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Optimal Admission Control for a Markovian Queue Under the Quality of Service Constraint

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***Abstract*— We study an optimal admission of arriving customers to a Markovian finite-capacity queue, e.g. M/M/c/N queue, with several customer types. The system managers are paid for serving customers and penalized for rejecting them. The rewards and penalties depend on customer types. The goal is to maximize the average rewards per unit time subject to the constraint on the average penalties per unit time. We provide a solution to this problem through a Linear Programming transformation and characterize the structure of optimal policies based on Lagrangian optimization. For a feasible problem, we show the existence of a 1-randomized trunk reservation optimal policy with the acceptance thresholds for different customer types ordered according to a linear combination of the service rewards and rejection costs. In addition, we prove that any 1-randomized optimal policy has this structure. In particular, we establish the structure of an optimal policy that maximizes the average rewards per unit time subject to the constraint on the blocking probability for one of the customer types or for a group of customer types pooled together, i.e., the QoS (Quality of Service) constraint. In the end, we also formulat the problem with multiple constraints and similar results hold.**

I. INTRODUCTION

The admission control problem for a queue in which the QoS (quality of service) is provided has broad applications in telecommunication, computer, service and engineering. One aspect of admission control is the ability to monitor, control, and enforce the use of resources and services with policy-based management. In this paper, we describe the structure of optimal admission policies to finite capacity queues, including $M/M/c/N$ queues, with a fixed number of customer types. At the arrival epoch a customer can be either rejected or accepted. The latter is possible only if the system is not full. Each customer type $i = 1, 2, \ldots, m$, where m is the number of customer types, is characterized by three parameters: a Poisson arrival rate λ_i, a reward r_i that a customer pays for the service, and the penalty c_i paid to a rejected customer. The service times do not depend on the customer types. The goal is to maximize the average rewards per unit time subject to the constraint that the average penalty per unit time does not exceed a certain number. Such problems arise, for example, when the goal is to maximize the average rewards per unit time subject to the quality of service (QoS) constraint.

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A 1-randomized trunk reservation policy ϕ is defined by m numbers M_i^ϕ, $0 \le M_i^\phi \le N-1$, $i = 1, \ldots, m$. Among these numbers $M_1^\phi, \ldots, M_m^\phi$, at most one number is non-integer and at least one number equals $N-1$. For a number M we denote by $\lfloor M \rfloor$ the integer part of M. If the system is controlled by the policy ϕ, a type i arrival will be admitted with probability 1 if it sees no more than $\lfloor M_i^\phi \rfloor$ customers in the system, it will be rejected if the number of customers in the system exceeds $\lfloor M_i^\phi \rfloor + 1$, and it will be accepted with the probability $(M_i^\phi - \lfloor M_i^\phi \rfloor)$ if there are exactly $\lfloor M_i^\phi \rfloor + 1$ customers in the system at the time of its arrival. In particular, if the number M_i^ϕ is an integer, a type i arrival will be admitted if and only if it sees no more that M_i^ϕ customers in the system. Thus, $M_i^\phi = N-1$ means that a type i arrival is admitted whenever the system is not full. A 1-randomized trunk reservation policy ϕ is called consistent with a function r' defined on the set $\{1, \ldots, m\}$ if $r'_i > r'_j$ implies $M_i^\phi \ge M_j^\phi$, $i, j = 1, \ldots, m$. If all the thresholds are integers, the 1-randomized trunk reservation policy is called a trunk reservation policy. We sometimes write M_i instead of M_i^ϕ for the thresholds when there is only one policy in the context and no confusion will occur.

In this paper, we prove that, if the problem is feasible, then there exists a 1-randomized trunk reservation policy which is consistent with the reward function

$$r'_i = r_i + \bar{u}_1 c_i, \tag{1}$$

where $\bar{u}_1 \ge 0$ is the first entry of the dual solution to the linear programming problem we formulated later in this paper. In addition, Theorem 3 shows that any 1-randomized stationary optimal policy is a 1-randomized trunk reservation policy which is consistent with r'.

Miller [19] studied a one-criterion problem for an $M/M/c/loss$ queue when $r_1 > r_2 > \cdots > r_m$. In this case, there exists an optimal non-randomized trunk reservation policy which is consistent with r. In other words, all the thresholds M_i are integers and $N-1 = M_1 \ge M_2 \ge \cdots \ge M_m$. Feinberg and Reiman [8] studied a constrained problem with $r_1 > r_2 > \cdots > r_m$ when the goal is to maximize average rewards per unit time subject to the constraint that the blocking probability for type 1 customers does not exceed a given level. Feinberg and Reiman [8] proved the existence of an optimal 1-randomized trunk reservation policy with $N-1 = M_1 \ge M_2 \ge \cdots \ge M_m$.

Instead of considering $M/M/c/loss$ or $M/M/c/N$ queues, Feinberg and Reiman [8] made a more general assumption that the service rate μ_n, when there are n customers in the system, does not decrease in n. This assumption holds for $M/M/c/N$ queues. In this paper, we also consider systems that satisfy this assumption.

This research was initially motivated by the following natural question: what is the solution for the problem with $r_1 > r_2 > \cdots > r_m$ when the goal is to maximize the average rewards per unit time subject to the constraint that the blocking probability for type j customers does not exceed a given number? This is a particular case of the problem considered in this paper when $c_j = {\lambda_j}^{-1}$ and $c_i = 0$, $i \neq j$. Therefore,

$$r'_i = \begin{cases} r_j + \bar{u}_1/\lambda_j, & \text{if } i = j; \\ r_i, & \text{otherwise.} \end{cases} \tag{2}$$

Since $\bar{u}_1 \geq 0$, in view of (2), we have $r'_j \geq r_j$. Thus Corollary 2 below implies that, when $r_1 > r_2 > \cdots > r_m$, for a feasible problem, there exists an optimal 1-randomized trunk reservation policy with $M_1 \geq \cdots \geq M_{j-1} \geq M_{j+1} \geq \cdots \geq M_m$ and $M_j \geq M_{j+1}$. In other words, the threshold for type j customers can increase. In the particular case when $j = 1$, studied by Feinberg and Reiman [8], the orders $r'_1 > r'_2 > \cdots > r'_m$ and $r_1 > r_2 > \cdots > r_m$ coincide and therefore we have that $M_1 = N - 1$ is the highest threshold. If the constraint limits the blocking probability for several customer types pooled together, then the optimal policy also has a simple structure described in Corollary 5. This corollary implies that, if we have the constraint on the blocking probability of the several most profitable customer types pooled together, then the optimal policy is again a 1-randomized trunk reservation policy consistent with r.

We remark that our main result, Theorem 3, is a stronger statement than just the existence of an optimal 1-randomized trunk reservation policy, which is Corollary 2. We prove that any randomized optimal stationary policy, that uses a randomization procedure in at most one state, has a 1-randomized trunk reservation form. We recall that for 1-constrained semi-Markov or continuous-time Markov decision processes describing the problem considered in this paper, when the problem is feasible, there exists a randomized stationary optimal policy that uses a randomization procedure in at most one state; see Feinberg [6] or [7].

In addition to the classical Miller's [19] problem formulation and its constrained version studied by Feinberg and Reiman [8], various versions and generalizations of the admission problem have been studied in the literature. Lippman [17] studied a problem with an infinite number of customer classes. Other early references can be found in the surveys by Crabill, Gross, and Magazine [4] and by Stidham [25]. Nguyen [20] considered a queueing system with two types of arrivals: one type is generated by a Poisson process and the other is an overflow process of an $M/M/m/m$ queue. Carrizosa, Conde and Munoz-Marquez [3] studied an M/G/c/loss queue with different service distributions for different customer types. There the control parameter is the probability to accept an arrival, if the system has available space. This probability depends on the arrived customer type and does not depend on the state of the system. Lewis, Ayhan and Foley [13], [14] investigated bias optimality. Lewis [12] studied a dual admission control scheme to an $M/M/1$ queue with the service times depending on customer types. Lin and Ross [15], [16] considered optimal admission control policies with a gatekeeper for M/M/1/loss queues where the gatekeeper can not know the busy-idle status of the server. Admission control problems with customers requiring multiple servers were considered by Kelly [10], Key [11], Ross and Yao [24], Papastavrou,Rajagopalan and Kleywegt [21], and Altman, Jimenez and Koole [1]. If service times depend on customer types or different types of customers require different numbers of servers, trunk reservation may not be optimal. Examples can be found in Ross [23, p. 137] and Altman, Jimenez and Koole [1]. However, a trunk reservation policy is asymptotically optimal under certain conditions; Hunt and Laws [9], Puhalskii and Reiman [22]. The survey on applications of MDP in communication networks by Altman [2] provides additional references on admission control.

This paper is organized as follows. We formulate the problem in Section II. Following Feinberg and Reiman [8], we formulate the problem as a unichain semi-Markov decision problem with one constraint and with finite state and action sets. We remark that the problem can also be formulated as a continuous-time Markov Decision Process; Miller [19]. In section II we also formulate the linear program (LP) that identifies an optimal policy and explain the meaning of the constant $\bar{u}_1$ as an element of the dual solution to this LP.

Section III is devoted to main results. Previously Feinberg and Reiman [8, Corollary 3.7] proved that if $r_1 > r_2 > \cdots > r_m$ then any optimal stationary policy has a trunk reservation form for an unconstrained problem. We study the unconstrained problem when $r_1 \geq r_2 \geq \cdots \geq r_m$. This case is important because even if we assume that $r_1 > r_2 > \cdots > r_m$, it is possible that $r'_i = r'_j$ in (1) for some $i, j = 1, \ldots, m$. Theorem 1 establishes the link between optimal policies and appropriate LPs. We describe the geometrical structure of the optimal solutions of related LPs in Theorem 2. Namely, we show that the optimal LP solution, which corresponds to a 1-randomized optimal policy, is a convex combination of two vectors corresponding to (non-randomized) stationary policies, and all these three policies differ at most at one point. In addition, the two non-randomized stationary policies are optimal for Lagrangian relaxation of the original problem. In Theorem 3 we present the structure of the optimal policies and in the following corollaries, various applications in Quality of Service constraint are considered and structures of optimal policies for these cases are formulated.

II. PROBLEM FORMULATION

We consider a controlled queue which is a generalization of an $M/M/c/N$ queue. The queue has space for at most N customers, where N is a given integer. When there are n customers in the queue, the departure rate is μ_n, $n = 1, \ldots, N$. The numbers μ_n, $n = 1, \ldots, N$, satisfy the condition $\mu_{n-1} \leq \mu_n$, where $\mu_0 = 0$ and $\mu_1 > 0$. In particular, for an $M/M/c/N$ queue, for some $\mu > 0$

$$\mu_i = \begin{cases} i\mu, & \text{if } i = 1, \dots, c, \\ c\mu, & \text{if } i = c+1, \dots, N. \end{cases}$$

There are $m = 1, 2, \dots$ types of customers arriving according to m independent Poisson processes with the intensities λ_i, $i = 1, \dots, m$, respectively. When a customer arrives, its type becomes known. When there are N customers in the system, the system is full and new arrivals are lost. If the system is not full, upon an arrival of a new customer, a decision of accepting or rejecting this customer is made. A positive reward r_i is collected upon completion of serving an accepted type i customer. A nonnegative cost c_i incurs due to the rejection or lost of an arriving type i customer. The service time of a customer does not depend on the customer type. Unless otherwise specified, we do not assume that $r_1 \geq r_2 \geq \cdots \geq r_m$.

Our goal is to maximize the average rewards the system collects per unit time, subject to the constraint on the average costs per unit time. In particular, we are interested in the problem when we want to maximize the average rewards per unit time subject to the blocking probability constraint for a certain type of customers. In the more particular case, when $r_1 > \cdots > r_m$ and the constraint is the blocking probability for type 1 customers, this problem was studied in Feinberg and Reiman [8].

Following Feinberg and Reiman [8], we model the problem via a semi-Markov decision process (SMDP) with the state space $I = \{0, 1, \dots, N-1\} \bigcup (\{0, 1, \dots, N\} \times \{1, \dots, m\})$. If the state of the system is $n = 0, \dots, N-1$, this means that a departing customer leaves n customers in the system. The state (n, i) means that an arrival of type i sees n customers in the system. Thus, the state space I represents the departure and arrival epochs.

The action set $A = \{0, 1\}$. For $n = 0, \dots, N-1$, and $i = 1, \dots, m$, we set $A(n, i) = A = \{0, 1\}$, where the action 0 means that the type i arrival should be rejected and the action 1 means that it should be accepted. We also set $A(N, i) = \{0\}$. In any state $n = 0 \dots, N-1$ there is no decision chosen. So, we set $A(n) = \{0\}$.

Let $\tau(s, a)$ denote the average time that the system spends in a state $s \in I$ if an action $a \in A(s)$ is chosen in this state. Let $p(s, s', a)$ be the transition probability from the state s to s' if action $a \in A(s)$ is chosen. For the notation simplicity, we write $\tau(n)$ and $p(n, s)$ instead of $\tau(n, 0)$ and $p(n, s, 0)$ respectively for $n = 0, \dots, N-1$, $s \in I$. Denote $\Lambda = \sum_{i=1}^{m} \lambda_i$.

We have $\tau(n) = (\mu_n + \Lambda)^{-1}$, where $n = 0, \dots, N-1$. Also, for $i = 1, \dots, m$,

$$\tau((n,i),a) = \begin{cases} \tau(n), & \text{if } \quad a = 0, \\ & \qquad n = 0, \dots, N, \\ \tau(n+1), & \text{if } \quad a = 1, \\ & \qquad n = 0, \dots, N-1. \end{cases}$$

For $n = 0, \dots, N-1$, $i = 1, \dots, m$,

$$p(n, s) = \begin{cases} \mu_n \tau(n), & \text{if } \quad s = n-1, \\ \lambda_i \tau(n), & \text{if } \quad s = (n, i), \\ 0, & \qquad \text{otherwise,} \end{cases}$$

and

$$p((n,i), s, a) = \begin{cases} p(n, s), & \text{if } \quad a = 0, \\ p(n+1, s), & \text{if } \quad a = 1. \end{cases}$$

For simplicity, let the reward be collected when an arrival is accepted. Therefore,

$$r(s, a) = \begin{cases} r_i, & \text{if } \quad s = (n, i), \\ & \qquad n = 0, \dots, N-1, a = 1, \\ 0, & \qquad \text{otherwise,} \end{cases}$$

and

$$c(s, a) = \begin{cases} c_i, & \text{if } \quad s = (n, i), \\ & \qquad n = 0, \dots, N, a = 0, \\ 0, & \qquad \text{otherwise.} \end{cases}$$

In summary, we have defined an SMDP with the state space I; action space A, set $A(s)$ of available actions at states $s \in I$; transition probability $p(s, s', a)$; average sojourn time $\tau(s, a)$ in a state $s \in I$ after an action a is chosen; reward function $r(s, a)$ and cost function $c(s, a)$.

Let $t_0 = 0$. If t_n is defined for some $n = 0, 1, \dots,$ we define t_{n+1} as the time epoch of either the next departure or arrival, whichever occurs first. Therefore $0 = t_0 < t_1 < \cdots$ is the sequence of jump epochs when the state of the system changes. A strategy π, which may be randomized and past-dependent, assigns actions a_n at epoch t_n to control the system. We define the long-run average rewards earned by the system as

$$W(z, \pi) = \liminf_{t \to \infty} t^{-1} \mathbb{E}_z^{\pi} \sum_{n=0}^{N(t)-1} r(x_n, a_n)$$

and the long-run average cost of the system as

$$C(z, \pi) = \limsup_{t \to \infty} t^{-1} \mathbb{E}_z^{\pi} \sum_{n=0}^{N(t)-1} c(x_n, a_n),$$

where z is an initial state, π is a strategy, x_n is the state at epoch t_n, $\mathbb{E}_z^{\pi}$ is the expectation operator for the initial state z and the strategy π, and $N(t) = \max\{n : t_n \leq t\}$ is the number of jumps by the epoch t.

A strategy is called a randomized stationary policy if assigned actions a_n depend only on the current state x_n. In addition, if a_n is a deterministic function of x_n, the corresponding strategy is called a stationary policy.

According to [8, p.471], the Unichain Condition holds for this model. The Unichain Condition means that any randomized stationary policy defines a Markov chain on the system's state space with one ergodic class and a (possibly empty) set of transient states. Under this condition, the objective functions $W(z, \phi)$ and $C(z, \phi)$ do not depend on the initial state $z \in I$ when ϕ is a randomized stationary policy. So, we shall write $W(\phi)$ and $C(\phi)$ instead of $W(z, \phi)$ and $C(z, \phi)$ respectively when ϕ is a randomized stationary policy. According to [6, Theorem 8.1(iv)], if the Unichain Condition holds and the problem (3),(4) is feasible for some z, then there exists a randomized stationary policy which is optimal for any initial state z and the value does not depend on z. Thus, our problem can be modelled as the following optimization problem with a randomized stationary policy ϕ as the variable:

$$\begin{aligned} &\text{maximize} \quad W(\phi) && (3)\\ &\text{subject to} \quad C(\phi) \le G. && (4)\end{aligned}$$

Since an action can be chosen only at the arrival epochs, a randomized stationary policy ϕ for our problem can be defined by $\phi(n,i)$, $n = 0,\dots,N-1$, $i = 1,\dots,m$: the probability of accepting an arrival of type i when there are n customers in the system.

A randomized stationary policy ϕ is called k-randomized stationary, $k = 0,1,2,\dots,$ if the number of states (n,i) where $0 < \phi(n,i) < 1$ is less than or equal to k. The notions of stationary and 0-randomized stationary policies coincide.

III. MAIN RESULTS

A. Unconstrained Problem

The major difference of our discussion here to the results in Feinberg and Reiman [8] is that, instead of considering strict inequalities among rewards, we allow different classes to have equal rewards. This is due to the reason that even if we assume that $r_1 > r_2 > \cdots > r_m$, it is possible that $r'_i = r'_j$ in (1) for some $i,j = 1,\dots,m$. The following lemmas cover the case $r_1 \ge \cdots \ge r_m > 0$. However, being motivated by constrained problems, for which it is possible that $r'_i < r'_{i+1}$, we do not specify these inequalities in the following lemmas.

Lemma 1. *Any stationary optimal policy φ for the unconstrained problem (3) is a trunk reservation policy consistent with the rewards r_i.*

Lemma 2. *Consider any randomized stationary optimal policy ϕ for the unconstrained problem (3). (i) For any i,j such that $r_i > r_j$, we have*

$$\phi(n,i) \ge \phi(n,j), n = 0,\dots,N-1, i,j = 1,\dots,m. \quad (5)$$

For each $n = 0,\dots,N-1$, if there exist $j = j_1, j_2,\dots,j_s$ such that $0 < \phi(n,j) < 1$ then $r_{j_1} = r_{j_2} = \cdots = r_{j_s}$.

(ii) There exists at least one customer type, say type j such that

$$\phi(n,j) = 1, n = 0,\dots,N-1; \quad (6)$$

(iii)

$$\phi(n,j) \ge \phi(n+1,j), n = 0,\dots,N-2, j = 1,\dots,m, \quad (7)$$

and for each $j = 1,\dots,m$, all probabilities $\phi(n,j)$, $n = 0,\dots,N-1$, except at most one, are equal to either 0 or 1.

B. Justification of the LP formulation

Consider the following Linear Program (LP) with variables (x,P), where $x = \{x(n,i) : n = 0,\dots,N-1, i = 1,\dots,m\}$, $P = (P_0,\dots,P_N)$.

$$\text{maximize}_{x,P} \sum_{i=1}^{m} \lambda_i r_i \sum_{n=0}^{N-1} x(n,i) \quad (8)$$

subject to

$$\sum_{i=1}^{m} \lambda_i c_i (1 - \sum_{n=0}^{N-1} x(n,i)) \le G, \quad (9)$$

$$\sum_{i=1}^{m} \lambda_i x(n,i) = \mu_{n+1} P_{n+1}, \quad (10)$$

$$n = 0,\dots,N-1,$$

$$\sum_{n=0}^{N} P_n = 1, \quad (11)$$

$$0 \le x(n,i) \le P_n, n = 0,\dots,N-1, \quad (12)$$

$$i = 1,\cdots,m.$$

For a vector (x,P) satisfying (9)-(12), consider a randomized stationary policy ϕ such that:

$$\phi(n,i) = \begin{cases} x(n,i)/P_n, & \text{if } P_n > 0, \\ & n = 0,\dots,N-1, \\ & i = 1,\dots,m; \\ \text{arbitray}, & \text{otherwise}. \end{cases} \quad (13)$$

Theorem 1. *(i) A randomized stationary policy ϕ is feasible for the problem (3), (4) if and only if (13) holds for a feasible vector (x,P) of the LP (8)-(12).*

(ii) If (x,P) is an optimal solution of the LP (8)-(12) then $P_n > 0$ for all $n = 0,1,\dots,N$.

(iii) A randomized stationary policy ϕ is optimal for the problem (3), (4) if and only if

$$\phi(n,i) = x(n,i)/P_n, n = 0,\dots,N-1, i = 1,\dots,m, \quad (14)$$

for an optimal solution (x,P) of the LP (8)-(12). In addition, if (x,P) is a basic optimal solution of the LP (8)-(12), then the policy ϕ defined in (14) is 1-randomized stationary optimal.

If $G \ge \sum_{i=1}^{m} \lambda_i c_i$, Theorem 1 implies the following result.

Corollary 1. *(i) If (x,P) is an optimal solution of the LP (8), (10)-(12) then $P_n > 0$ for all $n = 0,1,\dots,N$.*

(ii) A randomized stationary policy ϕ is optimal for the problem (3) if and only if (14) holds for an optimal solution (x,P) of the LP (8), (10)-(12). In addition, if (x,P) is a basic optimal solution of the LP (8), (10)-(12), then the policy ϕ defined in (14) is non-randomized stationary optimal.

C. Lagrangian Relaxation and Geometric Properties of Optimal Policies

In view of (11) and (12), the feasible region of the LP (8)-(12) is bounded. Therefore, this LP has an optimal solution, if it is feasible. If the LP (8)-(12) is feasible, we consider an arbitrary optimal dual solution $(\bar{u},\bar{v})$, $\bar{u} = (\bar{u}_1,\dots,\bar{u}_{2mN+1})$

and $\bar{v} = (\bar{v}_1, \ldots, \bar{v}_{N+1})$, where $\bar{u}$ corresponds to all inequality constraints and $\bar{v}$ corresponds to equality constraints, and introduce the following LP:

$$\begin{aligned} \text{maximize}_{x,P} \quad & \sum_{i=1}^{m} \lambda_i (r_i + \bar{u}_1 c_i) \sum_{n=0}^{N-1} x(n,i) - \\ & \bar{u}_1 (\sum_{i=1}^{m} \lambda_i c_i - G) \\ \text{subject to} \quad & (10) - (12). \end{aligned} \tag{15}$$

Here $\bar{u}_1$ is also called the Lagrangian multiplier with respect to constraint (9). Notice that most of the contemporary LP solvers use interior point methods and calculate the primal and dual solutions simultaneously, therefore we do not formulate the dual LP in this paper.

Lemma 3. *If the LP (8)-(12) is feasible then: (i) any optimal solution of the LP (8)-(12) is an optimal solution of the LP (15), and (ii) the optimal values of objective functions for these two LPs are equal.*

Lagrangian relaxation in general refers to using weak duality theorem in obtaining lower bounds for non-linear programming problem. We use this term "relaxation" here in the sense that the optimal set is enlarged after the transformation, although the optimal value remain the same.

We notice that for any randomized stationary policy ϕ there is a unique solution P^{ϕ} of the following birth-and-death equations:

$$(\sum_{i=1}^{m} \lambda_i \phi(n,i)) P_n = \mu_{n+1} P_{n+1}, \quad n = 0, \ldots, N-1, \tag{16}$$

$$\sum_{n=0}^{N} P_n = 1. \tag{17}$$

In other words, P_n^{ϕ} is the limiting probability that there are n customers in the system when the randomized stationary policy ϕ is used.

In addition, we define

$$x^{\phi}(n,i) = \phi(n,i) P_n^{\phi}, \quad n = 0, \ldots, N-1, \ i = 1, \ldots, m. \tag{18}$$

Then (x^{ϕ}, P^{ϕ}) satisfies (10)-(12) and therefore it is a feasible solution of the LP (8), (10)-(12). In view of Theorem 1(i), a randomized stationary policy ϕ is feasible for the problem (3), (4) if and only if (x^{ϕ}, P^{ϕ}) is a feasible solution of the LP (8)-(12). In addition, according to Theorem 1(iii), a randomized stationary policy is optimal for the problem (3), (4) if and only if (x^{ϕ}, P^{ϕ}) is optimal for the LP (8)-(12). In particular, according to Corollary 1, a randomized stationary policy ϕ is optimal for the unconstrained problem (3) if and only if the vector (x^{ϕ}, P^{ϕ}) is optimal for the LP (8), (10)-(12).

The following theorem links geometrically the optimal solutions of the LP (8)-(12) to feasible vectors of the LP (8), (10)-(12).

Theorem 2. *Let ϕ be a 1-randomized stationary optimal policy for the problem (3), (4). If there exists a state (n_0, i_0) with $0 < \phi(n_0, i_0) < 1$, consider two stationary policies ϕ' and ϕ'' that coincide with ϕ at all states except the state (n_0, i_0) and $\phi'(n_0, i_0) = 0$, $\phi''(n_0, i_0) = 1$. Then for some $0 < \alpha < 1$,*

$$(x^{\phi}, P^{\phi}) = \alpha (x^{\phi'}, P^{\phi'}) + (1 - \alpha)(x^{\phi''}, P^{\phi''}). \tag{19}$$

D. Main Theorem and Its Applications

Theorem 3. *Any 1-randomized stationary optimal policy for the problem (3), (4) is a 1-randomized trunk reservation policy, which is consistent with the reward function $r'_i = r_i + \bar{u}_1 c_i$, $i = 1, \ldots, m$, where $\bar{u}_1 \geq 0$ is the Lagrangian multiplier with respect to constraint (9).*

Since our problem is an average reward SMDP with one constraint and the Unichain Condition holds, due to [6], if a feasible policy exists, then there exists a 1-randomized stationary optimal policy. Therefore, the previous theorem implies the the following corollary.

Corollary 2. *If the problem (3), (4) is feasible, then there exists an optimal 1-randomized trunk reservation policy which is consistent with r'.*

Let

$$c_i = \begin{cases} 1/\lambda_j, & i = j, \\ 0, & \text{otherwise}. \end{cases} \tag{20}$$

According to [8, p. 471], for the costs c_i defined by (20), the average cost $C(z, \pi)$ has the meaning of the blocking probability for type j customers. Therefore, the problem of maximizing the average rewards per unit time subject to the constraint that the blocking probability for type j customers does not exceed q is equivalent to the problem (3), (4) with the cost function c defined in (20).

The following corollary describes the structure of optimal policies when the objective is to maximize the average rewards per unit time subject to the constraint on the blocking probability for type j customers.

Corollary 3. *Consider a special case of the problem (3), (4) with the constraint on the blocking probability of type j customers, $j = 1, \ldots, m$. If this problem is feasible then any 1-randomized stationary optimal policy is 1-randomized trunk reservation consistent with the reward function r' defined in (2) and satisfying the properties that $r'_i = r_i$ if $i \neq j$ and $r'_j \geq r_j$.*

In particular, when $j = 1$, Corollary 3 implies the following statement.

Corollary 4. *Consider a special case of the problem (3), (4) with the constraint on the blocking probability of the most profitable customers (type 1). If this problem is feasible then any 1-randomized stationary optimal policy is a 1-randomized trunk reservation policy consistent with the rewards r_i.*

In particular, for the case $r_1 > r_2 > \ldots > r_m$, Corollary 4 coincides with the main result in Feinberg and Reiman [8].

If the cost constraint limits the blocking probability for several customer types pooled together, say, for customer types belonging to a set J, $J \subset \{1, \ldots, m\}$, then we define $\Lambda_J = \sum_{j \in J} \lambda_j$ and

$$c_i = \begin{cases} 1/\Lambda_J, & \text{if } j \in J, \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

Then the combined blocking probability for customers in the set J under policy π and initial state z is $C(z, \pi)$ with the function c_i defined by (21).

The following corollary describes the structure of optimal policies when the objective is to maximize the average rewards per unit time subject to the constraint on the combined blocking probability for several customer types.

Corollary 5. *Consider a special case of the problem (3), (4) with the constraint on the combined blocking probability for customer types belonging to a set J, $J \subset \{1, \ldots, m\}$. If this problem is feasible then any 1-randomized stationary optimal policy is a 1-randomized trunk reservation policy consistent with a function r', where*

$$r'_i = \begin{cases} r_i + \bar{u}_1/\Lambda_J, & \text{if } i \in J, \\ r_i, & \text{otherwise,} \end{cases} \tag{22}$$

and it satisfies the properties: (i) $r'_i = r_i$ if $i \notin J$, (ii) $r'_i \geq r_i$ if $i \in J$, and (iii) $r'_i \geq r'_j$ if $i, j \in J$ and $r_i \geq r_j$.

IV. Further Results on Multiple Constraint Problems

Further more, we considered the admission control problem with multiple constraints and similar results hold. We briefly give the formulation and main results here. For each customer type $i = 1, 2, \ldots, m$, where m is the number of customer types, besides the Poisson arrival rate λ_i, a reward r_i, there is a K dimension penalty vector $C_i = (c_i^1, c_i^2, \ldots, c_i^K)$ paid to a rejected type i customer. The goal is to maximize the average rewards per unit time subject to K constraints that the average penalty vector per unit time does not exceed a certain constant vector. That is, we consider the following optimization problem:

$$\text{maximize} \quad W_0(\phi) \tag{23}$$

$$\text{subject to} \quad W_k(\phi) \leq G_k, \quad k = 1, \ldots, K. \tag{24}$$

Theorem 4. *Any K_m-randomized stationary optimal policy for the problem (23), (24) is a K_m-randomized trunk reservation policy, which is consistent with the reward function $r'_i = r_i + \bar{u}^K C_i$, $i = 1, \ldots, m$, where $\bar{u}^K = (\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_K) \geq 0$ is the vector of Lagrangian multipliers.*

Such problems arise, for example, when the goal is to maximize the average rewards per unit time where each type of customers has its own quality of service (QoS) constraint.

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