Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Optimal Model Simplification of Quasirational Distributed Systems and their Use for Rapid Design of Simple Controllers

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Abstract

The optimal model simplification technique which was originally proposed for the simplification of systems having simple delays in the numerator transfer functions has been extended to systems having several numerator delays such as quasirational and related systems. The design of simple feedback controllers for such systems has engaged the attention of researchers for sometime. In this work, it is shown that by using the internal model control method, the optimal simplified models can be used to parametrize simple feedback controllers for these systems. This procedure facilitates the deployment of a very effective and powerful controller design technique for quasirational and related systems.

Keywords

Quasirational distributed systems, time delay systems, optimal model simplification, internal model control, feedback controller design

1. Introduction

The approximation of complex models by simple ones is commonplace in control systems analysis and design [1]. Approximation of high order systems by low order ones with time delay is often used in process control [2,3]. One application of these simple models is in the implementation of internal model and Smith predictor control schemes [4]. Appropriate choice of the time delay in the simplified model can also ensure that both the complex and simplified models possess approximately equal amount of phase lags at critical frequencies.

It is sometimes also necessary to accurately approximate a complex model with or without time delay by a simpler model having no time delay, especially for the purpose of analysis and control system design using state variable methods. This paper considers the optimal approximation of complex models with several delay terms in the numerator transfer function by either simplified models that are delay free or those having a single delay in the numerator transfer function. This is an extension of earlier work [5] which dealt with systems with a single delay. The results of this work facilitate the analytical evaluation of the weighted error squared integral for systems having several delay terms in their numerator transfer functions. Application of the method in this paper not only produces optimal simplified models but also ensures that existing process flowsheeting and design programs, which are incapable of handling models possessing several delays in the numerator, can be used.

Simple effective control of quasirational distributed systems (QRDS) has engaged the interest of researchers for some time [6,7].These systems are encountered in processes modelled by hyperbolic partial differential equations. The QRDS transfer function represents a large class of processes such as tubular heat exchangers, particle size distribution in fluidized bed calciners and crystallizers. Such systems may have an infinity of right half-plane zeros which precludes their use for deriving simple feedback control schemes. Furthermore both their time and frequency responses can be very peculiar, thus precluding simple analysis [2,3,8] that are valid for systems without multiple numerator delays. Previously [6,7], both the Smith predictor and the generalized Smith predictor structures had been proposed for the control of these systems. In this work it is shown that the application of optimal model simplification facilitates the determination of simpler and more effective feedback controllers for such systems through internal model controller parametrization. The optimal simplification method always produces stable and accurate reduced models of desired order and structure unlike other methods such as moment matching which may yield unstable or relatively inaccurate simplified models.

2. Description of the Problem

2.1 Computation of Optimal Simplified Model

Consider a stable linear time-invariant system with time delay whose input-output description is given by

$$Y(s) = G(s)U(s) \tag{1}$$

where U(s), Y(s) and G(s) are respectively the Laplace transforms of the input variable u(t), the output response y(t) and the impulse response g(t). The objective is to compute a stable simplified delayed or delay-free model $G_r(s)$ given by

$$Y_r(s) = G_r(s)U(s) \tag{2}$$

such that for a suitable input, the integral of the weighted squared error between the outpout time responses of the original and simplified models

$$J = \int_0^\infty h(t)(y(t) - y_r(t))^2 dt \tag{3}$$

is minimized. The weight h(t) which is a suitable function of time bestows desirable properties on the response $y_r(t)$. For example a weight which is a polynomial in t penalizes the error at long times and usually forces $y_r(t)$ to approach y(t) speedily. The weight specified in this work is of the form

$$h(t) = t^k \tag{4}$$

where k is a positive integer. J is known as the integral of the squared error, ISE, whenever k is zero. For the purposes of this work

$$U(s) = \frac{u_0}{s} + u_1 + \frac{u_2}{u_3 s + 1} \tag{5}$$

where u_0, u_1, u_2 and u_3 are constants. The problem then is to compute $G_r(s)$ such that (3) is minimized subject to the constraint that all the poles p_i of $G_r(s)$ lie in the open left half plane,

$$Re(p_i) < 0 \tag{6}$$

with zero steady state error in the output responses, that is,

$$e(t) = y(t) - y_r(t) = 0 \text{ for } t \to \infty \tag{7}$$

where $Re(p_i)$ denotes the real part of the complex number p_i . The steady state constraint (7) implies

$$G(0) = G_r(0) \tag{8}$$

Whenever $u_0 \neq 0$ in (5) thereby ensuring that (3) is finite. This is therefore a parameter optimization problem which can be efficiently solved by a suitable optimization procedure provided that the stability constraint (6) is satisfied during the course of the optimization and (3) can be cheaply computed. This is done by using Parseval's identiy

$$J = \int_0^\infty t^k e^2(t)dt = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} F(s)E(-s)ds \tag{9}$$

where $i = \sqrt{-1}$ and

$$F(s) = (-1)^k \frac{d^k E(s)}{ds^k} \tag{10}$$

J cannot be directly evaluated using recursive algorithms [9], but the simple reformulation explained below solves this problem.

2.2 Computation of the Integral of Time-Weighted Squared Error

The complex model is given by the transfer function of the form

$$G(s) = \frac{P_1(s)e^{-d_1 s} + P_2(s)e^{-d_2 s} + P_3 e^{-d_3 s}}{Q(s)} \quad (11)$$

where the $P_i(s)$ are polynomials in s of finite degree and of real coefficients, while the simplified model has the form

$$G_r(s) = \frac{N(s)e^{-\tau s}}{D(s)} = \frac{(c_{m-1}s^{m-1} + \ldots c_0)e^{-\tau s}}{s^m + d_{m-1}s^{m-1} + \ldots d_0} \quad (12)$$

where N(s) and D(s) are polynomials of finite degrees with real coefficients. Both G(s) and $G_r(s)$ are assumed strictly proper. The error E(s) can be decomposed into the steady state and transient components

$$E(s) = \left(\frac{m_1}{s} + z_1(s)\right)e^{-d_1 s} + \left(\frac{m_2}{s} + z_2(s)\right)e^{-d_2 s} + \left(\frac{m_3}{s} + z_3(s)\right)e^{-d_3 s} - \left(\frac{m_r}{s} + z_r(s)\right)e^{-d_r s} \quad (13)$$

where $m_i = u_0 P_i(0)/Q(0)$, $i = 1,2,3$ and $m_r = G(0)u_0 = m_1+m_2+m_3$

In an earlier work, Taiwo [5] has shown how J in (9) may be evaluated when E(s) contains two terms. The same procedure can be applied to (13) which now contains four terms, without loss of generality. This step facilitates the expression of J as a function of the reduced model unknown parameters. The minimization of J then facilitates the computation of the optimal $G_r(s)$.

2.3 The Design of Simple Feedback Controllers

The internal model control method [4] is used to parametrize the simple feedback controllers obtained in this work. By assuming a step input and using the optimal reduced model as the plant model, the feedback controller c is given by

$$c = \frac{q}{1 - G_r q} \quad (14)$$

where q is the (strictly) proper IMC controller which has been augmented by the IMC filter of the form

$$f = \frac{1}{(\lambda s + 1)^n} \quad (15)$$

3. Illustrative Examples

Example 1

This is taken from Jerome and Ray [7], with

$$G(s) = \frac{2(2s+1)e^{-2s} - (2s+1)}{3s+1} \quad (16)$$

This system has inverse response characteristic and the response takes off immediately. During the computation of the optimal simplified model, τ in (12) is specified as zero and m is specified as 2. The ISE for this model is 0.3939. The parameters of the computed optimal approximant are given by $[d_1, d_0, c_1]$ = [2.2393,1.2891,-3.567]. Here G(0) = 1 hence $c_0 = d_0$. Upon using IMC parametrization, and choosing the filter time constant as 2, the feedback controller is given by

$$c(s) = \frac{0.231}{0.735s+1}\left(1 + \frac{1}{1.737s} + 0.4466s\right) \quad (17)$$

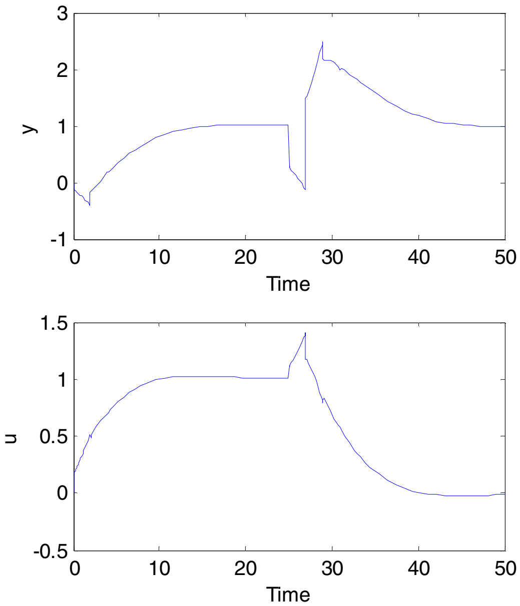

Fig. 1. Closed-loop step responses of (16) with controller (17) to simultaneous reference and disturbance step inputs, where a unit step reference change is applied at time zero and a unit step load disturbance going through the process transfer function is introduced at time 25s.

The satisfactory responses of the system output y and the controller output u are displayed in Fig. 1. Note that since the simplified model has a right half plane zero, it was necessary to factorize it as $G_r=G_{r+}G_{r-}$ where G_{r+} contains the right half plane zero and is either all pass or at least unity at zero frequency. It is when Gr_+ is all pass that the controller (17) is obtained. In fact it was impossible to obtain a stable closed loop system when the alternative factorization and a first order filter are used to parametrize the feedback controller. It is also important to note here that the second order Pade approximant for (16) is unstable. The closed loop in [7] gives jerky and staircase-like step responses.

Example 2.
This is one of the examples used by Jerome and Ray [7]. The transfer function is given by

$$G(s) = \frac{(4s+2)e^{-8s} - (s+1)e^{-6s}}{(s+1)(2s+1)} \qquad (18)$$

The second order with time delay optimal simplified model computed for this complex model was relatively accurate with an ISE of 0.06157. A first order plus time delay optimal simplified model with an ISE of 0.38948 was also computed. The parameters of this model are $c_0 = d_0 = 1.3047$ and $\tau = 8.4657$. This model will be used to parametrize the feedback controller in view of its simplicity and relative accuracy. It is worth noting here that no first order plus time delay model having real coefficients could be obtained for this complex model using moment matching. However a second order plus time delay model with an ISE of 0.2149 was computed. From the ISE values it is noted that this simplified model is much less accurate than the optimal one of the same order

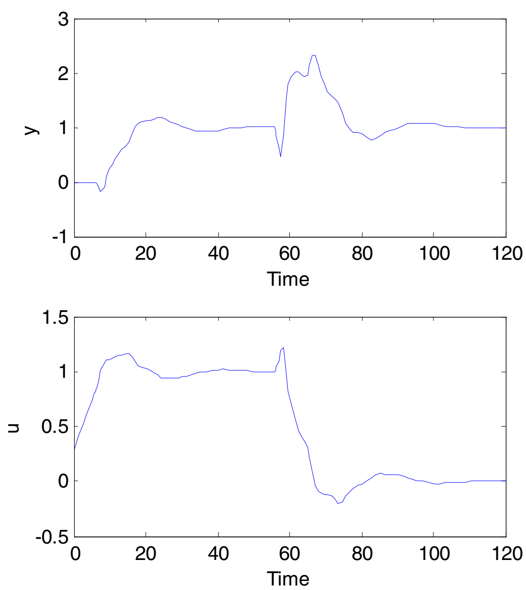

Fig. 2. Closed-loop step response of (18) with controller (19) to simultaneous reference and disturbance step inputs, where a unit step reference change is applied at time zero and a unit step load disturbance going through the process transfer function is introduced at time 50s.

One of the objectives of this work is to use IMC to parametrize simple controllers from the optimal simplified models. Hence it is expedient to approximate the delay in the first order simplified model by a 1/1 Pade approximant before proceeding with the controller parametrization. The controller obtained in this way (we used the same method of factorization for the non-minimum phase transfer function as described in example 1) is given by

$$c(s) = \frac{0.418}{1.238s+1}(1+\frac{1}{5s}+0.721s) \qquad (19)$$

The closed loop response of the system with this controller is given in Fig. 2. This corresponds to a filter time constant in q of 3.5. A second order delay free simplified model was also computed for this system. This can also be used for controller parametrization. The first order plus time delay model has, however, been used here as it is relatively more accurate.

Example 3.
The process considered here is a packed tubular reactor as described elsewhere [7]. The transfer function of this process is given by

$$G(s) = (-0.25894)\frac{(-0.07266s+1)e^{-127.7s}}{(348.11s^2+26.7s+1)} + 2.74207e^{-176s} \qquad (20)$$

As the time delay dominates here, it is expedient to seek a simplified model with time delay. Both the first order plus time delay and the second order plus time delay optimal simplified models give about the same ISE (0.9094). Hence the first order plus time delay simplified model will be used to parametrize the feedback controller. Again a 1/1 Pade approximant is used to represent the delay of the reduced model which is given by

$$G_r(s) = \frac{124.1515e^{-176s}}{s+50} \qquad (21)$$

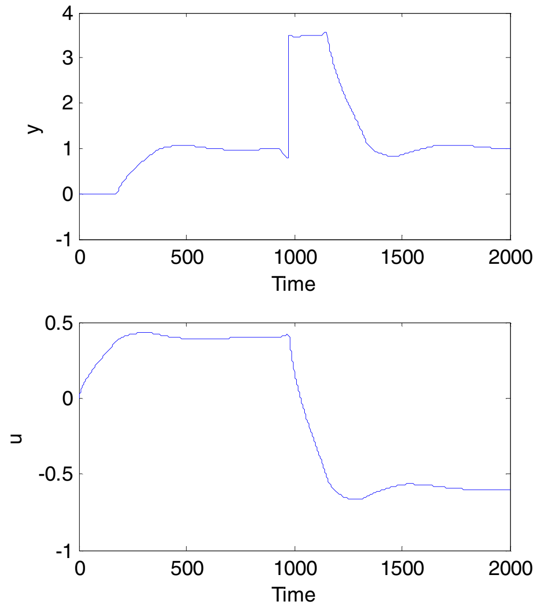

Fig. 3. Closed-loop step response of (20) with controller (22) to simultaneous reference and disturbance step inputs, where a unit step reference change is applied at time zero and a unit step load disturbance going through the process transfer function is introduced at time 800s.

The simple feedback controller obtained here is given by

$$c(s) = \frac{0.1385}{27.5s+1}(1+\frac{1}{88s}+0.02s) \qquad (22)$$

. This corresponds to choosing a filter time constant of 80 for q. The closed loop step responses of the system with this controller are shown in Fig. 3. The moment matching method did not yield a first order plus time delay simplified model with real coefficients for this plant. However the second order simplified model has a rather large ISE of 24.146.

4. Conclusions

This work has proposed a direct and simple method for parametrizing feedback controllers for systems having many delay terms in the numerator transfer functions such as quasirational distributed systems. The key step in the proposed procedure is to get an accurate model of the process which can be used for feed back controller parametrization using the IMC method. For this purpose it was necessary to extend the original method of Taiwo [5] which hitherto could only be used for the reduction of systems with a single delay in the numerator. It is also important to note the speed which this procedure offers for feedback controller design such that the closed loop system satisfies such additional requirements as robustness, since this requires only an appropriate choice of the IMC filter. In general, the closed loop responses given here are favourable to earlier ones [7] obtained for these systems. What is even more important is that the controllers are simpler, typically involving a PID structure with a first order filter. The closed loop systems have the desired servo and regulator characteristics as depicted by the simulated responses to simultaneous step changes in reference and disturbance inputs. It is a pleasant surprise that even for these complex models, a delay free second order or a first order plus time delay optimal simplified model led to the parametrization of very simple but effective feed back controllers. Higher order optimal simplified models were also computed for all the examples, and these could be used for feedback controller parametrization whenever more stringent performance or constraints are specified for the closed loop systems.

Acknowledgement

This work was done while the first author was a visiting professor to the Institute of Petroleum Engineering, Technische Universität Clausthal and sponsored by the Alexander von Humboldt Foundation.

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