdots&0\end{bmatrix},\quad b=\begin{bmatrix}0\\0\\\vdots\\0\\1\end{bmatrix},\quad h=\begin{bmatrix}1\\0\\\vdots\\0\\0\end{bmatrix}. \quad (3.2)$$

The equation (3.1) can be written in the following state space form

$$\begin{aligned}\dot{x} &= Ax+b\{f(x)+d(t)+\tau(t)\}\\ y &= h^T x\end{aligned} \quad (3.3)$$

Using the modified saturation nonlinearity (2.2), the above system is converted to

$$\begin{aligned}\dot{x} &= Ax+b\{f(x)+d(t)+u(t)+\delta(t)\}\\ y &= h^T x\end{aligned} \quad (3.4)$$

Define the desired state vector $x_d(t)$ as

$$x_d(t)=[y_d,\dot{y}_d,\cdots,y_d^{(n-1)}]^T. \quad (3.5)$$

Assumption 1 (Bounded Desired Trajectory): The desired trajectory $x_d(t)$ is bounded and continuous, and $\|x_d(t)\|\le Q$ with Q a known scalar bound.

It is assumed that nonlinear function $f(x)$ contains parameter uncertainties including known and unknown states and that only output y is measurable. The objective of the controller and compensator designs is to estimate the unknown system states and find a control law u to compensate for the actuator saturation.

IV. NN-Based Observer Design

A nonlinear observer that estimates unknown states of system (3.4) is given by

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x}+b\{\hat{f}(\hat{x})+\hat{\delta}(t)+u(t)\}+L(y-h^T\hat{x})+v_o(t)\\ \hat{y} &= h^T\hat{x}\end{aligned}, \quad (4.1)$$

where $\hat{x}$ is an estimate of the state vector x, functions $\hat{f}$ and $\hat{\delta}$ are estimates of f and δ respectively. A robust term $v_o(t)$ is added to accommodate the function estimation error and unknown disturbances. The observer matrix gain $L=[l_1 \;\; l_2 \;\; \cdots \;\; l_n]^T$ is chosen so that $A_s=A-Lh^T$ is stable *i.e.*, for a symmetric positive definite matrix Q, there exists a symmetric positive definite matrix P satisfying the following equations

$$\begin{aligned}A_S^T P+PA_S &= -Q\\ Pb &= h\end{aligned}. \quad (4.2)$$

Dynamic or recurrent NNs are capable to identify or estimate internal states of an unknown nonlinear dynamical system. Applying NN universal approximation property, there exist a NN with some ideal weights V_o and W_o that closely approximates the unknown function $f+\delta$

$$f+\delta=W_o^T\sigma(V_o^T x_{oNN})+\varepsilon_o. \quad (4.3)$$

As in [6], a two layer NN has been used. The first layer weights V are selected randomly and will not be tuned; the second layer weights W are tunable. Implemented NN is actually an approximation of the ideal NN (4.3), and is given by

$$\hat{f}+\hat{\delta}=\hat{W}_o^T\sigma(V_o^T\hat{x}_{oNN}), \quad (4.4)$$

Since not all states are available, input to NN includes the known and estimated states.

The NN weights approximation error is given by

$$\tilde{W}_o=W_o-\hat{W}_o. \quad (4.5)$$

Assumption 2 (Bounded Ideal NN Weights): The ideal NN weights W_o are bounded so that $\|W_o\|\le W_{oM}$, with W_{oM} a known bound.

Substituting (4.3) and (4.4) into (3.4) and (4.1) respectively yields

$$\begin{aligned}\dot{x} &= Ax+b\{W_o^T\sigma(V_o^T x_{oNN})+\varepsilon+d(t)+u(t)\}\\ y &= h^T x\end{aligned} \quad (4.6)$$

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x}+b\{\hat{W}_o^T\sigma(V_o^T\hat{x}_{oNN})+u(t)\}+L(y-h^T\hat{x})+v_o(t)\\ \hat{y} &= h^T\hat{x}\end{aligned} \quad (4.7)$$

Subtracting (4.7) from (4.6) yields the estimation error dynamics

$$\dot{\tilde{x}} = A_S\tilde{x} + b\left\{\tilde{W}_o^T\sigma(V_o^T\hat{x}_{oNN}) + W_o^T\tilde{\sigma} + \varepsilon_o + d\right\} - v_o(t) \quad (4.8)$$
$$\tilde{y} = h^T\tilde{x}$$

where $\tilde{\sigma} = \sigma(V_o^T x_{oNN}) - \sigma(V_o^T\hat{x}_{oNN})$. Based on the above ideal bounded weight assumption for W_o and the property of the NN activation function, it is obvious that $W_o{}^T\tilde{\sigma}$ is bounded.

Corollary 1: Term $W_o^T\tilde{\sigma}$ is bounded such that $\left\|W_o^T\tilde{\sigma}\right\| \le \varepsilon_{oM}$, with ε_{oM} a known bound.

Then disturbance term $\varepsilon = W_o^T\tilde{\sigma} + \varepsilon_o$ is also bounded such that $\|\varepsilon\| \le \varepsilon_M$ where ε_M is a known bound. Then, the estimation error dynamics can be expressed as

$$\dot{\tilde{x}} = A_S\tilde{x} + b\left\{\tilde{W}_o^T\sigma(V_o^T\hat{x}_{oNN}) + \varepsilon + d\right\} - v_o(t)$$
$$\tilde{y} = h^T\tilde{x} \quad . \quad (4.9)$$

***Theorem 1 (Tuning of NN Observer)*:** Given the system in (3.4) with saturation imposed and Assumptions 1 and 2, choose the dynamic NN estimator equation (4.1), and the robustifying term as

$$v_o = k_r\tilde{y} + k_W\left\|\hat{W}_o\right\|_F\tilde{y}, \quad (4.10)$$

with scalars $k_r > 0$ and $k_W > 0$. Let the estimated NN weights be provided by the NN tuning algorithm

$$\dot{\hat{W}}_o = \Gamma\sigma(V_o^T\hat{x}_{oNN})\tilde{y} - h\Gamma\|\tilde{y}\|\hat{W}_o, \quad (4.11)$$

where $\Gamma = \Gamma^T > 0$ is a constant matrix representing the learning rates of the NN and h is a small scalar positive design parameter. By properly selecting the control gains and the design parameters, the estimation error $\tilde{x}$ and the NN weights $\hat{W}_o$ are uniformly ultimately bounded.

Proof: Choose Lyapunov function candidate as

$$L_o = \frac{1}{2}\tilde{x}^T P\tilde{x} + \frac{1}{2}tr(\tilde{W}_o^T\Gamma^{-1}\tilde{W}_o) \quad (4.12)$$

Differentiating yields

$$\dot{L}_o = \frac{1}{2}(\dot{\tilde{x}}^T P\tilde{x} + \tilde{x}^T P\dot{\tilde{x}}) + tr(\tilde{W}_o^T\Gamma^{-1}\dot{\tilde{W}}_o) \quad . \quad (4.13)$$

Whence substitution from error equation (4.9), and using (4.2) yields

$$\begin{aligned}\dot{L}_o &= -\frac{1}{2}\tilde{x}^T Q\tilde{x} + \tilde{x}^T Pb\varepsilon + \tilde{x}^T Pbd - \tilde{x}^T Pv_o + \\ &\quad tr\left\{\tilde{W}_o^T\left[\Gamma^{-1}\dot{\tilde{W}}_o + \sigma(V_o^T\hat{x}_{oNN})(Pb)^T\tilde{x}\right]\right\} \\ &= -\frac{1}{2}\tilde{x}^T Q\tilde{x} + \tilde{x}^T Pb\varepsilon + \tilde{x}^T Pbd - \tilde{x}^T Pv_o + \\ &\quad tr\left\{\tilde{W}_o^T\left[\Gamma^{-1}\dot{\tilde{W}}_o + \sigma(V_o^T\hat{x}_{oNN})\tilde{y}\right]\right\}\end{aligned} \quad (4.14)$$

Applying the NN tuning rules and robust term (4.10), equation (4.14) is equivalent to

$$\begin{aligned}\dot{L}_o &= -\frac{1}{2}\tilde{x}^T Q\tilde{x} + \tilde{x}^T Pb\varepsilon + \tilde{x}^T Pbd - K_r\tilde{x}^T P\tilde{y} - \\ &\quad K_W\left\|\hat{W}_o\right\|_F\tilde{x}^T P\tilde{y} + h\|\tilde{y}\|tr\left\{\tilde{W}_o^T\hat{W}_o\right\} \\ &= -\frac{1}{2}\tilde{x}^T Q\tilde{x} + \tilde{x}^T Pb\varepsilon + \tilde{x}^T Pbd - K_r P(Pb)^T\|\tilde{x}\|^2 \\ &\quad - K_W\left\|\hat{W}_o\right\|_F P(Pb)^T\|\tilde{x}\|^2 + h\|\tilde{y}\|tr\left\{\tilde{W}_o^T\hat{W}_o\right\}\end{aligned} \quad (4.15)$$

Let $P_{\min}$, $Q_{\min}$ be the smallest eigenvalues and $P_{\max}$, $Q_{\max}$ the largest eigenvalues of P, Q. Then (4.15) yields

$$\begin{aligned}\dot{L}_o &\le -\frac{1}{2}Q_{\min}\|\tilde{x}\|^2 + P_{\max}(\varepsilon_N + bD)\|\tilde{x}\| + \\ &\quad h\left\|(Pb)^T\right\|\|\tilde{x}\|tr\left\{\tilde{W}_o^T\hat{W}_o\right\}\end{aligned} \quad (4.16)$$

Using the inequality,

$$tr\left[\tilde{X}^T(X - \tilde{X})\right] \le \left\|\tilde{X}\right\|_F\|X\|_F - \left\|\tilde{X}\right\|_F^2, \quad (4.17)$$

(4.16) can be written as

$$\begin{aligned}\dot{L}_o &\le -\frac{1}{2}Q_{\min}\|\tilde{x}\|^2 + P_{\max}(\varepsilon_N + bD)\|\tilde{x}\| + \\ &\quad h\left\|(Pb)^T\right\|\|\tilde{x}\|(\left\|\tilde{W}_o\right\|_F\|W_o\|_F - \left\|\tilde{W}_o\right\|_F^2) \\ &\le -\|\tilde{x}\|\left\{\begin{aligned}&(\frac{1}{2}Q_{\min}\|\tilde{x}\| + h\left\|(Pb)^T\right\|(\left\|\tilde{W}_o\right\|_F - \frac{1}{2}W_{oM})^2 \\ &- P_{\max}(\varepsilon_N + bD) - \frac{1}{4}h\left\|(Pb)^T\right\|W_{oM}^2\end{aligned}\right\}\end{aligned} \quad (4.18)$$

which is guaranteed to remain negative as long as

$$\|\tilde{x}\| \ge \frac{\frac{h}{4}\left\|(Pb)^T\right\|W_{oM}^2 + P_{\max}(\varepsilon_N + bD)}{\frac{1}{2}Q_{\min}}, \quad (4.19)$$

or

$$\left\|\tilde{W}_o\right\|_F \ge \frac{1}{2}W_{oM} + \left(\frac{P_{\max}(\varepsilon_N + bD)}{h\left\|(Pb)^T\right\|} + \frac{1}{4}W_{oM}^2\right)^{\frac{1}{2}}. \quad (4.20)$$

V. NN-Based Controller Design

Let define the state tracking error vector, $\hat{e}(t)$ as

$$\hat{e}(t) = \hat{x}(t) - x_d(t) \quad , \quad (5.1)$$

and a filtered tracking error as

$$\hat{r} = K^T\hat{e}, \quad (5.2)$$

where $K=[k_1,k_2,...k_{n-1},1]^T$ is appropriately chosen coefficient vector [17], so that $\hat{e}\to 0$ exponentially as $\hat{r}\to 0$. Then, the time derivative of the filtered tracking error can be written as

$$\dot{\hat{r}}=f(x)+u+\delta+d+Y_d\,, \tag{5.3}$$

where $Y_d=-y_d^{(n)}+\sum_{i=1}^{n-1}k_i\hat{e}_{i+1}$ is a known function of the tracking error and the desired state.

Applying the feedback linearization method choose the tracking control law as

$$w=-\hat{f}(\hat{x})-Y_d+v_c-K_v\hat{r}\ , \tag{5.4}$$

where $\hat{f}(\hat{x})$ is the fixed approximation of function $f(\cdot)$. The functional estimation error is given by

$$\tilde{f}=f-\hat{f}\ . \tag{5.5}$$

Assumption 3 (Bounded Estimation Error): The nonlinear function f is assumed unknown, but a fixed estimate $\hat{f}$ is assumed known, such that the functional estimation error $\tilde{f}$ satisfies

$$\left\|\tilde{f}\right\|\le f_M \tag{5.6}$$

with f_M a known bound.

As in [6], [16], approximation $\hat{f}$ is fixed in this paper and will not be adapted. Robust term v_c is chosen for the disturbance rejection. The feedback gain K_v is a scalar. The control law u is then a combination of the tracking controller and the saturation compensator, as shown in the Figure 1.

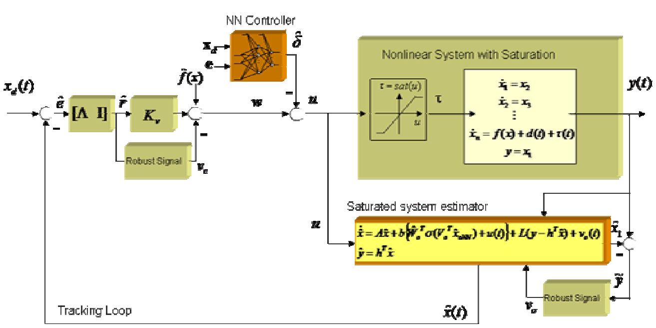

Figure 1. Saturation compensation based on a NN observer scheme.

The control signal is given by

$$u=w-\hat{\delta}\ \ , \tag{5.7}$$

where $\hat{\delta}$ is the approximation of modified saturation nonlinear function $\delta(x)$.

Using NN function approximation property, there exists a NN that closely approximates the modified saturation nonlinear function $\delta(x)$

$$\delta=W_c^T\sigma(V_c^T\hat{x}_{cNN})+\varepsilon_c\ . \tag{5.8}$$

Two layer NN is used where the second layer weights W_c are tunable and the first layer weights V_c are selected randomly and will not be tuned. Implemented NN is given by

$$\hat{\delta}=\hat{W}_c^T\sigma(V_c^T\hat{x}_{cNN})\ , \tag{5.9}$$

where the NN weights approximation error is

$$\tilde{W}_c=W_c-\hat{W}_c \tag{5.10}$$

Input to the NN saturation compensator is chosen as $\hat{x}_{cNN}\equiv[x_d,\hat{e}]^T$.

Assumption 4 (Bounded Ideal NN Weights): The ideal NN weights W_c are bounded so that $\|W_c\|\le W_{cM}$, with W_{cM} a known bound.

Using control law (5.7) and (5.4), and substituting into (5.3) overall closed-loop error dynamics is

$$\dot{\hat{r}}=\tilde{f}+\tilde{W}_c^T\sigma(V_c^Tx_{cNN})+v_c+d-K_v\hat{r}+\varepsilon_c\ . \tag{5.11}$$

Theorem 2 (Tuning of NN Observer and Compensator): Given the system (3.4) with saturation nonlinearity and Assumptions 1-4, choose the dynamic NN estimator (4.1) and the robustifying term as

$$v_o=k_r\tilde{y}+k_W\left\|\hat{W}_o\right\|_F\tilde{y}\,, \tag{5.12}$$

with scalars $k_r>0$ and $k_W>0$. Choose the tracking control law (5.7), the saturation compensator (5.9), and the robustifying term as

$$v_c(t)=-(f_M(x)+D(x))sign(\hat{r})\,, \tag{5.13}$$

where the $f_M(x)$, $D(x)$ are bounds on the functional estimation error and disturbance, and *sign*(.) is standard sign function. Let the estimated NN weights for the observer and controller be provided by the NN tuning algorithms

$$\dot{\hat{W}}_o=\Gamma\sigma(V_o^T\hat{x}_{oNN})\tilde{y}-h\Gamma\|\tilde{y}\|\hat{W}_o\,, \tag{5.14}$$

$$\dot{\hat{W}}_c=S\sigma(V_c^Tx_{cNN})\hat{r}-kS|\hat{r}|\hat{W}_c\,, \tag{5.15}$$

where $\Gamma=\Gamma^T>0$ and $S=S^T>0$ are constant matrices representing the learning rates of the NN, h and k are small scalar positive design parameters. By properly selecting the control gains and the design parameters, the estimation error $\tilde{x}$, the filtered error $\hat{r}(t)$, the NN compensator weights $\hat{W}_c$, and the NN observer weights $\hat{W}_o$ are uniformly ultimately bounded.

Proof: Choose Lyapunov function candidate as

$$L=L_o+L_c\ \ , \tag{5.16}$$

where L_o is defined in (4.12) and the L_c is given by

$$L_c=\frac{1}{2}\hat{r}^2+\frac{1}{2}tr(\tilde{W}_c^TS^{-1}\tilde{W}_c)\ . \tag{5.17}$$

Differentiating yields

$$\dot{L}_c = \hat{r}\dot{\hat{r}} + tr(\widetilde{W}_c^T S^{-1}\dot{\widetilde{W}}_c) \quad (5.18)$$

Whence substitution from (5.11) yields,

$$\begin{aligned}\dot{L}_c &= -K_v\hat{r}^2 + \hat{r}\widetilde{f} + \hat{r}\widetilde{W}_c^T\sigma(V_c^T x_{cNN}) + \hat{r}v_c + \hat{r}d + \hat{r}\varepsilon_c + tr(\widetilde{W}_c^T S^{-1}\dot{\widetilde{W}}_c) \\ &= -K_v\hat{r}^2 + \hat{r}(\widetilde{f} + v_c + d + \varepsilon_c) + tr(\widetilde{W}_c^T(S^{-1}\dot{\widetilde{W}}_c + \sigma(V_c^T x_{cNN})\hat{r}))\end{aligned} \quad (5.19)$$

Applying the NN compensator tuning rules, equation (5.19) is equivalent to

$$\dot{L}_c = -K_v\hat{r}^2 + \hat{r}(\widetilde{f} + v_c + d + \varepsilon_c) + k|\hat{r}|tr(\widetilde{W}_c^T\hat{W}_c) \quad (5.20)$$

Using (5.13) one has

$$\begin{aligned}\dot{L}_c \le& -K_v\hat{r}^2 + k|\hat{r}|tr(\widetilde{W}_c^T(W_c - \widetilde{W}_c)) - |\hat{r}|f_M \\ & -|\hat{r}|D + |\hat{r}|d + |\hat{r}|\widetilde{f} + |\hat{r}|\varepsilon_N\end{aligned} \quad (5.21)$$

Using the inequality (4.17) yields

$$\begin{aligned}\dot{L}_c &\le -K_v|\hat{r}|^2 + k|\hat{r}|(\|\widetilde{W}_c\|_F\|W_c\|_F - \|\widetilde{W}_c\|_F^2) + |\hat{r}|\varepsilon_N \\ &\le |\hat{r}|\left\{-K_v|\hat{r}| + k(\|\widetilde{W}_c\|_F W_{cM} - \|\widetilde{W}_c\|_F^2) + \varepsilon_N\right\}. \\ &= |\hat{r}|\left\{-K_v|\hat{r}| - k(\|\widetilde{W}_c\|_F - \frac{1}{2}W_{cM})^2 + \frac{1}{4}kW_{cM}^2 + \varepsilon_N\right\}\end{aligned} \quad ,(5.22)$$

Combining terms L_o and L_c, the $\dot{L}$ becomes

$$\begin{aligned}\dot{L} &= \dot{L}_o + \dot{L}_c \\ &\le -\|\widetilde{x}\|\left\{\begin{aligned}&(\frac{1}{2}Q_{\min}\|\widetilde{x}\| + h\|(Pb)^T\|(\|\widetilde{W}_o\|_F - \frac{1}{2}W_{oM})^2 \\ &-P_{\max}(\varepsilon_N + bD) - \frac{1}{4}h\|(Pb)^T\|W_{oM}^2\end{aligned}\right\} \\ &+ |\hat{r}|\left\{-K_v|\hat{r}| - k(\|\widetilde{W}_c\|_F - \frac{1}{2}W_{cM})^2 + \frac{1}{4}kW_{cM}^2 + \varepsilon_N\right\}\end{aligned} \quad (5.23)$$

which is guaranteed to remain negative as long as the following inequalities hold

$$|\hat{r}| \ge \frac{\frac{k}{4}W_{cM}^2 + \varepsilon_N}{K_v}, \quad (5.24)$$

$$\|\widetilde{W}_c\|_F \ge \left(\frac{\sqrt{\frac{k}{4}W_{cM}^2 + \varepsilon_N} + \frac{1}{2}W_{cM}}{k}\right)^{\frac{1}{2}}. \quad (5.25)$$

$$\|\widetilde{x}\| \ge \frac{\frac{h}{4}\|(Pb)^T\|W_{oM}^2 + P_{\max}(\varepsilon_N + bD)}{\frac{1}{2}Q_{\min}}, \quad (5.26)$$

$$\|\widetilde{W}_o\|_F \ge \frac{1}{2}W_{oM} + \left(\frac{P_{\max}(\varepsilon_N + bD)}{h\|(Pb)^T\|} + \frac{1}{4}W_{oM}^2\right)^{\frac{1}{2}}. \quad (5.27)$$

VI. SIMULATION OF NN SATURATION COMPENSATOR

The simulation was performed to verify the effectiveness of the proposed NN compensator using a "generalized pendulum" system [6], [17] with actuator saturation nonlinearity.

To show and focus on the functionality of adaptive NN saturation compensator, we choose the robust term $v_o = 0$ $v_c = 0$ in the following simulation.

We consider a nonlinear system of a pendulum type with bounded disturbance $d(t) = 2\cos(t)$

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= -5x_1^3 - 2x_2 + d + \tau \\ y &= x_1\end{aligned} \quad . \quad (6.1)$$

The control input τ is constrained by the saturation nonlinearity characterized by the parameters

$$\tau_{\max} = 5, \tau_{\min} = -5, m = 1 \quad . \quad (6.2)$$

In this paper, the NN observer with sigmoidal activation function has six, eleven and one neurons at the input, hidden and output layers, respectively. The first-layer weights V_o are selected randomly [10], [16]. They are uniformly randomly distributed between –1 and +1. These weights represent the stiffness of the sigmoid activation function. The threshold weights for the first layer v_0 are uniformly randomly distributed between -20 and +20. The second layer weights W_o are initialized to zero.

The NN compensator with sigmoidal activation function has four, fifteen and one neurons at the input, hidden and output layers, respectively. The first-layer weights V_c are selected randomly. They are uniformly randomly distributed between –1 and +1. The threshold weights for the first layer v_c are uniformly randomly distributed between -20 and +20. The second layer weights W_c are initialized to zero. The outer tracking PD controller will make the whole system stable subject to the saturated constraints before the NN saturation compensators start learning and control.

Tracking loop controller parameters are chosen as $K_v = 4$, $K = [2, 1]^T$. The initial conditions $[x_1(0), x_2(0)]^T$ are set to $[0, 0]^T$; the initial conditions $[\hat{x}_1(0), \hat{x}_2(0)]^T$ are $[0.05, 0.05]^T$, and desired trajectory is given by $x_1(t) = \sin(t), x_2(t) = \cos(t)$.

The NN observer weights tuning parameters are chosen as

$$h = 0.002, \quad \Gamma = 2 \quad . \quad (6.3)$$

The NN saturation compensator weights tuning parameters are chosen as

$$k = 0.0001, \quad S = 5 \quad . \quad (6.4)$$

The performance of the NN observer is shown in Figure 2.

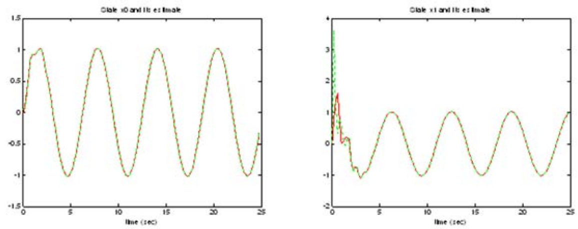

Figure 2. Performance of NN observer.

The position tracking errors and control signal are shown in the Figure 3 and Figure 4 with and without the saturation compensator.

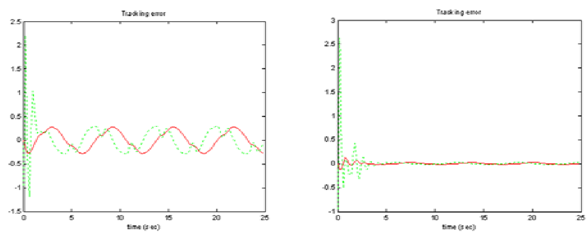

Figure 3. Tracking errors e_1 (solid) and e_2 (dotted) without NN saturation compensator (left) and with saturation compensator (right).

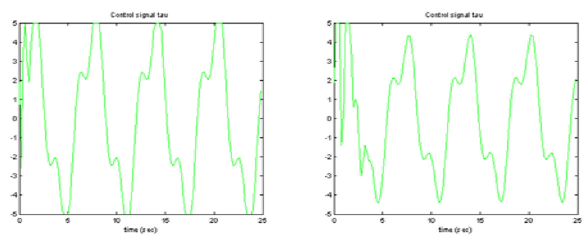

Figure 4. Control signal τ without NN saturation compensator (left) and with saturation compensator.

VII. CONCLUSIONS

In this paper, an observer based NN saturation compensator for a class of nonlinear systems with unknown nonlinearity has been presented. Further research will be conducted on the more challenging MIMO systems with actuator saturation nonlinearity.

VIII. REFERENCES

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