Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

VSC of a servo-actuated ATR90-type pantograph

Benedetto Allotta, Alessandro Pisano, Luca Pugi, Elio Usai, *Member, IEEE*

***Abstract*— One of the problems in high-speed-train transportation systems is related to the current collection quality, that can dramatically decrease because of oscillations of the pantograph-catenary system. In the recent literature this problem has been addressed by means of active pantographs. In order to reduce the costs, a common requirement of the train companies is that the control system could be applied to the actually used pantographs. In this paper we present the preliminary results about the possible implementation of VSC techniques on a servo-actuated symmetric pantograph that can be obtained by modifying a passive high-speed pantograph Ansaldo ATR90, currently used by Italian Railways. We consider the equivalent mechanical characteristics of the catenary as uncertainties to compensate for, and the use of a robust, nonlinear, control scheme is proposed. Recent results about the influence of fast actuators in second-order sliding mode control schemes are exploited to avoid the destroying effect of a resonant actuator and the system performance are verified by simulation, under the hypothesis of knowing the actual contact force. The contact force results to be very close to the desired value in various operating conditions.**

INTRODUCTION

ONE of the main problem in high-speed train transportation systems is related to the interaction between the train pantograph and the overhead equipment, the catenary.

During the train run, the pantograph deforms the catenary and oscillatory motions are induced. As the train speed increases, these oscillations can become larger and larger and the loss of contact between the pantograph head and the collector wire can occur.

Because of its economic relevance, the complex behavior of the catenary/pantograph system gained the attention of many researchers and engineers since several decades [1]-[5]. These studies pointed out that satisfactory performances, at high speed, can be achieved by means of active pantographs, that require relatively small investments.

In particular, it has been shown that better performance can be achieved if approximate models of the catenary/pantograph system are considered to design or tune the controllers [2],[6],[7]. Most authors represented the pantograph as a linear system [6],[8],[9]; furthermore, some authors assumed that the complex dynamics of the catenary could be well approximated by a linear mechanical system with space-varying lumped parameters and used such an approximation in the controller design [7].

Actually, there is a large uncertainty in almost all the proposed models; therefore, the use of robust control techniques was proposed [9]-[11]. We consider the equivalent model of the catenary as an uncertainty to compensate for by means of Variable Structure Control (VSC) systems with Sliding Modes (SM) [11], which are recognized as an effective and simple control approach to uncertain control systems [12].

We showed that the robust output-feedback approach in [11], which implies the design of a high-order observer/controller, presents hard practical implementation problems for the considered pantograph, obtained by modifying a passive high-speed pantograph Ansaldo ATR90. In fact the actuator can be modeled as a second order system with a small damping, so that the implementation of the output-feedback control scheme in [11] requires the estimation of the second derivative of the available output, with very large values both of itself and of its Lipshitz constant, and very large values of the control gain. Therefore, to achieve acceptable real sliding behaviors a very small sampling time is required.

In order to avoid the need of any differentiator, recent results about the influence of fast actuators in second-order sliding mode control (2-SMC) schemes [13]-[16] are exploited in the design of the controller. In particular, a generalized formulation of the “sub-optimal” VSC (2-VSC) [17], [18] and a proper filter are used to attenuate the undesired peaking of the contact force and to shape the residual oscillations that appear during the approximate sliding regime that is established.

Simulations showed the effectiveness of the proposed output-feedback control scheme in different operating conditions.

I. A MODEL OF THE PANTOGRAPH-CATENARY SYSTEM

An exact model of the overhead suspension system is very difficult to define because it is a distributed system, and its parameters are very hard to evaluate exactly. For this reason, simplified models with lumped time-varying parameters have been introduced [7]-[9]. They are based on a the local, in space, characterization of the equivalent mechanical parameters (i.e., mass, stiffness, and dumping coefficient) of the contact wire.

These parameters present a periodic behavior varying along each span, with smaller variations due to the droppers. This allows one to represent the equivalent parameters by a combination of harmonic functions [8]

Manuscript received March 5, 2005. This work was supported in part by the Italian Minister for Education, University and Research (MIUR) under the project PRIN BS123456.

E. Usai and A.Pisano are with the Dipartimento di Ingegneria Elettrica ed Elettronica, Università di Cagliari, 09123 Cagliari, Italy (E. Usai corresponding author, phone: +39-070-675-5784; fax: +39-070-675-5782; e-mail: eusai@diee.unica.it, pisano@diee.unica.it).

B. Allotta and L. Pugi are with the Dipartimento di Energetica “S. Stecco”, Università di Firenze, Italy.

0-7803-9568-9/05/$20.00 ©2005 IEEE

$$
\begin{aligned}
m_c(x) &= m_{c_0} + m_{c_1}\cos\left(\tfrac{2\pi}{L}x\right) + m_{c_2}\cos\left(\tfrac{4\pi}{L}x\right),\\
b_c(x) &= b_{c_0} + b_{c_1}\cos\left(\tfrac{2\pi}{L}x\right) + b_{c_2}\cos\left(\tfrac{4\pi}{L}x\right),\\
k_c(x) &= k_{c_0} + k_{c_1}\cos\left(\tfrac{2\pi}{L}x\right) + k_{c_2}\cos\left(\tfrac{4\pi}{L}x\right),
\end{aligned}
\tag{1}
$$

where L is the span length and x the distance from a tower.

The contact between the supply wire and the contact shoe is modeled with a lumped high stiffness and a low damping, and a nonlinear saturation constraint can be introduced to simulate the loss of contact. The symmetric pantograph can be well modeled by means of a linear lumped-parameters system affected by the input forces (i.e., the constant force applied to the lower frame by the pneumatic positioning system of the pantograph, and the control forces acting on the lower or on the upper frame) and by the constraint forces (i.e., the contact force) [8], [9].

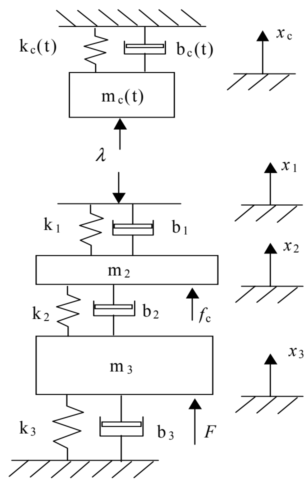

Fig. 1. A lumped-parameters model of the pantograph-catenary system.

The dynamics of the system in Fig. 1 can be represented by a set of linear dynamic systems possibly coupled by means of the contact force constraint , which assumes only positive values when x_1=x_c (i.e., the pantograph shoe is in contact with the supply wire)

$$
m_c x_c + b_c x_c + k_c x_c = \lambda \tag{2}
$$

$$
\begin{cases}
x_1 = x_c & \text{if } \lambda > 0\\
k_1(x_1 - x_2) + b_1(x_1 - x_2) = 0 & \text{if } \lambda = 0\\
m_2 x_2 + b_2 x_2 + k_2 x_2 = k_2 x_3 + b_2 x_3 + f_c - \lambda & \\
m_3 x_3 + (b_2 + b_3)x_3 + (k_2 + k_3)x_3 = k_2 x_2 + b_2 x_2 + F &
\end{cases}
\tag{3}
$$

$$
\lambda = \max\{k_1(x_2 - x_c) + b_1(x_2 - x_c);0\} \tag{4}
$$

During operation (i.e., when the contact force is positive), the dynamics of the pantograph-catenary system can be represented as a linear time-varying system derived by combining equations (2)-(4), with the actual position of the train along the span evaluated by integration of the train speed $v(t)$, i.e., $x(t) = \int_0^t v(\tau)d\tau$

$$
\begin{aligned}
\mathbf{z}(t) &= \mathbf{A}(t)\mathbf{z}(t) + \mathbf{B}\mathbf{f}(t)\\
y(t) &= \mathbf{C}\mathbf{z}(t)
\end{aligned}
, \tag{5}
$$

where $\mathbf{z} = [x_1, x_1, x_2, x_2, x_3, x_3]^T$, $x_1 \equiv x_c$, $x_1 \equiv x_c$, is the state vector, $\mathbf{f}$=$[f_c,F]^T$ is the control vector, $\mathbf{A}$ is a time-varying matrix, $\mathbf{B}$ and $\mathbf{C}$ are constant matrix and vectors with proper dimensions, and y is the contact force, i.e.,

$$
\mathbf{A}(t) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0\\
-\frac{k_c(t)+k_1}{m_c(t)} & -\frac{b_c(t)+b_1}{m_c(t)} & \frac{k_1}{m_c(t)} & \frac{b_1}{m_c(t)} & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0\\
\frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{b_1+b_2}{m_2} & \frac{k_2}{m_2} & \frac{b_2}{m_2}\\
0 & 0 & 0 & 0 & 0 & 1\\
0 & 0 & \frac{k_2}{m_3} & \frac{b_2}{m_3} & -\frac{k_2+k_3}{m_3} & -\frac{b_2+b_3}{m_3}
\end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix}
0 & 0\\
0 & 0\\
0 & 0\\
\frac{1}{m_2} & 0\\
0 & 0\\
0 & \frac{1}{m_3}
\end{bmatrix}
\qquad
\mathbf{C} = \begin{bmatrix} -k_1 & -b_1 & k_1 & b_1 & 0 & 0 \end{bmatrix}
$$

A standard pneumatic actuator has been used to apply the needed pre-load force, F, on the pantograph frame. This actuator is designed to apply a constant force that compensates for the gravitation forces and impose the desired constant contact force in the worst case condition. It can be then assumed that $F = const$.

The contact force is regulated by means of a wire actuator acting on the sliding bows. The actuator dynamics can be represented by a linear second order system with a small damping and a large resonance peak at about 500 rad/s; therefore

$$
f_c + 2\zeta\omega_n f_c + \omega_n^2 f_c = \omega_n^2 K_a u \tag{6}
$$

where u is the actuator command.

Taking into account previous considerations about the forces acting on the pantograph frame, system (5)-(6) can be reduced into the following normal form [12]

$$
\begin{aligned}
y(t) &= \varphi(\mathbf{y}(t), \mathbf{w}(t), t) + \gamma \cdot u(t),\\
\mathbf{w}(t) &= \boldsymbol{\psi}(\mathbf{y}(t), \mathbf{w}(t), t),
\end{aligned}
\tag{7}
$$

with γ=$K_a b_1 \omega_n^2/m_2$, and

$$
\boldsymbol{\psi} = \begin{bmatrix}
0 & 1 & 0 & 0 & 0\\
-\frac{k_c}{m_c} & -\frac{b_c}{m_c} & 0 & 0 & 0\\
\frac{k_1}{b_1} & 1 & -\frac{k_1}{m_1} & 0 & 0\\
0 & 0 & 0 & 0 & 1\\
\frac{b_2 k_1}{b_1 m_3} & \frac{b_2}{m_3} & \frac{b_1 k_2 - b_2 k_1}{b_1 m_3} & -\frac{k_2+k_3}{m_3} & -\frac{b_2+b_3}{m_3}
\end{bmatrix}\mathbf{w} + \begin{bmatrix} 0\\ 0\\ 0\\ 0\\ \frac{1}{m_3} \end{bmatrix} F + \begin{bmatrix} 0\\ \frac{1}{m_c}\\ \frac{1}{b_1}\\ 0\\ \frac{b_2}{b_1 m_3} \end{bmatrix} y
$$

$\mathbf{y}$ represents the phase output vector, whose elements are the contact force (i.e., the system output y) and its first and second time derivatives, and $\mathbf{w} \in R^5$ is the internal state vector.

The state variables of the overall system (5)-(6), $[\mathbf{z}^T, f_c, f_c]^T$, are related with those of the normal form representation (7), $[\mathbf{w}^T, y, y, y]^T$, by the following affine relationship, which is invertible if $b_1 \neq 0$:

$$
\begin{bmatrix} \mathbf{w}\\ \mathbf{y} \end{bmatrix} = \left[\begin{array}{c|cc}
\mathbf{P} & 0 & 0\\
\hline
\mathbf{C} & 0 & 0\\
\mathbf{CA} & \mathbf{CB}_1 & 0\\
\mathbf{C}(\mathbf{A}+\mathbf{A}^2) & \mathbf{CAB}_1 & \mathbf{CB}_1
\end{array}\right]
\begin{bmatrix} \mathbf{z}\\ f_c\\ f_c \end{bmatrix} + \left[\begin{array}{c}
\mathbf{0}\\
\hline
0\\
\mathbf{CB}_2\\
\mathbf{CAB}_2
\end{array}\right] F
\tag{8}
$$

with

$$\mathbf{P} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_2 \end{bmatrix} \qquad \mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2]$$

The last two rows of (8) point out that the relative degree of system (5) depends on the actual values of the damping effects, characterized by the values of parameters b_1 and b_2. In fact it results $\mathbf{CB}_1 = b_1/m_2$, $\mathbf{CB}_2 = 0$, $\mathbf{CAB}_2 = b_1 b_2/(m_3 m_2)$, $\mathbf{CAB}_1 = k_1/m_2 - b_1\,(b_1/m_c + (b_1+b_2)/\,m_2)/\,m_2$. It is clear that a small damping effect corresponds to a small control gain, and therefore strong control efforts are usually needed. Disregarding the damping effects would cause the relative degree of the system to increase, so leading to a different control structure whose effectiveness on the real system should be carefully analyzed.

Taking into account that the input-output relationship in (7) is derived by substitution of (8) into the following

$$\begin{aligned} y = {} & \mathbf{C}(\mathbf{A} + 2\mathbf{AA} + \mathbf{AA} + \mathbf{A}^3)\mathbf{z} + \mathbf{C}(2\mathbf{A} + \mathbf{A}^2)\mathbf{B}_2 F + \\ & \mathbf{C}(2\mathbf{A} + \mathbf{A}^2 - \omega_n^2 \mathbf{I}_6)\mathbf{B}_1 f_c + \mathbf{C}(\mathbf{A} - 2\xi\omega_n \mathbf{I}_6)\mathbf{B}_1 f_c + \\ & K_a \omega_n^2 \mathbf{CB}_1 u \end{aligned} \tag{9}$$

and from the explicit form of $\mathbf{\Psi}$, it results that the vector field $\mathbf{\Psi}$ and the function φ in (7) are time-varying since they are affected by the catenary equivalent parameters, m_c, b_c and k_c, in eq. (1). These time-varying parameters never vanish, and they can be assumed to be sufficiently smooth function on time.

By eq. (7) it results that the internal dynamics of the approximate linear model of the catenary/pantograph system is BIBO stable. Therefore, on the basis of the previous considerations about the time-varying terms, it can be claimed that all the functions and variables in eq. (7) are bounded, i.e., the following inequalities are fulfilled

$$\begin{aligned} & |\varphi(\boldsymbol{y}, \boldsymbol{w}, t)| \le \Phi(\|\boldsymbol{y}\|, \|\boldsymbol{w}\|), \\ & \left\| \begin{bmatrix} z(t) \\ f_c(t) \\ f_c(t) \end{bmatrix} \right\| \le \Omega(\|\boldsymbol{y}\|, \|\boldsymbol{w}\|) = P\|\boldsymbol{w}(t)\| + Q \sup_{(t_0,\infty)} \|\boldsymbol{y}(t)\|, \end{aligned} \tag{10}$$

where Φ is a known, positive definite, convex function, P and Q are positive known constants and t_0 is the initial time instant.

II. A Robust Output-Feedback Control System

The required task for the control system is to solve a regulation problem, i.e., to maintain the actual contact force to a desired constant value y_{ref}. Consider the output error $e(t) = y(t) - y_{\text{ref}}$, it is trivial to show that the error dynamics, in the normal form, is represented by the same set of differential equations than the controlled system, provided that the substitutions $y(t) = e(t) + y_{\text{ref}}$, $y^{(i)}(t) = e^{(i)}(t)$ (i=1,2,...) are performed. The regulation problem of the contact force between the pantograph head and the contact wire is, therefore, reduced to an output-feedback stabilization problem for an uncertain, possibly nonlinear, system.

Recently some robust output-feedback control approaches based on the use of variable structure controllers were presented in the literature [19]-[21]. The common aim is to steer to zero, possibly in a finite time, a proper linear combination of the output error and its time derivatives (the sliding variable). Once the sliding variable is constrained to zero, the error system performs as an autonomous, stable, linear system of reduced-order. The main differences are in how the error derivatives are estimated.

Oh and Khalil [19] solved the problem of estimating the needed time derivatives of the error signal by a High-Gain Observer (HGO), and used a classical VSC [12] as the controller. Differently, Bartolini *et al.* [20] and Levant [21] suggested the use of Higher-Order Sliding Modes (HOSM) to avoid both some differentiation step and the destroying propagation of the measurement noise during numerical differentiation.

In fact, HOSM are characterized by steering to zero the sliding variable and its first *h-1* derivatives, *h* being the sliding order [22], [23]. This implies that the sliding variable σ could be defined as a proper linear combination of the error signal and its derivatives, up to the $(r-h)^{\text{th}}$-order, r being the relative degree [12] between the system output, i.e., the sliding variable σ and the discontinuous control.

The direct application of the latter approach, with the use of a 2-VSC with chattering avoidance features [22]-[25], to the considered system would imply the definition of the sliding surface as follows:

$$c = e + c_1 e + c_0 e, \tag{11}$$

with c_k (k=0,*1*) coefficients of a Hurwitz polynomial.

Equation (11) requires the estimation of the first and second derivative of the available signal $e = y - y_{\text{ref}}$. They are fast varying during the transient, when the contact force y is not yet regulated, and, therefore, the Lipschitz constant of e is very large, as can be evaluated by (9). Furthermore, the second order sliding dynamics, which derives from differentiation of (11) taking into account (7)-(9), can be described by the following relationship

$$c(t) = \varphi_4(z, f_c, f_c, F, u) + \gamma u \tag{12}$$

that has the drift term φ_4, whose explicit form is not reported for the sake of conciseness, depending, among the others, on terms like $\mathbf{Az}$ and $\mathbf{A}^4\mathbf{z}$. These fact makes the bound of φ_4 very large.

The above considerations imply that huge values of both the differentiator and controller gains should be used to implement an output-feedback VSC scheme as in [11].

Even if , theoretically, the output-feedback VSC approach can guarantee the asymptotic regulation of the contact force, the practical application of the described approach in this case could be not effective. In fact, the theoretic performance comes from the assumption that the discontinuous control u switches at infinite frequency. This is not possible either because the controller is digitally implemented or because some hysteretic relay is used to implement the VSC. In these cases the invariant set of the closed-loop system is a boundary layer of the sliding surface, i.e., after a finite time transient it results

$$|\sigma| \le K_0 \tau^2, \qquad |\sigma| \le K_1 \tau, \qquad \forall t \ge T \tag{13}$$

where τ and T are the equivalent switching delay and the reaching time respectively [22,26].

The huge values of the constants K_0 and K_1, due both to the resonance peak of the actuator and to the parameters of the pantograph/catenary system, require almost unrealistic values of τ to achieve the desired accuracy.

A. An approximate sliding mode controller

The control scheme we are going to present is based on the hypothesis that the presence of fast actuators, or sensors, would result in a singular perturbation of the ideal sliding mode [13]-[16]. Therefore, we can design the controller disregarding the presence of the actuator, and implementing a chattering avoidance scheme.

In fact, such a scheme can be applied to relative degree one systems, as in our case, and it needs a 2-SMCs with a cascade first order filter (often a simple integrator), so that the relative degree between the sliding variable and the discontinuous control is two. Then the normal form for the system without the actuator would be the following

$$\begin{aligned} \dot{y}(t) &= \varphi_1(y(t),\mathbf{w}(t),t) + \gamma_1 f_c(t), \\ \dot{\mathbf{w}}(t) &= \boldsymbol{\psi}(y(t),\mathbf{w}(t),t), \end{aligned} \tag{14}$$

where the relationship between vector $\mathbf{z}$ and $[\mathbf{w}^T, y]^T$ comes from (8) not considering the last two rows, and the input-output first order dynamics derives from the following

$$\dot{y} = \mathbf{CAz} + \mathbf{CB}_1 f_c + \mathbf{CB}_2 F \tag{15}$$

with $\mathbf{CB}_1 = b_1/m_2$ and $\mathbf{CB}_2 = 0$.

In [15] it was shown that the use of the “Generalized Sub-Optimal” 2-SMC [17], [18] with unmodeled linear fast actuators of any order guarantees the reaching of a boundary layer like that in (13), τ being the fast actuator’s time constant. Furthermore, the parameters of such a controller can be also tuned to counteract the peaking phenomenon [18], and to shape the frequency and magnitude of the system oscillations within the boundary layer (13) [16].

The above results, and the considerations in Section II about the bounds of the system dynamics, suggest the use of the “Generalized Sub-Optimal” 2-VSC with constant parameters guaranteeing the monotonic behavior of the sliding variable as the controller, i.e., [17]

$$\begin{aligned} &\dot{f}_c = -V\,\mathrm{sgn}(e - \beta e_M) \\ &V > \frac{\overline{\Phi}}{\gamma(2\beta - 1)} \qquad\qquad (e,\dot{e}) \in \Omega \\ &\beta \in (0.5,1] \end{aligned} \tag{16}$$

where e_M is the most recent extremal point of the error variable $e(t)=y(t)-y_{ref}$, whose vanishing defines the sliding surface, and $\overline{\Phi}$ is an upper bound of the uncertain drift term of the second order dynamics of $e(t)$ in the set □, which can be estimated by the last row of (8).

Tuning the 2-VSC parameters by means of the worst case design, as stated by conditions on V in (16) is usually less effective than deriving them by means of preliminary simulations. In fact, the latter usually gives better results since the estimated bounds for the uncertainty are very conservative, and imply a very large control authority. This will result in a larger cost and larger values of constants K_0 and K_1 in (13).

In order to consider the actuator as a singular perturbation of the ideal system, the resonant dynamics of the wire actuator should be compensated by means of a pre-filter. In the considered case the wire actuator has been identified by the harmonic response [9] drawn in Fig.2.

It is evident the large resonant peak, at a high frequency, that could amplify some of the harmonic components generated by the finite frequency switching of the VSC such that the real sliding could be destroyed.

The pre-filter should be designed in such a way that the actuator is a low-pass filter whose bandwidth is large enough to allow the control to compensate the harmonic disturbances due to the variation of the catenary parameters. Taking into account the standard span length ($L = 60$ m) and the usual maximal velocity of high-speed train on Italian railway systems ($v = 350$ km/h), the spectrum of the disturbances to compensate for with the control is within the range 0÷10 rad/s.

Therefore, in the specific case a simple first-order low-pass filter with time constant $\tau_f = 0.1$ s as the pre-filter suffices (Fig.3).

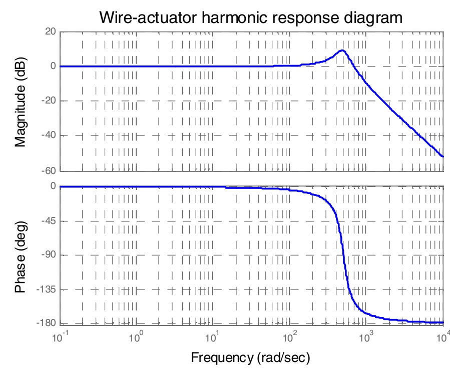

Fig. 2. The wire actuator harmonic response

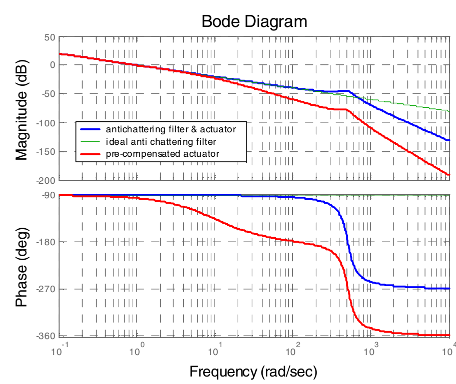

Fig. 3. The pre-compensation design

Figure 3 shows that the pre-filter drastically reduces the influence of the resonant wire actuator (i.e., the compensated frequency response is always under the straight line representing the simple integrator introduced for chattering avoidance purposes) while leaving the harmonic response values almost unchanged within the working frequency range 0÷10 rad/s (i.e., only a very small gain variation and an acceptable phase shift are present in the considered frequency range).

The overall system dynamics is then represented by the set of differential equations (14), (6) with the following

$$0.1u + u = v, \qquad (19)$$
$$v = -V\,\mathrm{sgn}\left(\left(y-\lambda_{ref}\right)-\beta\left(y-\lambda_{ref}\right)_{|M}\right),$$
which represent the nonlinear dynamic controller driving the wire actuator command signal u.

The convergence of the system into the boundary layer (13) is assured by choosing parameter V sufficiently high [17], while the parameter β can be tuned to shape the frequency and the magnitude of the residual oscillations of the contact force, taking into account that higher values of □ imply smaller oscillations at a higher frequency, but a slower transient as well [16].

III. Simulation Results

The effectiveness of the proposed controller has been tested by extensive simulations on the linear model of an ATR90 pantograph for high-speed trains combined with a linear time varying model of the catenary. The system parameters in (5) have been computed, or measured, to be: k_1=100000 Nm^{-1}, m_2=7.6 kg, m_3=9.25 kg, k_2=3421Nm^{-1}, k_3=0 Nm^{-1}, b_1=5000 Nm^{-1}s, b_2=20 Nm^{-1}s, b_3=100 Nm^{-1}s; while the catenary has been modeled according to (1) with m_{c_o}=195 kg, m_{c_1}=100 kg,b_{c_o}=b_{c_1}=240 Nm^{-1}s, k_{c_o}=7000 Nm^{-1}, k_{c_1}=3360 Nm^{-1}, b_{c2}=k_{c2}=m_{c2}=0, L=60 m; and the actuator parameters in (6) have been identified: ω_h=508.0354 rad/s, ζ=0.1772, K_a=1. A pre-load constant force F=140 N was assumed, and the wire strength f_c has been forced to be always not positive (the wire can only be stretched) by a proper saturation block.

Simulations have been carried out by means of MATLAB-Simulink® using the Runge-Kutta integration method with integration step h=10^{-4}.

A first test has been carried out assuming that the actuator is a pure gain. Therefore the system dynamics reduces to (14) and controller (16) has been directly used with parameters V=3000 and β=0.9. The extremal values are estimated with a first difference algorithm with delay 10^{-4}. Without affecting the validity of the proposed control scheme, simulations have been performed assuming a constant acceleration from 0 to 350 km/h, although this is an unrealistic acceleration performance. The contact force is almost perfectly regulated to the desired value □$_{ref}$=100 N *(± 0.04 N)* (Fig. 4).

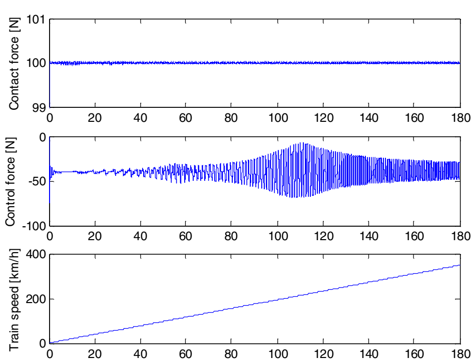

Fig. 4. Test 1, no actuator dynamics

Then the actuator with the pre-filter has been introduced and the simulation was repeated without changing the 2-VSC parameters (Fig. 5). Also in this case the contact force is well regulated to the desired value y_{ref}=100 N *(± 1 N)*, with a small detriment of the performance.

Finally a measurement noise with uniform distribution between ±2 N was introduced (Fig. 6). In all tests the train speed profile was the same as in the lower plot of Fig. 4.

It is evident that the decreasing of the performance is not dramatic; in fact the deviation of the contact force from the desired value is greater than the measurement error only at the main resonance frequency when the train speed is about 200 km/h. The controller tuning, based on the ideal case, is still effective also in perturbed cases showing the robustness of the 2-VSC and of the proposed control scheme.

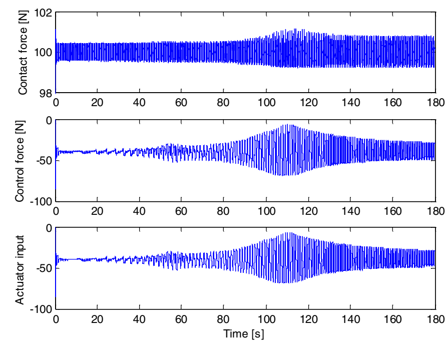

Fig. 5. Test 2, pre-compensated actuator

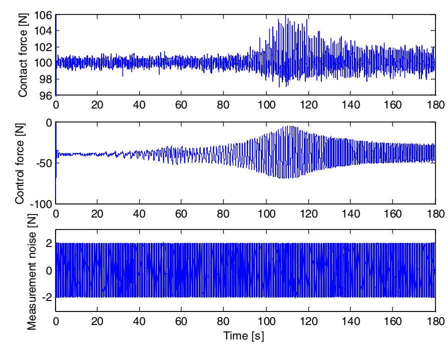

Fig. 6. : Test 3, pre-compensated actuator with noisy measurements

IV. Conclusions

The paper presents the application of a robust control scheme, based on the generalized sub-optimal second-order sliding mode controller, to the regulation problem of the contact force in high-speed railway system. The actuator is pre-compensated by a properly designed filter in cascade with the 2-VSC, so that the overall pre-filter and actuator can be considered as a singular perturbation of the ideal second order sliding mode. The proposed approach results to be simple and effective, and robust to the measurement noise as well.

References

[1] Gostling R.J., Hobbs A.E.W., The Interaction of Pantograph and Overhead Equipment: Practical applications of a New Theoretical Method, Proc Instn Mech Engrs, 197C, pp. 61-69, 1983.

[2] Eppinger S.D., O'Connor N.D., Seering W.P., Wormley D.N., Modeling and Experimental Evaluation of Asymectric Pantograph

Dynamics, J. of Dynamic Systems, Measurement, and Control - Trans. ASME, 110, pp.168-174, 1988.
[3] Galeotti G., Galanti M., Magrini S., Toni P. Servo actuated railway pantograph for high-speed running with constant contact force, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, 207 (1), p 37-49), 1993.
[4] Poetsch G., Evans J., Meisinger R., Kortum W., Baldauf W., Veitl A., Wallaschek J., Pantograph/Catenary Dynamics and Control, Vehicle System Dynamcs, 28, pp. 159-195, 1997.
[5] Diana G., Fossati F., Resta F., High Speed Railway: Collecting Pantographs Active Control and Overhead Lines Diagnostic Solutions, Vehicle System Dynamics, 30, pp. 69-84, 1998.
[6] O'Connor N.D., Eppinger S.D., Seering W.P., Wormley D.N., Active Control of High-Speed Pantograph, J. of Dynamic Systems, Measurement, and Control - Trans. ASME, 119, pp.1-4, 1997.
[7] Wu T.X., Brennan M.J., Active Vibration Control of a Railway Pantograph, Proc. Instn Mech Engrs,. 211 (F), pp. 117-130, 1997.
[8] Balestrino A., Bruno O., Landi A., Sani L., Innovative Solutions for Overhead Catenary-Pantograph Systems: Wire Actuted Control and Observed Contact Force, Vehicle System Dynamics, 33, pp. 69-89, 2000.
[9] Allotta B., Papi M., Pugi L., Toni P., Violi A.G., Experimental campaign on a Servo-Actuated Pantograph, Proc. 2001 IEEE/ASME Int.l Conf. on Advanced Intelligent Mechatronics, vol.1, pp.237-42, 2001.
[10] Makino T., Yoshida K., Seto S., Makino K., Running Test on Current Collector with Contact Force Controller for High-Speed Railways, JSME International Journal - Series C, 40 (4), pp.671-680, 1997.
[11] Pisano A., Usai E., Output-Feedback Regulation of the Contact-Force in High-Speed train Pantographs, ASME Journal of Systems, Measurement, and Control, 126, pp. 82-87, march 2004.
[12] Slotine J.-J.E., Li W., Applied Nonlinear Control, Prentice-Hall International, Englewood Cliffs, New Jersey, 1991.
[13] Fridman L., An averaging approach to chattering, IEEE Trans. on Automatic Control, 46, pp. 1260-1265, 2001.
[14] Boiko I., Castellanos M.I., Fridman L., Analysis of Second Order Sliding Mode Algorithms in the Frequency Domain, Proc. of 42th Conference on Decision and Control, Maui, Hawaii, pp.4575-4580, 2003.
[15] Boiko I, Fridman L, Pisano A, Usai E., "Performance analysis of second-order Sliding-mode control systems with fast actuators", Proc. of the 8th Workshop on Variable Structure Systems (VSS'04), Vilanova i la Geltru, Spain, September 2004
[16] Boiko I, Fridman L, Pisano A, Usai E., "Shaping the amplitude and frequency of chattering in 2-SMC with dynamic actuators", Proc. of the 8th Workshop on Variable Structure Systems (VSS'04), Vilanova i la Geltru, Spain, September 2004
[17] Bartolini G., Pisano A., Punta E., Usai E., A survey of applications of second-order sliding mode control to mechanical systems, International Journal of Control, 76 (9/10), pp. 875-892, 2003
[18] Bartolini G., Pisano A., Usai E., Global Stabilization and Peaking Reduction for Nonlinear Uncertain Systems with Unmodelled Actuactor Dynamics, IEEE Trans. on Automatic Control, 46 (11), pp. 1826-1832, 2001
[19] Oh S., Khalil H.K., Nonlinear Output-Feedback Tracking Using High-gain Observer and Variable Structure Control, Automatica, 33, pp. 1845-1856, 1997.
[20] Bartolini G. Levant A., Pisano A. Usai E., Higher-Order Sliding Modes for the Output-Feedback Control of Nonlinear Uncertain Systems, Variable Structure Systems: Towards the 21st Century, X. Yu and J.-X. Xu LNCIS 274, pp. 83-108, Springer, Berlin, 2002
[21] Levant A. Universal Output-feedback SISO Controllers, Asian Journal of Control, 5 (4), pp. 484-497, 2003
[22] Levant A., Sliding order and sliding accuracy in sliding mode control, International Journal of Control, 58, pp. 1247-1263, 1993.
[23] Emel'yanov S.V., Korovin S.K., Levantosky L.V. (Levant A.), Higher order sliding modes in the binary control systems, Soviet Physics, Doklady, 31, no. 4, pp. 291-293, 1986
[24] Bartolini G., Ferrara A., Levant A., Usai E., On Second Order Sliding Mode Controllers, in Variable Structure Systems, Sliding Mode and Nonlinear Control, K.D. Young and U. Ozguner eds, LNCI-247, Springer-Verlag, pp. 329-350, 1999.
[25] Bartolini G., Ferrara A, Usai E., Chattering Avoidance by Second-Order Sliding Mode Control, IEEE Trans. on Automatic Control, 43 (2), pp. 241-246, 1997
[26] Bartolini G., Pisano A., Usai E., Digital second-order sliding mode control for uncertain nonlinear systems, Automatica, 37, pp. 1371-1377, 2001