Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Fault Tolerant Control of a Civil Aircraft Using a Sliding Mode Based Scheme

Halim Alwi and Christopher Edwards

Abstract— **This paper presents a sliding mode control scheme for reconfigurable control of a civil aircraft. The controller is based around a state-feedback sliding mode scheme where the nonlinear unit vector term is allowed to adaptively increase when the onset of a fault is detected. Compared to other fault tolerant controllers which have been implemented on this model, the controller proposed here is relatively simple and yet is shown to work across the entire 'up and away' flight envelope. Unexpected deviation of the switching variables from their nominal condition triggers the adaptation mechanism.**

I. Introduction

The safety of aircraft passengers has been and will continue to be an important issue in the commercial aviation industry. The increasing importance of Fault Tolerant Control (FTC) has helped stimulate a growing body of research work in the area. A recent paper by Zhang & Jiang [16] provides good insight, classification and a bibliographical review of FTC in general – especially so-called 'active' FTC [9]. In terms of flight control applications, a survey paper by Huzmezan & Maciejowski [5], describes the latest development in this sub-area. The papers by Hess & Wells [4] and Shtessel *et al* [10], represent some of the most important recent research in the field of flight control using sliding mode techniques. The insensitivity and robustness properties of sliding modes to certain types of disturbance and uncertainty [2], [13] make it attractive for applications in the area of flight control and fault tolerant control. The work by Hess & Wells [4] argues that sliding mode control has the potential to become an alternative to reconfigurable control and has the ability to maintain the required performance without requiring fault detection and isolation (FDI). This represents a so-called 'passive' approach to FTC [9]. Alternatively Shtessel *et al* [10] use sliding mode ideas with online reconfiguration of the sliding surface boundary layer to control the aircraft in the presence of faults.

In this paper, a sliding mode controller is designed for application to a large passenger transport aircraft. The design of the switching surface uses a new idea inspired from the sliding mode literature and the design includes integral action to incorporate a tracking requirement [2]. A novel adaptive gain is used in the nonlinear part of the control law which reacts to the occurrence of a fault and attempts to keep the switching function as close as possible to zero thus maintaining tracking performance. If total failure of an actuator is detected a switch is made to a 'back-up' control surface (in this case the stabilizer) and the linear component of the control law remains unchanged.

This work is supported by an EPSRC grant GR/S70876.

Halim Alwi and Chris Edwards are with the Control and Instrumentation Research Group, Department of Engineering, University of Leicester, University Road, Leicester, LE1 7RH, UK ha18,ce14@le.ac.uk

II. Sliding Mode Controller Design

Consider the nth order linear time invariant system with m inputs subject to so-called matched uncertainty given by

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t) + B_p \xi(t, u, x_p) \quad (1)$$

where $A_p \in \mathbb{R}^{n\times n}$ and $B_p \in \mathbb{R}^{n\times m}$ with $1 \leq m < n$. The function $\xi(t, u, x_p)$ is assumed to be unknown but bounded and generally represents any uncertainty in the system, but in the context of this paper, models the effect of actuator faults. Without loss of generality it can be assumed that the input distribution matrix B_p has full rank. Define a switching function $s : \mathbb{R}^n \rightarrow \mathbb{R}^m$ to be

$$s(t) = S_p x_p(t) \quad (2)$$

where $S_p \in \mathbb{R}^{m\times n}$ is of full rank and let $\mathcal{S}$ be the hyperplane defined by $\mathcal{S} = \{x_p \in \mathbb{R}^n \ : \ S_p x_p = 0\}$. If a control law can be developed which forces the closed loop trajectories onto the surface in finite time and constrains the states to remain there, then an ideal sliding motion is said to have been attained [2]. Furthermore the surface can always be designed so that $S_p B_p = I_m$. The selection of the sliding surface is the first part of any design. The second aspect is the synthesis of a control law to guarantee that the surface is reached in finite time and subsequently maintained. The one that will be used as a starting point here is essentially the unit vector controller of Ryan & Corless [12]. Consider the uncertain system from (1) and suppose

$$\|\xi(t, x_p, u)\| \leq k\|u\| + \alpha(t, x_p) \quad (3)$$

where $0 \leq k < 1$ is a known constant and $\alpha(\cdot)$ is a known function. The proposed control law is given by $u(t) = u_l(t) + u_n(t)$ where the linear component is given by

$$u_l(t) = -(S_p B_p)^{-1}\left(S_p A_p - \Phi S_p\right) x_p(t) \quad (4)$$

where $\Phi \in \mathbb{R}^{m\times m}$ is any stable design matrix. The nonlinear component is defined to be

$$u_n(t) = -\rho(t, x_p)(S_p B_p)^{-1} \frac{P_2 s(t)}{\|P_2 s(t)\|} \quad \text{for } s(t) \neq 0 \quad (5)$$

where $P_2 \in \mathbb{R}^{m\times m}$ is a symmetric positive definite (s.p.d.) Lyapunov matrix for Φ. It will be assumed S_p has been chosen so that $S_p B_p = I_m$. The function $\rho(t, x_p)$, which depends only on the magnitude of the uncertainty, is any function satisfying

$$\rho(t, x_p) \geq \left(k\|u_l\| + \alpha(t, x_p) + \eta\right)/(1 - k) \quad (6)$$

where $\eta > 0$ is a design parameter. In the above formulation the parameter k in (3) could model the effect of an actuator fault: consider

$$\dot{x}_p(t) = Ax_p(t) + Bu(t) - BKu(t) + B\zeta(x_p, t) \quad (7)$$

with $K = \text{diag}(k_1, \ldots, k_m)$ where the k_i are scalars which satisfy $0 \leq k_i < 1$ and $\zeta(x_p, t)$ represents purely state dependent uncertainty. These scalars model a decrease in effectiveness of a particular actuator: so if $k_i = 0$ the ith actuator is working perfectly whereas if $k_i > 0$ some level of fault is present. Since by assumption $k_i < 1$ this excludes the possibility of the actuators failing completely (this issue will be addressed in detail separately later in the paper). If $k = \max\{k_1, \ldots, k_m\} < 1$ then the model of the faulty system given in (7) can be expressed in the more usual form of (1) and the controller can overcome completely the effect of the actuator faults.

In the majority of implementations and indeed in most of the published literature, the gain $\rho(\cdot)$ from (6) associated with the unit vector component in the control law, is fixed or a function of the states/outputs and the overall control signal. Notable exceptions are [15] and the references therein. Here the gain will be allowed to be adaptive. Furthermore because only faults in certain channels are going to be considered, a slightly different structure to that in (6) will be considered. This is facilitated by the choice of S_p for which $S_pB_p = I_m$ which effectively decouples the components of the sliding surface and associates with each a particular control input. As shown earlier, in (6) the denominator has the term $(1-k)$ and consequently as $k \to 1$ (and the effectiveness of the actuator becomes zero) arbitrarily large gains $\rho(\cdot)$ in (6) are required to maintain sliding. In a fault free situation it is not necessary and indeed is not advisable to have a large gain on the switched term – therefore ideally the term $\rho(\cdot)$ should adapt to the onset of a fault and react accordingly. Here the proposed control structure has the form

$$u_i = L_i x_p(t) - (\rho_i(t) + \eta_i)\text{sign}(s_i(t)) \quad (8)$$

where the η_i are positive constants and the gains $\rho_i(\cdot)$ in each of the control channels are assumed to satisfy

$$\dot{\rho}_i(t) = \alpha_i D(|s_i(t)|) - \beta_i \rho_i(t), \qquad \rho_i(0) > 0 \quad (9)$$

where α_i and β_i are positive constants, L_i is the ith row of

$$L = -(S_pB_p)^{-1}(S_pA_p - \Phi S_p) \quad (10)$$

and $s_i(t)$ is the ith component of $s(t) = S_px(t)$. The function $D : \mathbb{R} \mapsto \mathbb{R}$ is the dead-zone function

$$D(z) = \begin{cases} 0 & \text{if } |z| < \epsilon \\ z - \text{sign}(z)\epsilon & \text{otherwise} \end{cases}$$

where ϵ is a positive scalar. This adaptation scheme is different to the one in [15]. The choice of the design parameters will be discussed later. If Φ is chosen as a diagonal matrix, P_2 from (5) is also diagonal and so the control law in (8) is identical to the control law in (4)-(6) except the gains $\rho_i(\cdot)$ act independently in each control channel. The following lemma shows that providing the uncertainty $\xi(\cdot)$ is bounded, the gain functions $\rho_i(\cdot)$ are also bounded.

Lemma 1: Consider the faulty system represented by (1) with the control law in (8); then provided the uncertainty is bounded, each of the components $\rho_i(t)$ remains bounded.

Proof

From the decoupled structure it follows that

$$\dot{s}_i = -\phi s_i - (\rho_i(t) + \eta_i)\text{sign}(s_i) + \xi_i(t, x, u) \quad (11)$$

where it has been assumed that $\Phi = \text{diag}(-\phi_1, \ldots - \phi_m)$ and the ϕ_i are positive scalars. Suppose $\xi(t, x, u)$ remains bounded so that

$$|\xi_i(t, x, u)| \leq \rho_{max,i} = const \quad \text{for } i = 1 \ldots m \quad (12)$$

Define

$$V(s) = \sum_{i=1}^{m} \left(\frac{1}{2} p_i s_i^2 + \frac{1}{2} \frac{p_i}{\alpha_i} (\rho_i(t) - \rho_{max,i})^2 \right)$$

where the scalars $p_i > 0$ for $i = 1 \ldots m$ and the α_i are the scalars from (9). Clearly $V(\cdot)$ is positive definite with respect to s. Taking derivatives along the trajectories and substituting from (11) and (9) gives

$$\begin{aligned} \dot{V} \;\; = \;\; & \sum_{i=1}^{m} \Big(-\phi_i p_i s_i^2 - p_i \eta_i |s_i| - p_i \rho_i |s_i| + p_i s_i \xi_i \\ & + \frac{p_i}{\alpha_i} (\rho_i(t) - \rho_{max,i})(\alpha_i D(|s_i|) - \beta_i \rho_i) \Big) \quad (13) \end{aligned}$$

Then from (12) it follows

$$\begin{aligned} \dot{V} \;\; \leq \;\; & \sum_{i=1}^{m} \Big(-\phi_i p_i s_i^2 - p_i \eta_i |s_i| - p_i \rho_i |s_i| + p_i |s_i| \rho_{max,i} \\ & + \frac{p_i}{\alpha_i} (\rho_i(t) - \rho_{max,i})(\alpha_i D(|s_i|) - \beta_i \rho_i) \Big) \quad (14) \end{aligned}$$

If $|s_i| > \epsilon$ then $D(|s_i|) = |s_i| - \epsilon$ and so substituting in (14) and simplifying terms yields

$$\dot{V} \leq \sum_{i=1}^{m} \Big(-\phi_i p_i s_i^2 - p_i \eta |s_i| - \frac{p_i}{\alpha_i} (\rho_i(t) - \rho_{max,i})(\epsilon + \beta_i \rho_i) \Big)$$

Notice by construction $\rho_i(t) \geq 0$ and so $(\epsilon + \beta_i \rho_i) > 0$. Therefore for $|s_i| > \epsilon$ and $\rho_i(t) \geq \rho_{max,i}$ it follows $\dot{V} < 0$ and so $\rho_i(t)$ and $|s_i|$ remain bounded. ■

III. FTLAB747 V6.1 / V6.5

The FTLAB747 software running under MATLAB has been developed for the study of fault tolerant control and FDI schemes. It represents an extensive first principles mathematical model of a Boeing B747-100/200 where the technical data and the underlying differential equations have been obtained from NASA. The software was originally developed at Delft University by van der Linden [14] and Smaili [11], and later modified for use in terms of fault detection and fault tolerant control by Marcos & Balas [7]. The capabilities of this software as a realistic platform to test FTC and FDI schemes is demonstrated by its subsequent use by many researchers: Ganguli *et al* [3], Maciejowski & Jones [8], and Zhou *et al* [17], have all used the software for their work.

In this paper only longitudinal control is considered. This is similar to the scenario considered in [3]. The controller is designed for an 'up and away' [3] flight envelope and the main objective is to obtain smooth tracking of flight path angle (FPA) and true airspeed (V_{tas}). The nominal fault-free sliding mode controller has first been designed using a linear model which is obtained from FTLAB747. The linearization obtained from FTLAB747 has been obtained based around an operating condition of 300,000 Kg, 184 m/s true airspeed, and an altitude of 4000m at half maximum thrust. The result is a 4th order model associated with pitch rate q, true airspeed V_{tas}, angle of attack α and pitch angle θ. For design purposes the flight path angle $(\theta - \alpha)$ has been used as a state instead of θ. The four individual engine thrusts have been aggregated to produce a single control input. The two other inputs represent elevator deflection and stabilizer deflection. In the following state-space representation, the three inputs have been individually scaled which results in

$$A_p = \begin{bmatrix} -0.6803 & 0.0002 & -1.0490 & 0 \\ -0.1463 & -0.0062 & -4.6726 & -9.7942 \\ 1.0050 & -0.0006 & -0.5717 & 0 \\ -0.0050 & 0.0006 & 0.5717 & 0 \end{bmatrix} \quad (15)$$

$$B_p = \begin{bmatrix} -1.5539 & 0.0154 \\ 0 & 1.3287 \\ -0.0398 & -0.0007 \\ 0.0398 & 0.0007 \end{bmatrix} \quad b_{st} = \begin{bmatrix} -1.5760 \\ 0 \\ -0.0398 \\ 0.0398 \end{bmatrix} \quad (16)$$

where the states represent pitch rate (rad/s), true airspeed (m/s), angle of attack (rad) and flight path angle (rad) respectively. The inputs associated with B_p are elevator deflection (rad) and total thrust (N) (scaled by 10^5). The vector b_{st} is the distribution matrix associated with the stabilizer. In normal operation the aircraft would be controlled using the thrust and elevator, however in the event of an elevator failure, the stabilizer will be used as 'back-up'. The controlled outputs are

$$C_p = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (17)$$

which represent flight-path angle and true airspeed.

A. State-feedback Control Laws

Integral action has been included to add a tracking facility. The uncertain faulty system from (7) has been augmented with integral action states $x_r \in \mathbb{R}^2$ satisfying

$$\dot{x}_r(t) = r(t) - C_p x_p(t) \quad (18)$$

where the differentiable signal $r(t)$ satisfies

$$\dot{r}(t) = \Gamma\,(r(t) - r_{ref}) \quad (19)$$

with $\Gamma \in \mathbb{R}^{2\times 2}$ a stable design matrix and r_{ref} is a constant demand vector. Augmenting the states from (7) with the integral action states and defining $x = \text{col}(x_r, x_p)$ it follows

$$\dot{x}(t) = Ax(t) + Bu(t) + B_r r(t) + B\xi(t, x, u) \quad (20)$$

where

$$A = \begin{bmatrix} 0 & -C_p \\ 0 & A_p \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ B_p \end{bmatrix} \quad B_r = \begin{bmatrix} I_2 \\ 0 \end{bmatrix} \quad (21)$$

If the pair (A_p, B_p) is controllable and (A_p, B_p, C_p) does not have any invariant zeros at the origin then (A, B) is controllable [2]. The proposed controller seeks to induce a sliding motion on the surface $\mathcal{S}_a = \{x \ : \ Sx = 0\}$ where $S \in \mathbb{R}^{2\times 6}$. The linear component of the control law from (4) is modified slightly to give $u_l(x, r) = Lx + L_r r$ with gains defined as

$$L = -(SB)^{-1}(SA - \Phi S) \quad (22)$$

$$L_r = -(SB)^{-1}(SB_r) \quad (23)$$

Details of the modifications are given in [2]. The first step in the design is the selection of the sliding surface matrix S. This has been chosen using a modified version of the quadratic cost function approach of [13] described in the appendix. Here the weighting matrix has been chosen as $Q = \text{diag}\{0.5, 0.5, 5, 20, 1, 1\}$. This results in

$$S = \begin{bmatrix} 0.204 & -0.001 & -0.652 & 0.008 & -0.569 & -0.889 \\ 0.000 & -0.119 & 0.000 & 0.753 & 0.000 & 0.000 \end{bmatrix}$$

The poles associated with the reduced order sliding motion are $\{-0.7079, -0.3522 \pm 0.3479i, -0.1581\}$. Choosing the stable matrix $\Phi = -I_2$ yields

$$L = \begin{bmatrix} -0.204 & 0.001 & 0.777 & -0.009 & 0.104 & 1.168 \\ 0.000 & 0.119 & 0.110 & -0.867 & 3.517 & 7.371 \end{bmatrix}$$

$$L_r = \begin{bmatrix} -0.2035 & 0.0013 \\ 0.0001 & 0.1190 \end{bmatrix}$$

from (22) and (23). The Lyapunov matrix $P_2 = \frac{1}{2} I_2$. The pre-filter matrix from (19) has been designed to be

$$\Gamma = \begin{bmatrix} -0.2400 & 0 \\ 0 & -0.1250 \end{bmatrix}$$

The nominal controller (using the elevator) has fixed gains $\rho_1(\cdot) = 0.1$ and $\rho_2(\cdot) = 0.05$. In the simulations the discontinuity in the nonlinear control term has been smoothed by using a fixed scalar $\delta = 0.01$ in the denominator (see for example chapter 3 in [2]). The initial fixed gains for the 'back-up' controller (using the stabilizer) are given by $\rho_{stab,1} = 0.4$ and $\rho_{stab,2} = 0.05$. Here the smoothing parameter is chosen as $\delta_{stab} = 0.1$. The larger value of δ_{stab} is used to accommodate the smaller positional movement and lower rate limits of the stabilizer. In this paper, only the gains in the elevator channel are allowed to adapt: the gains associated with the thrust channel are fixed as above. The adaptation parameters are $\alpha_1 = 1000$ and $\beta_1 = 5$ and the tolerance $\epsilon = 0.0001$. When employing the adaptive gain for the nominal controller from (8) the upper and lower limits for $\rho_1(t)$ are $\bar{\rho}_1 = 0.7$ and $\underline{\rho}_1 = \eta_1 = 0.1$.

As can be seen from (6) as $k \to 1$ the inequality cannot hold for finite values of $\rho(\cdot)$. In the case of total failure $k = 1$ and an alternative control surface (in this case the stabilizer) must be employed. The input distribution matrix associated with this 'back-up' surface b_{st} from (16) replaces the first column of the existing input distribution matrix B_p in (16). However b_{st} can be uniquely decomposed as

$$b_{st} = B_p\lambda + b_p, \qquad b_p \in \mathbb{R}^4$$

where $\lambda \in \mathbb{R}^2$ and $b_p \in \mathcal{N}(B_p)$ i.e. the null-space of B_p. The differential equations associated with the aircraft system with thrust and horizontal stabilizer as inputs are

$$\dot{x} = Ax(t) + BRu(t) + b_a u_1(t) + B(\xi(\cdot) + f_a(t)) \quad (24)$$

where $R = \begin{bmatrix} \lambda & e_2 \end{bmatrix}$ with $e_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}^{\mathrm{T}}$ and b_a is an augmented version of b_p to account for the integral action states. The scalar $u_1(t)$ is the first component of $u(t)$ and $f_a(t)$ represents the effect of the failed actuator. Thus (24) effectively replaces (1) during an elevator failure scenario. *Using the same control law as before* provided $\rho(\cdot)$ is large enough to overcome the effect of R then sliding will still be achieved and the sliding motion will be governed by

$$\dot{x}(t) = (I - B(SB)^{-1}S)Ax(t) + (I - B(SB)^{-1}S)b_a u_1(t)$$

In this particular case $b_a \neq 0$ and represents unmatched disturbances/uncertainy and so the sliding motion will be affected by the change in actuator. At the design phase, the effect of u_1 has been minimized in an $\mathcal{L}_2$ [6] sense: for details see the appendix.

The control logic that has been used here is that the elevator gain $\rho_1(\cdot)$ is allowed to adapt according to (9) to account for faults in the elevator. However if $\rho_1(\cdot)$ becomes too large – indicating the potential onset of total elevator failure – then the control signal is switched from the elevator and directed to the stabilizer. Here the switch to the 'back-up' horizontal stabilizer occurs if $\rho_1(\cdot)$ exceeds $\bar{\rho}_1 = 0.7$.

IV. SIMULATIONS

The controller is designed for longitudinal axis control in the 'up and away' flight envelope [3]. The settling time when there is no failure should be 20sec for FPA and 45sec for V_{tas}. If a failure occurs, the tracking requirement is 30sec for FPA with no difference in the V_{tas} tracking. These specifications are taken from [3]. The simulations presented in this paper are all based on the *full non-linear model*. For the 'up and away' flight condition, the elevator is used to track FPA demands. As in [3], this paper only considers elevator failures. In such a situation the alternative control surface which can be used is the stabilizer.

A. Controller simulation - no failures

In this section, simulations are presented for the nominal controller designed as described in §III-A. The simulation covers the entire 'up and away' flight region. A series of 3 degree FPA and 10m/s V_{tas} commands are issued during the simulation to take the aircraft through the entire envelope. Figure 1 shows the results associated with the controller designed for the elevator and thrust, whilst Figure 2 shows the performance with the stabilizer as the control surface (together with thrust). In these responses the adaptive gain in each channel has been fixed throughout the simulation. Figures 1-2 show that both the controllers are able to maintain satisfactory tracking performance over the range of the 'up and away' flight region even though these conditions become increasingly far away from the nominal trimmed flight condition. Figure 2 shows that the 'back-up' controller (using the stabilizer) is also stable and able to maintain tracking albeit with a slower response to FPA to avoid the lower position and rate limits of the actuator.

B. Controller simulations - changes in effectiveness gain

This section shows how the controller with an adaptive gain $\rho(t)$ as defined in §II copes with different percentages of faults as modelled in (7). The 'effectiveness gain' has been implemented as a simple unknown gain between the output of the controller block and the actuator dynamics. Figure 3 shows comparisons of the adaptive gain controller with $k_1 = 0, 0.5, 0.6, 0.7, 0.8, 0.9$. Overall FPA tracking is still possible even at a 90% fault. Figure 4 shows that at 60% failure, the gain reaches the maximum allowable set gain ($\bar{\rho}_1$=0.7). Thus at this point it would be possible for a warning signal to be sent to the pilot or an automatic change to the 'back-up' controller could be initiated.

C. Elevator total failure simulations

This section shows the results of nonlinear simulations when the elevator develops floating and/or lock type actuator failures [3]. These simulate total failure of the elevator and therefore require stabilization of the aircraft using the 'back-up' controller which uses the (horizontal) stabilizer. The failure is set to occur during the climb (pitch up) manoeuvre at 10sec for both failure scenarios [3]. To simulate a floating actuator type of failure, the elevator signal is replaced with the angle of attack [3]. This simulates the ineffectiveness of the elevator to provide a moment and therefore the aircraft is unable to perform a pitch manoeuvre. Figure 5 shows that FPA tracking performance is slightly degraded and the response to a command is much slower. Figure 7 shows that the failure is detected at 10.44 sec when the adaptive gain reaches its maximum set value. Some peaks can be seen in the stabilizer signal (Figure 6) after activation due to the sudden change of control signal, but this stabilizes after a few seconds. *Once the controller is switched to the stabilizer, that surface is used for the remainder of the simulation.* To simulate lock failures, the elevator position is held at its value at 10sec. Fig 8 shows that, as before, the FPA tracking is slightly slower. Failure is detected at 12.45 sec and the stabilizer is activated (Figure 10). A peak occurs in the stabilizer signal but disappears after a few seconds (Figure 9). Overall tracking performance is maintained.

V. CONCLUSIONS

This paper has presented a sliding mode control scheme for reconfigurable control of a civil aircraft. As in the work of [3] only longitudinal control with a fault and/or failure occurring in the elevator channel has been considered. The controller is based around a state-feedback sliding mode scheme and the gain associated with the nonlinear term is allowed to adaptively increase when the onset of a fault is detected. Compared to other FTC schemes which have been implemented on this model, the controller proposed here is simple and yet is shown to still work across the entire 'up and away' flight envelope. It is not scheduled across any variables and its structure remains fixed (except for the adaptive gain associated with the nonlinear switching term). Unexpected deviation of the switching variable from its nominal condition initiates the adaptation mechanism. Total failure can also be detected from the switching function and has in this example been used to trigger the use of a 'back-up' control surface. A range of realistic fault scenarios have been considered and the results of simulations using the full nonlinear aircraft model have been presented.

REFERENCES

[1] Boyd S.P., El Ghaoui L., Feron E., Balakrishnan V., *Linear Matrix Inequalities in Systems and Control Theory*, SIAM, 1994.

[2] Edwards C and Spurgeon S.K., *Sliding Mode Control: Theory and Applications*. Taylor & Francis, 1998.

[3] Ganguli S, Marcos A, Balas G.J, *"Reconfigurable LPV Control Design for Boeing 747-100/200 Longitudinal Axis",* American Control Conference, Anchorage, Alaska, May 2002;

[4] Hess R.A , Wells S.R. *"Sliding mode control applied to reconfigurable flight control design"* Journal of Guidance, Control, and Dynamics, v 26, n 3, May-June 2003, p 452-62

[5] Huzmezan M and Maciejowski J, *"Reconfigurable Flight Control Methods and Related Issues - A Survey"* Technical report prepared for the DERA under the Research Agreement no. ASF/3455, Department of Engineering, University of Cambridge, August 1997.

[6] Khalil H.K., *Nonlinear Systems*, Prentice Hall, Englewood Cliffs NJ., 1992.

[7] Marcos A, Balas G.J, *"A Boeing 747-100/200 Aircraft Fault Tolerant and Fault Diagnostic Benchmark",* Technical report AEM-UoM-2003-1, Department of Aerospace and Engineering mechanics, University of Minnesota, USA, June, 2003.

[8] Maciejowski J.M. , Jones C N. *"MPC Fault-Tolerant Control Case Study: Flight 1862",*, SAFEPROCESS 03, Washington, 2003.

[9] Patton R.J, 'Robustness in model-based fault diagnosis: the 1997 situation,' *IFAC Annual Reviews*, vol. 21, pp. 101–121, 1997.

[10] Shtessel Y, Buffington J, Banda S, *"Multiple timescale flight control using reconfigurable sliding modes",* Journal of Guidance, Control, and Dynamics, v 22, n 6, Nov-Dec, 1999, p 873-883

[11] Smaili M.H *"FLIGHTLAB 747: Benchmark for Advance Flight Control Engineering",* technical report, Technical University Delft, The Netherlands, 1999

[12] Ryan E.P and Corless M., *"Ultimate boundedness and asymptotic stability of a class of uncertain dynamical systems via continuous and discontinuous control"*, IMA Journal of Mathematical Control and Information, v 1, 1984, p 223–242.

[13] Utkin V.I., *Sliding Modes in Control Optimization*, Springer-Verlag, Berlin, 1992.

[14] van der Linden C.A.A.M. *"DASMAT: Delft University Aircraft, Simulation Model and Analysis Tool",* Technical Report, LR-781 Technical University Delft, The Netherlands, 1996.

[15] Wheeler G., Su C., Stepanenko Y., *A sliding mode controller with improved adaptation laws for the upper bounds on the norm of uncertainties*, Automatica, vol 34, 1657-1661, 1998.

[16] Zhang Y, Jiang J. in *"Bibliographical Review on Reconfigurable Fault Tolerant Control Systems"*, Proceedings of the IFAC Symposium SAFEPROCESS 03, Washington, 2003, pp. 265–276.

[17] Zhou K, Rachinayani P.K, Liu N, Ren Z, Aravena J. *"Fault Diagnosis and Reconfigurable Control for Flight Control System with Actuator Failures",* 43rd IEEE Conf on Decision and Control, December 14-17, 2004, Atlantis, Paradise Island, Bahamas

APPENDIX

Consider the problem of minimising for system (20) the quadratic performance index

$$J = \frac{1}{2}\int_{t_s}^{\infty} x(t)^{\mathrm{T}} Q x(t)\ dt \tag{25}$$

where Q is a s.p.d matrix and t_s is the time at which the sliding motion commences. It is assumed that the system in (20) is already in regular form [13] so that

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$

where $A_{11} \in \mathbb{R}^{n\times n}$ and $B_2 \in \mathbb{R}^{m\times m}$. Also assume the matrix Q from equation (25) has a block diagonal structure $Q = \text{diag}(Q_1^{\mathrm{T}}Q_1, Q_2^{\mathrm{T}}Q_2)$ where $Q_2^{\mathrm{T}}Q_1 = 0$ and the matrix $Q_2^{\mathrm{T}}Q_2 \in \mathbb{R}^{m\times m}$ is nonsingular. It follows that

$$J = \frac{1}{2}\int_{t_s}^{\infty} x_1(t)^{\mathrm{T}}Q_1^{\mathrm{T}}Q_1 x_1(t) + x_2(t)^{\mathrm{T}}Q_2^{\mathrm{T}}Q_2 x_2(t)\ dt \tag{26}$$

where $x = \text{col}(x_1, x_2)$ and $x_1 \in \mathbb{R}^n$. Because of the assumption of regular form, the differential equation constraint (20), whilst sliding, may be written as

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}x_2(t) \tag{27}$$

where $Mx_1 + x_2 = 0$. The hyperplane design problem can therefore be viewed as one of choosing an appropriate M and thus can be interpreted as a standard linear quadratic regulator (LQR) problem. Details can be found in [2]. However as argued in [1] (page 114), the LQR problem can also be posed as an LMI optimization: i.e. Minimise trace(X^{-1}) subject to

$$\begin{bmatrix} A_{11}X + XA_{11}^{\mathrm{T}} - A_{12}N - N^{\mathrm{T}}A_{12}^{\mathrm{T}} & (Q_1X - Q_2N)^{\mathrm{T}} \\ Q_1X - Q_2N & -I \end{bmatrix} < 0 \tag{28}$$

where $N = MX$. In the 'back-up' case the sliding mode will be affected by the unmatched uncertainty in (24). In regular form this equation can be represented as

$$\dot{x}_1(t) = (A_{11} - A_{12}M)x_1(t) + b_1u_1(t) \tag{29}$$

where b_1 is a sub-vector of b_a from (24). The objective is to minimize the effect of u_1 on the performance of the system in equation (29) in an $\mathcal{L}_2$ sense. Under the constraint that a common Lyapunov function for both the LQR and the $\mathcal{L}_2$ gain problems is sought, from the Bounded Real Lemma [1], the $\mathcal{L}_2$ gain between x_1 and u_1 is less than γ if

$$\begin{bmatrix} A_{11}X + XA_{11}^{\mathrm{T}} - A_{12}N - N^{\mathrm{T}}A_{12}^{\mathrm{T}} & b_1 & X \\ b_1^{\mathrm{T}} & -\gamma I & 0 \\ X & 0 & -\gamma I \end{bmatrix} < 0 \tag{30}$$

The formal overall optimization problem used here is given by the following:

Minimise $(a_1\text{trace}(Z) + a_2\gamma)$ subject to

$$\begin{bmatrix} -Z & I_n \\ I_n & -X \end{bmatrix} < 0, \quad X, Z > 0 \tag{31}$$

in addition to (28) and (30). Here a_1 and a_2 are positive scalars which determine the relative weighting between the LQR and $\mathcal{L}_2$ problem. This represents a convex optimization problem in terms of X, Z, N and γ and can be solved using standard LMI packages. The matrix which determines the hyperplane is computed as $M = NX^{-1}$ and finally

$$S = \begin{bmatrix} M & I_m \end{bmatrix} \tag{32}$$

For the approach adopted in §II this matrix must then be scaled to ensure $SB = I_m$.

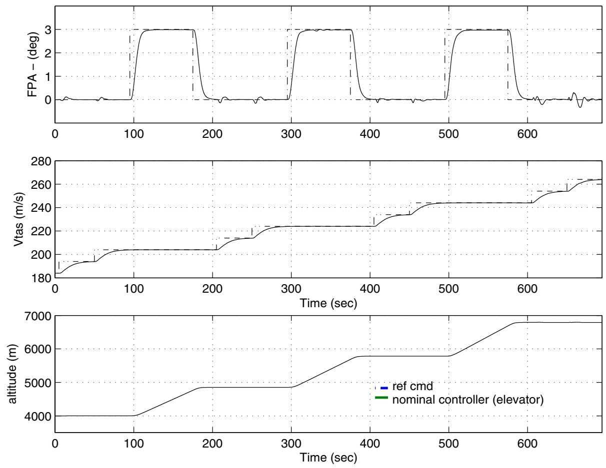

Fig. 1. Nominal fault free performance

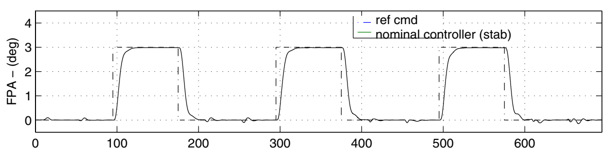

Fig. 2. Nominal fault free performance (stabilizer)

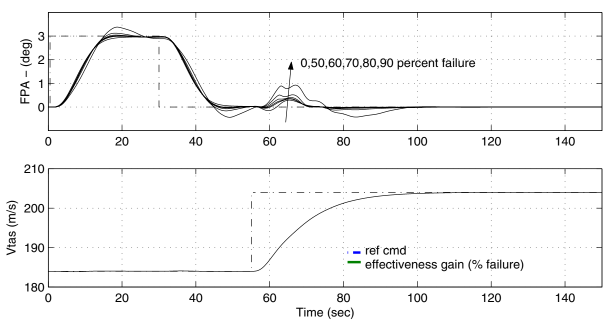

Fig. 3. Responses for different effectiveness gains

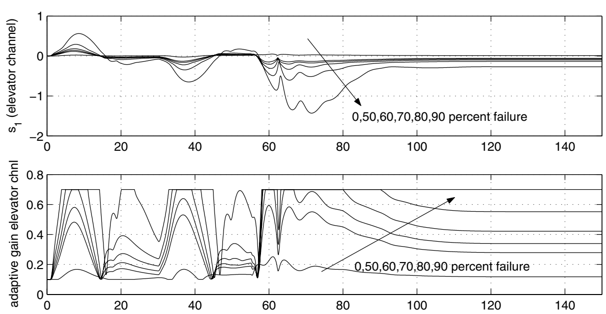

Fig. 4. Evolution of the switching function s_1 and adaptive gain ρ_1

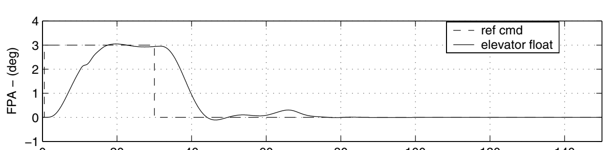

Fig. 5. Float failure: output responses

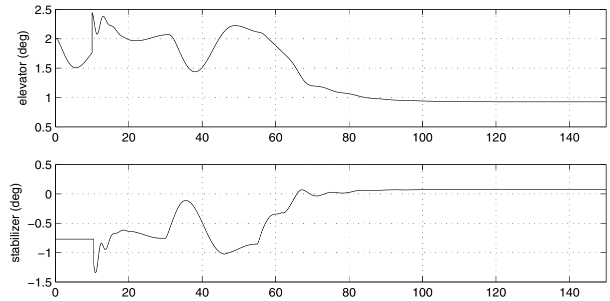

Fig. 6. Float failure: control signals

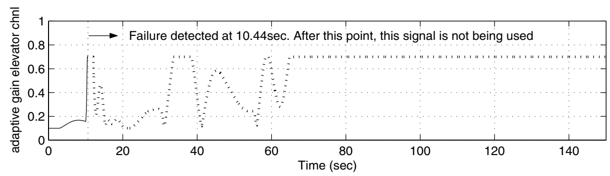

Fig. 7. Float failure: the adaptive gain ρ_1

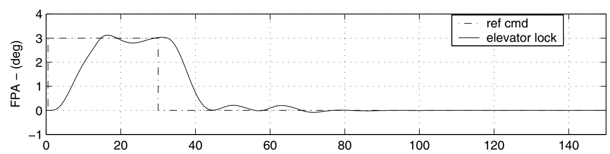

Fig. 8. Lock failure: output response

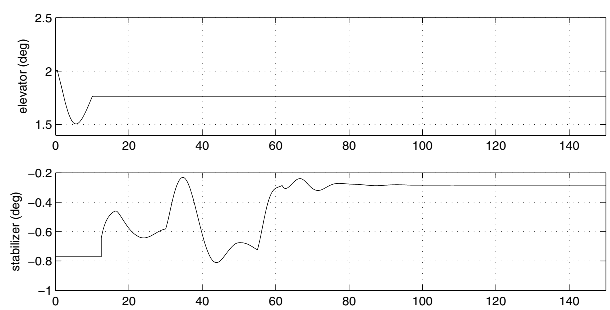

Fig. 9. Lock failure: control signals

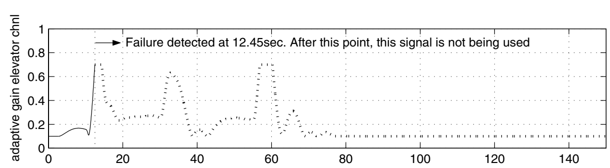

Fig. 10. Lock failure: adaptive gain