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Local and long-range interactions for distributed control of a group of robots

Arturo Buscarino, Luigi Fortuna, Mattia Frasca and Alessandro Rizzo

Abstract— **In this paper we present a distributed control scheme for a group of cooperating roving robots. It is based on the model introduced by Vicsek [1], which models the kinetics of self-propelled particles, able to undergo a phase transition from a disordered state to an ordered state, involving a net transport phenomenon. We examine the case in which the noise term is not zero and provide a simple explanation for this case. Moreover, the properties of small-world topology are considered in order to include long-range connections between the robots. This leads to an improvement of system performance in presence of noise. Finally, a suitable strategy based on this model to control exploration and transport is introduced.**

I. INTRODUCTION

Complex systems are common in nature. These systems are made up by a large number of simple particles cooperating and sharing information. Examples of complex system are bird flocks, the internet and so on [2].

Many particle systems exhibit a complex behavior during transition between ordered and disordered particle distribution phases. Furthermore, particle systems are able to model the collective motion of organisms, such as birds in a flock or fish in a school, having a spontaneous breaking of the symmetry state to assume an ordered phase in which all members of even large dimension flocks move with the same mean velocity. The direction of motion is not fixed *a priori* but each flock spontaneously takes an arbitrary direction to move in. Another important aspect of these particle systems is that each member is a self-propelled particle.

There are several studies on flocking and spontaneous breaking of symmetry, many of them using the concept of the physics of phase transition in equilibrium systems, such as cooperative behavior and scale invariance. These concepts are useful to understand the properties of non-equilibrium systems, like self-organization criticality involved in phase transition behavior, typical of equilibrium systems.

The model investigated in this paper is the self-driven particle system described by Vicsek et al. [1]. In [1] it is shown, by means of simulations, that the proposed model exhibits a kinetic phase transition from no transport to finite net transport through spontaneous symmetry breaking of the rotational symmetry.

We consider Vicsek's model for a robotic application with the aim of controlling the cooperative behavior of robots with local connections realized through sensors able to collect information from neighbor robots.

Recently, technology involved in control techniques for single robots has been pushed to a high level of robustness as long as computational and communication capabilities are growing up. These enhancements make possible to equip each robot with sensors, wireless transmission systems and a processor, dedicated to the processing of information collected by sensors through the communication system [3]. In this perspective, each robot is able to control its movement by exploiting information coming from the other robots acting in the same environment. Thus, the development of distributed algorithms for controlling the robots is becoming very important to achieve a global coordinated and cooperative behavior without the use of central coordination.

A cooperative control system consists of a population of autonomous components, which interact with the aim of controlling and coordinating their behavior [4]. It is clear that classical control theory cannot be applied to cooperative robots as it is, but needs to fit into the framework of distributed systems. Furthermore, cooperative control problems cannot be solved by considering the two aspects as independent. For example, communication and information sharing introduce delays, which worsen system performance and tend to make the control system unstable. Consequently, these delays must be taken into account in the design of the control system.

Therefore, designing a cooperative control algorithm involves a wise merging of the two previously described points of view, combining in an effective way strategies from both control theory and distributed computation [4].

The behavior of the Vicsek's model in absence of noise (i.e. without random errors in the headings) has been investigated in several papers [5], [6], [7]. To describe the interactions between the particles of the Vicsek's model a simple undirected graph, called interaction graph, can be used. The particles represent the vertices of the graph. The edges of the graph are defined so that an edge exists if the two vertices are in a local radius of interactions. Since the particles are free to move on the plane, the related graph is time-variant. Thus, the graph describing particle interactions also changes over time. The convergence of the individual particle motions to a common heading depends on the properties of this time-variant graph [5], [6], [7]. For instance, in [6] it is shown that convergence is achieved if the union of all interaction graphs started from any time is connected.

In this paper we give a qualitative analysis of the influ-

A. Buscarino, L. Fortuna and M. Frasca are with Università degli Studi di Catania, Dipartimento di Ingegneria Elettrica Elettronica e dei Sistemi, Viale A. Doria 6, 95125 Catania, Italy. Alessandro Rizzo is with Politecnico di Bari, Dipartimento di Elettrotecnica ed Elettronica, Via Re David 200, 70125 Bari, Italy. Email: mfrasca@diees.unict.it

ence of noise in the system performance. Motivated by our analysis, we then extend the model to include long-range connections between the robots and we show how these connections improve the system performance in presence of noise.

The introduction of these long-range connections is further motivated by the following considerations. The Vicsek's model can be viewed as a particular kind of locally connected network in which the connections change during observation. For example, in [5] interactions between robots are modelled by using a time-dependent relationship graph. Thus, the whole system can be modelled as a switched linear system, whose properties can be studied with respect to the switching signal [5]. In network theory, the introduction of particular communication topology, such as scale-free or small-world [12], enhance significantly the performance of the system in terms of synchronization and cooperation. In particular small-world networks are built by starting from a lattice network and adding some long-range connections [8]. The proposed approach takes inspiration from the properties of such systems to create a network of connections between robots which is conceptually similar to the small-world network: it will be shown that in this way it is possible to achieve a better and faster coordination of the robots.

II. EFFECT OF NOISE

The model in [1] presents a fixed number of particles moving in a two dimensional space with a constant absolute velocity and random initial direction and position. The only rule governing the model is that at each simulation step a given particle driven with a constant absolute velocity assumes the average direction of motion of the particles in its neighborhood of radius r with some random perturbation added.

Let r be the interaction radius, L (units are normalized with respect to the radius r) the dimension of the plane in which robots move (with periodic boundary conditions), N the number of robots, v the velocity and θ the direction of motion. The velocity is constant in module and is updated in direction through the $\theta_i(t)$ angle for each particle. The position and the orientation of each particle is updated according to:

$$\begin{aligned} \mathbf{x}_i(t+1) &= \mathbf{x}_i(t) + \mathbf{v}(t)\Delta \mathrm{t} \\ \theta_i(t+1) &= \langle \theta_i(t) \rangle_r + \Delta\theta \end{aligned} \tag{1}$$

where $\langle \theta_i(t) \rangle_r$ is the average of the direction of particles inside the neighborhood of the i-th particle and the time unit has been fixed to $\Delta t = 1$. The average direction is given by the angle $\arctan(\frac{\langle \sin\theta_i(t) \rangle_r}{\langle \cos\theta_i(t) \rangle_r})$. $\Delta\theta$ is a random number chosen with uniform probability from the interval $[-\frac{\eta}{2}, \frac{\eta}{2}]$.

The values of noise parameter η and particle density $\rho = \frac{N}{L^2}$ affect the behavior of the system: for small values of density and noise the particles tend to form groups moving toward random directions; increasing density and noise the particles move randomly with some correlation; increasing density but maintaining small the noise it is possible to observe an ordered motion of all the particles towards the same spontaneously chosen direction. The last case is the most interesting because it shows a kinetic phase transition due to the fact that particles are driven with a constant absolute velocity [1]. To characterize the ordered phase it is possible to define an order parameter: the average normalized velocity, which can be calculated with the following expression:

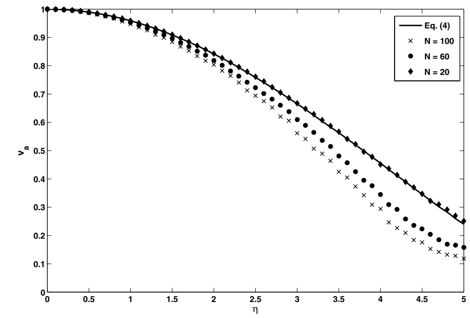

Fig. 1. Order parameter v_a vs η for different N and for the ideal case (Eq. (4), continuous line). The density has been fixed at $\rho = 4$ for all the cases. The other parameters are $r = 1$, $\Delta t = 1$ and $L = \sqrt{(N/\rho)}$. Initial conditions are random. v_a is averaged over 15 runs.

$$v_a = \frac{1}{N\nu} \left| \sum_{i=1}^{N} v_i \right| \tag{2}$$

where ν is the constant module of the velocity of particles. Clearly, $v_a = 0$ indicates that the robots are not coordinated (there is no net transport), while $v_a = 1$ indicate that all the robots move in the same direction (the net transport is maximum). In fact, $v_a = 1$ holds when $\theta_i = \bar{\theta}$ for all i.

As stated in the previous section, in this paper the group of robots able to locally share information about the direction of motion is modelled with a system of moving particles, which evolves according to Vicsek's model.

The typical behavior of v_a versus the noise parameter is shown in Fig. 1 (dotted curve, $N = 60$ and x-mark curve, $N = 100$), the other two curves are discussed below.

The behavior of the Vicsek's model in absence of noise has been investigated in several papers. In particular, in [5] it is proved that all particles shall move in the same heading, if the graphs describing their interactions are periodically linked together, i.e. if the union of any T sequential graphs is connected for some given T. A weaker condition is proved in [6], where the sufficient condition for the coordination of all agents is that the graphs are finally linked together, i.e. if the union of the graphs started from any time is connected. However, for the case of $\eta \neq 0$ there are no simple models. In this paper we study some qualitative issues of the Vicsek's model in presence of noise.

In order to introduce our model illustrating the behavior of the system with respect to the noise parameter, let us consider what occurs in the case when each robot knows the heading of all the other robots, i.e. when all the robots are packed forming a unique group. In this case $\langle \theta_i \rangle = \Theta$ for each robot and, since $v_i = \nu e^{j\theta_i}$, v_a is given by the following expression:

$$v_a = \frac{1}{N}\left|\sum_{i=1}^{N} e^{j\theta_i}\right| = \frac{1}{N}\left|\sum_{i=1}^{N} e^{j\Theta}e^{j\Delta\theta}\right| \tag{3}$$

and thus

$$v_a = \frac{1}{N}\left|\sum_{i=1}^{N} e^{j\Delta\theta}\right| \tag{4}$$

The curve v_a vs η corresponding to equation (4) is shown in Fig. 1. This is a sort of best case model in which all the robots lie in the neighborhood of each robot: thus every robot has information on all other robots. In this case the robots move in a packed group, maximizing v_a. The average velocity v_a depends only on the noise term, which tends to degrade the performance of the whole system. This case represents an ideal case occurring with high density in a small environment.

In real cases, although the robots may form a group, not all the robots will belong to the neighborhood of each robot and the effects of the random perturbations on θ_i will be treated differently depending on the specific neighborhood. Therefore, in real cases $\langle\theta_i\rangle_r$ will depend on i, and v_a will be smaller than the value predicted by eq. (4).

When, keeping constant the density, the number of robots is decreased, the robots will move on a smaller space and each of them will interact with a number of robots greater than the case of larger N. Thus, decreasing N and maintaining constant ρ the performance of the system approaches the ideal case. This is shown in Fig. 1, where it can be noticed that decreasing N the average velocity v_a increases.

Motivated by these considerations and by the robotics application, we introduce the possibility of having long-range connections. In fact, from a technological point of view long-range connections can be easily implemented on groups of roving robots. At each time step, some randomly selected robots are allowed to have a few long-range connections with other randomly chosen robots. Therefore, each of these robots does not have to know which robot is communicating with, nor its position. The difference with the original Vicsek's model is that a robot can choose an additional 'neighbor' which can be at a distance greater than its interaction radius and share information with it.

Introducing these new interactions is motivated by the positive effects that long-range (or weak ties) connections have on complex systems, in which a small percentage of such links leads to the so-called small-world effect [12], which improves system performance in terms of both local and global information exchange (for instance, the average path length decreases). The idea underlying this new model is that in this way the robots are much more connected, by exploiting almost the same radius of communication, plus a few long-range links. Consequently, the effect should be that of a massive connection without the overhead of communication imposed by this configuration. Thus, the v_a curve should be pushed toward the ideal one. The results discussed in the next section confirm our initial intuition.

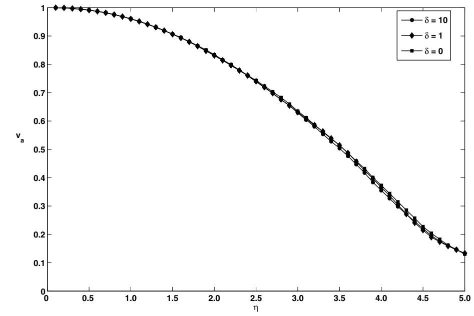

Fig. 2. Order parameter v_a vs η when long-range connections are introduced. The various curves are relative to different values of δ. The other parameters are $N = 100$, $L = 5$, $\rho = 4$, $r = 1$, $\nu = 0.03$, Δt = 1. The simulation time is 200 steps beyond the time necessary to reach convergence time in the case $\eta = 0$. Average results over 5 runs are shown.

III. LONG-RANGE CONNECTIONS

In order to evaluate the effects of the introduction of long-range connections, several simulations with different parameters are carried out. First of all, the effects of long-range connections between robots have been evaluated taking into account the case reported in [1]. In particular $N = 100$ robots are allowed to move on a plane of dimension $L = 5$, thus the density is $\rho = 4$. The number of long-range connections at each time step has been indicated as δ and ranges from 0 to 10, in order to keep the number of long-range connections small. The case $\delta = 0$ corresponds to the original Vicsek's model, where interactions are only local. The performance of the system has been evaluated by taking into account the order parameter v_a with respect to increasing values of η. For each value of η, 5 runs have been considered and results averaged.

The results are shown in Fig. 2, where it is clear that the performance is not strongly improved by increasing the value of δ. The best improvements is obtained when the noise η ranges from 1.5 to 3.5. This is clearly shown in Fig. 3 where the percentage enhancement of v_a with respect to the original model are shown as a function of δ. The best performance has been obtained by using ten long-range connections per step. Moreover, it can be noticed that the curve shifts up when the number of robots increases. Since these simulations were carried out with a fixed density, when N increases, so L does and the effects of long-range interactions become more important.

Another important effect of long-range connections is that the time required to reach the steady value of v_a decreases. Although it is not simple to quantify this effect, especially in the case of $\eta \neq 0$, we consider the trend of v_a and defined a convergence time to account for this effect. We define the convergence time (T_c) as the time necessary to reach the 90% of the steady value of v_a. An example is reported in Fig. 4, where the trend of the average velocity $v_a(t)$ is shown.

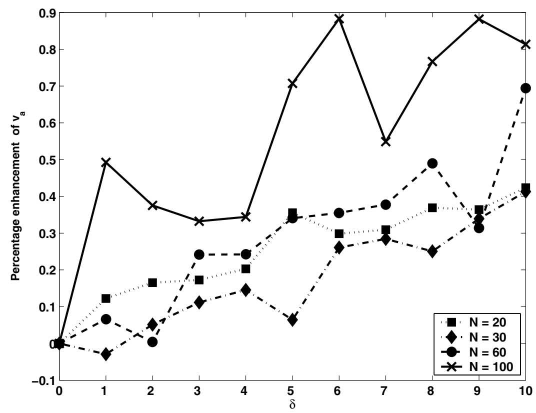

Fig. 3. Percentage enhancement of system performance in terms of average velocity varying δ for the case $\rho = 4$. Different curves are related to different values of N. The results are averaged for $1.5 \leq \eta \leq 3.5$.

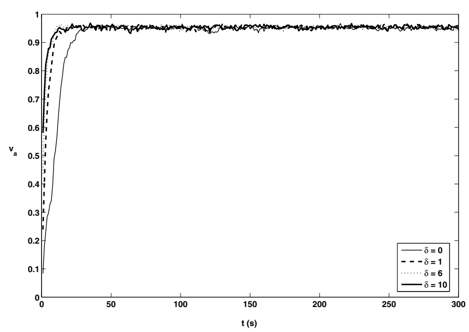

Fig. 4. Time evolution of the average velocity for the case $N = 100$, $\rho = 4$, $\eta = 1.0$, with different δ. Convergence time: $T_c = 23s$ for $\delta = 0$; $T_c = 10s$ for $\delta = 1$; $T_c = 9s$ for $\delta = 6$; $T_c = 7s$ for $\delta = 10$.

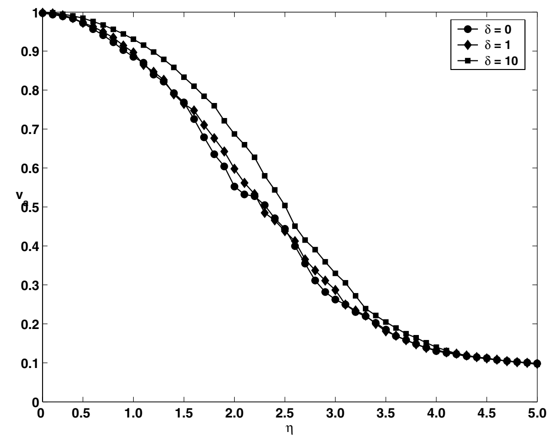

Fig. 5. Order parameter v_a vs η. Curves are relative to different values of δ. Results of 15 runs are shown. The other parameters are $N = 100$, $L = 10$, $\rho = 1$, $r = 1$, $\nu = 0.03$, $\Delta t = 1$. The simulation time is 200 steps beyond the time necessary to reach convergence in the case $\eta = 0$.

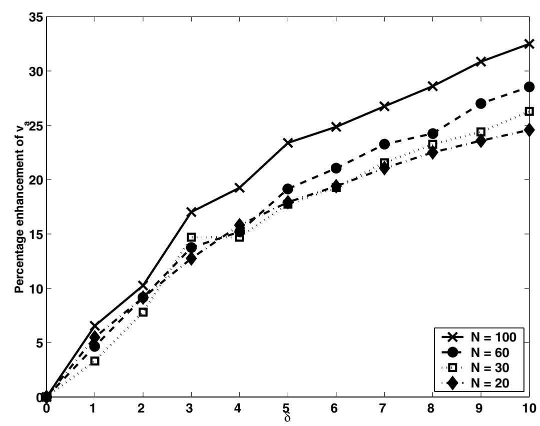

Fig. 6. Percentage enhancement of system performance in terms of average velocity varying δ for the case $\rho = 1$. Different curves are related to different values of N. The results are averaged for $1.5 \leq \eta \leq 3.5$.

The case $N = 100$ and $\rho = 4$ is taken into account. As it can be noticed, the time needed by the N robots to reach the coordination when $\delta = 10$ is less than that of the case $\delta = 0$. The case $\delta = 1$ leads to a small (often dependent on the realization) improvement in the system performance.

These considerations allow us to conclude that in the case of $\rho = 4$ the system shows similar values of v_a independently of the value of δ, but there is a small improvement on the time necessary to reach the steady speed. The behavior of the system has been further investigated with respect to different values of the density, leading to the conclusion that long-range connections introduce clear advantages when the density decreases. This is a very important point since, as shown in [1], v_a decreases with ρ. Therefore, the performance achieved is poorer as long as the density ρ decreases. For these cases the introduction of long-range connections leads to the greatest benefits. This behavior is due to the fact that long-range connections are important when the environment in which the robots move is larger. In fact, the results illustrated in [5] highlight that the connectivity of the interaction graph is fundamental for convergence. On the other hand, it is well-known from network theory that long-range connections lead to the small-world effect, in which a characteristic path length scales as $\log N$ and the clustering coefficient is high. In the case of Vicsek's model long-range connections help robots to share information about their headings even if they are not neighbors. This is particularly important in large fields.

Fig. 5 shows the effects of long-range connections when $\rho = 1$, where it can be noticed that the improvement is much more significant. This is further remarked in Fig. 6, where the percentage enhancement of v_a versus δ is shown. The best case occurs for a population of $N = 100$ robots, where the increase of the average velocity is of about 35%. Since, as shown in Fig. 5, the highest improvements occur for $\eta = 1.5 \div 3.5$, the increase of the average velocity has been evaluated in this range. Also in this case the higher the number of robots, the greater the improvements. The best case is obtained with $\delta = 10$.

Also in the case of low density, long-range connections decrease the time needed to reach an ordered behavior. Fig. 7 shows the trend of the average velocity v_a for $N = 100$ and $\eta = 1$. Three curves corresponding to three value of δ are shown. Moreover, Fig. 8 shows the convergence time

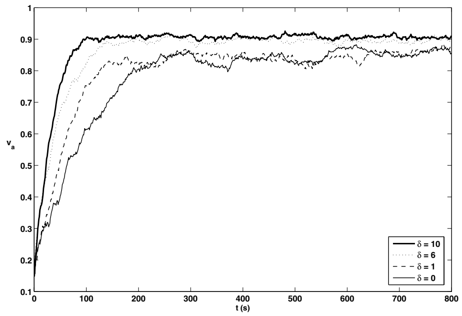

Fig. 7. Time evolution of the average velocity for the case $N = 100$, $\rho = 1$, $\eta = 1.0$ for different values of δ. Convergence time: $T_c = 173s$ for $\delta = 0$; $T_c = 107s$ for $\delta = 1$; $T_c = 93s$ for $\delta = 6$; $T_c = 61s$ for $\delta = 10$

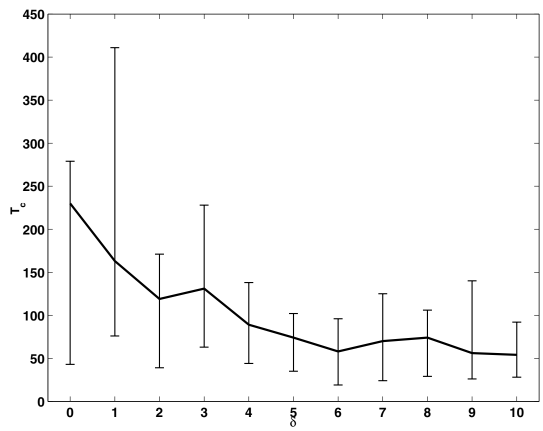

Fig. 8. Time to reach the steady velocity for increasing values of δ. The other parameters are $N = 100$, $\rho = 1$, $\eta = 0$. The error bar relative of the 15 different runs is also shown.

T_c versus δ: it is possible to notice that the introduction of long-range connections significantly reduces the time needed to reach the steady condition.

IV. EXPLORATION AND TRANSPORT

In this section we show how the model discussed above can be used with the aim of performing either the exploration of a surface or a coordinated behavior (transport).

It has been already shown how robots can move in a coordinated way. By introducing a new parameter in the model represented by Eq. (1), exploration of a given surface may be also achieved. Namely, if we consider the following model

$$\begin{aligned} \mathbf{x}_i(t+1) &= \mathbf{x}_i(t) + \mathbf{v}(t)\Delta \mathrm{t} \\ \theta_i(t+1) &= \alpha\langle\theta_i(t)\rangle_r + \Delta\theta \end{aligned} \tag{5}$$

with a nonzero value for η (e.g. $\eta = 1$); if $\alpha = 1$ robots will move toward a spontaneous chosen direction, while if $\alpha = 0$ robots will move in a random manner exploring the surface. Therefore, with a very simple control parameter both exploration and transport can be achieved on a simple model for distributed control.

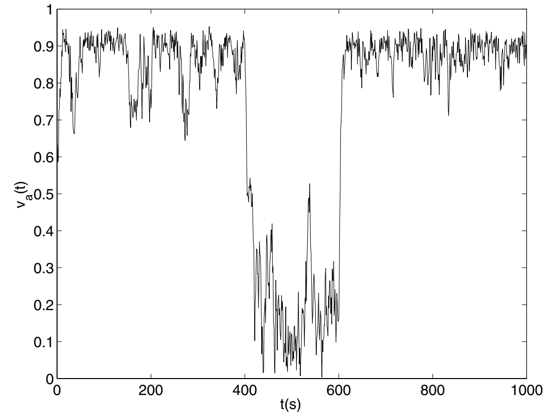

Fig. 9. Average velocity $v_a(t)$ for the exploration/transport model.

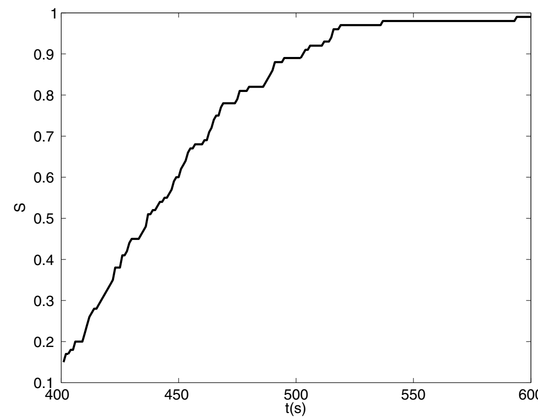

Fig. 10. Surface (normalized with respect to the total surface L^2) explored by the robots when $\alpha = 0$.

Fig. 9 shows the results obtained by simulating a group of $N = 20$ robots for $T = 1000s$. For $0 \leq t \leq 400$, α is fixed at $\alpha = 1$, thus robots move in a coordinated group. For $400 < t \leq 600$, α is fixed at $\alpha = 0$ and the robots move exploring the surface (in fact v_a decreases). The surface (normalized with respect to the total area L^2) explored during this time interval is shown in Fig. 10 as a function of the time t. For $600 < t \leq 1000$, α is fixed at $\alpha = 1$, and the robots again move in a coordinated group. Fig. 11 shows the robot trajectories (referred to the last 20 time steps) at different time steps. As can be noticed at $t = 400s$ (Fig. 11(a)) and at $t = 1000s$ (Fig. 11(c)) robot trajectories are oriented along a common heading, while at $t = 600s$ (Fig. 11(b)) robots follow uncorrelated trajectories.

V. CONCLUSIONS

This paper focuses on a distributed control scheme for a group of interacting robots and shows that the introduction of few long-range interactions between the robots improves the system performance without increasing the cost associated with a global communication scheme.

The simulation results discussed in this paper show that system performance is effectively improved in terms of both average velocity of the group of robots and average time

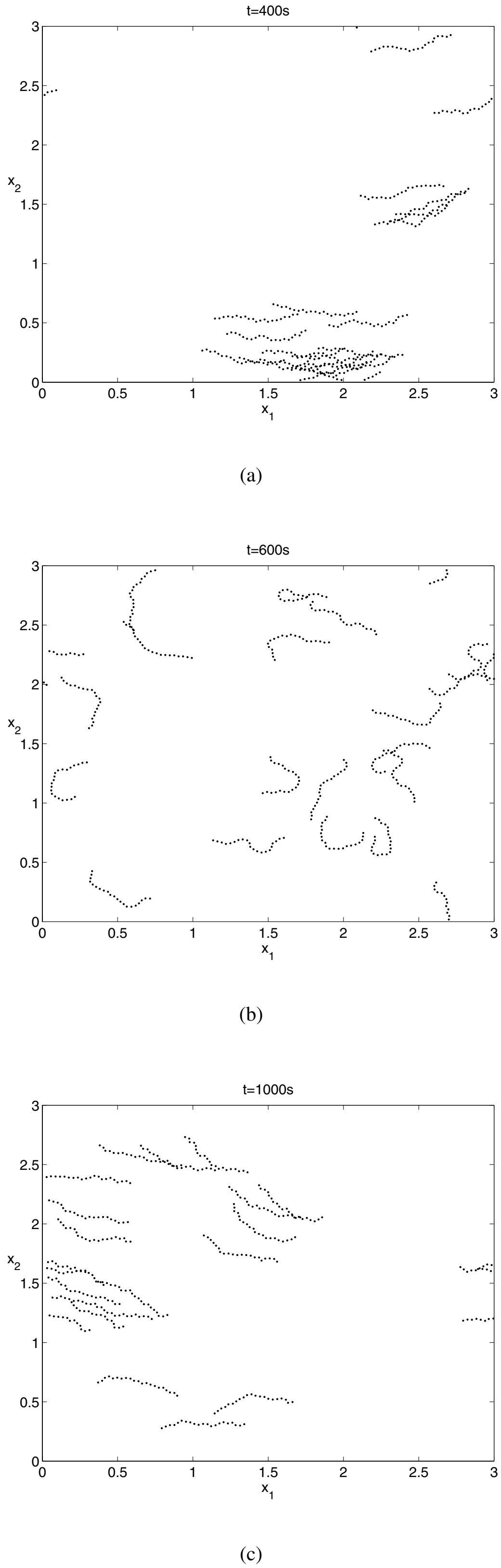

Fig. 11. Behavior of model (5). Last 20 positions of the robots (grey circles) and actual positions of the robots (black circles) at different time steps: (a) $t = 400s$; (b) $t = 600s$; (c) $t = 1000s$.

required to organize the group in a coordinated behavior. In real applications of cooperative robotics these factors can be very important (for instance as escape mechanism to rapidly move all the robots in a coordinated manner).

Keeping the robotics application in mind, we modified the original Vicsek's model to control the group of robots and make them able to switch between two activities, namely exploration and transport on a surface. Considering a family of robots which inspect an industrial plant, in this case an escape mechanism able to synchronize all the robots and to take out the robots as soon as possible is needed. In this scenario, the capability of synchronization and the short time needed to reach the synchronized state are important requirements which are fulfilled by the proposed strategy.

With the aim of the robotics application, a further topic to be investigated is the behavior of the system in presence of a few informed individuals, i.e. individuals aware of the heading to be assumed under a specific circumstance.

VI. ACKNOWLEDGMENTS

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