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Levitation and Propulsion Control of a Magnetic Levitation System with Two Degrees of Freedom

R. Castro-Linares and D. Estrada-Durán

Abstract— **The design and implementation of a control scheme for the levitation height and angular position reference tracking in a two degrees of freedom levitation system is presented. The levitation system prototype is described and a nonlinear model that reflects its dynamic behaviour is obtained. In particular, the force to current relationship that is needed to implement the control scheme is proposed and experimentally validated. The control scheme design is based on the sliding mode technique and it was implemented on the prototype, thus allowing to evaluate its peformance. The limitations of the power circuits associated to the prototype were also evaluated through some experiments.**

I. INTRODUCTION

During the last few years, the search of new technological alternatives that improve the functional acting of the traditional systems of transport has propitiated an increase of the study of the levitation systems (see for example [5], [6]).

In this work, a magnetic levitation system with two degrees of freedom is presented. The system was designed and built at the Department of Electrical Engineering of CINVESTAV-IPN, Mexico City, Mexico, with the purpose of studying this kind of systems and improving, at the same time, its performance via the design and implementation of diverse control methods. The system is similar to the one developped at the Laboratory of Automation and Control of the Polythecnic School at the University of São Paulo, Brazil [1], [2], with some important differences between them. For example, in the system built at CINVESTAV-IPN the meausurement of the height and angular position of the moving coil was improved by using optical shaft encoders. Also, the relationship that relates the propulsion and levitation forces to the currents applied to the fixed coils was experimentally found and validated; this allowed to obtain a more exact model of the system. In particular, this model was used to design and implement a control scheme, based on the sliding mode technique, in order to have reference tracking for the height and angular position of the moving coil. Some experiments were carried out to verify the performance of the closed-loop system. These experiments allowed to evaluate the limitations due to the power circuits associated to the system.

The paper is organized as follows. In section II, the magnetic levitation system is described. The system dynamic equations and the force to current relationship is presented in section III. In section IV, the control scheme design is given. Some experiments that were carried out are described in section V. Finally, a conclusion is presented in section V.

R. Castro-Linares is with the Department of Electrical Engineering, CINVESTAV-IPN, México D.F, México rcastro@cinvestav.mx
D. Estrada-Durán is with the Department of Electrical Engineering, CINVESTAV-IPN,México D.F, México destrada@cinvestav.mx

II. SYSTEM DESCRIPTION

The magnetic levitation and propulsion system is conformed by 39 air nucleus coils that form a circumference, called *fixed coils*. There is another air nucleus coil that can be moved over the fixed coils. This coil, called *moving coil*, is placed at the extremity of an aluminum bar supported in the middle by a pivot with two degrees of freedom. On the opposite extremity of the bar there is a counterweight whose position can be adjusted so that the moving coil may be magnetically attracted or repeled. In the system described here, the counterweight was placed near the pivot so that a repulsion force is needed to levitate the moving coil; a photograph of the system appears in Fig. 1 while a schematic diagram is shown in Fig. 2. Thus, in order to have levitation and propulsion a constant current is applied to the moving coil and convenient currents are generatded and applied, by means of a control scheme, to the fixed coils positioned next to the moving coil. In this work, six of the fixed coils were used to generate those forces.

The magnetic system is controlled by a personal computer (PC) and a data acquisition board installed in the computer. The data acquisition board used is the 6024E electronic board together with the 6711 analogue output board, both manufactured by National Instruments, that have six analog outputs and 2 digital input ports of 8 bits each one. The height of levitation and the position of the moving coil are measured by means of two optical shaft encoders, US Digital 2048 [7], which have a low weight and negligeable shaft viscous friction (due to the ball bearings included in the encoders); the signals from the encoders are connected to one of the digital input ports. The analog outputs of the 6711 board have a resolution of 24 $bits$ and a voltage range of 0 $volts$ to 2.5 $volts$. The voltages generated by the the computer through these analog outputs are converted into proportional current values with the aid of signal conditioning and power electronic circuits. Only six current fixed coils are controlled at any instant in order to have levitation and propulsion in the moving coil.

A detailed description of the system as well as the hardware and software tools developed can be found in [4].

III. SYSTEM MODEL

A. Dynamic Equations

The dynamic equations of the system described in section II were obtained using a Lagrangian approach. The general-

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Fig. 1. Photography of the system.

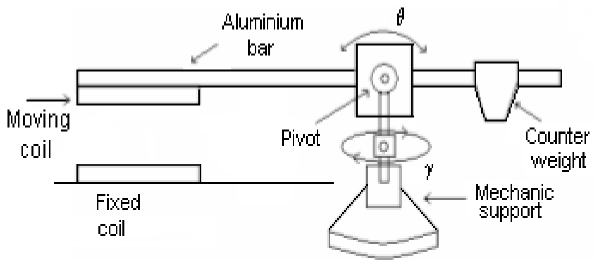

Fig. 2. Schematic view of the magnetic system.

ized coordinates of the system are

$$q = [q_1 \ q_2]^T = [\gamma \ \theta]^T$$

Thus the kinetic energy is

$$\begin{aligned} T &= \frac{1}{2}[(m_1L_1^2 + m_2L_2^2 + m_3L_3^2)\dot{\theta}^2 \\ &\quad + (m_1L_1^2\cos^2\theta + m_2L_2^2\cos^2(\theta+180) \\ &\quad + m_3L_3^2\cos^2\theta)\dot{\gamma}^2] \qquad (1) \end{aligned}$$

while the potential energy is

$$V = m_1gL_1\sin\theta + m_2gL_2\sin(\theta+180) + m_3gL_3\sin\theta \quad (2)$$

where m_1 is the mass of the moving coil, m_2 is the mass of the counterweight, m_3 is the mass of the aluminum bar, L_1 is the longitude from the pivot center to the center of mass of the moving coil, L_2 is the longitude from the pivot center to the center of mass of the counterweight, L_3 is the longitude from the pivot center to center of mass of the aluminum bar and g is the gravity coefficient. γ is the angular displacement that determines the position of the moving coil and θ is the angular displacement that determines the levitation of this coil. F_γ is the *propulsion force* and F_θ is the *levitation force*. The Lagrangian is defined as

$$L = T - V$$

The system dynamic equations are then obtained from the Euler-Lagranmge equations with external generalized force

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} + K\dot{q} = \tau \qquad (3)$$

where

$$\tau = \begin{bmatrix} F_\gamma L_1 \\ F_\theta L_1 \end{bmatrix}$$

and $K\dot{q}$ is a term due to the viscous friction defined as

$$K\dot{q} = \begin{bmatrix} k_2 & 0 \\ 0 & k_1 \end{bmatrix}$$

with k_1 and k_2 being the viscous friction coefficients associated to the displacements θ and γ, respectively. The values of these coefficients are small due to the ball bearings used in the mechanism and the ones already included in the encoders, and were obtained experimentally. Finally, from (1) and (2), the Euler-Lagrange equation (3) takes the form

$$\begin{aligned} F_\gamma L_1 &= [(m_1L_1^2 + m_3L_3^2)(\cos^2\theta) \\ &\quad + m_2L_2^2\cos^2(\theta+180)]\ddot{\gamma} \\ &\quad - [(m_1L_1^2 + m_3L_3^2)(\sin 2\theta) \\ &\quad + m_2L_2^2\sin 2(\theta+180)]\dot{\gamma}\dot{\theta} \\ &\quad + k_2\dot{\gamma} \qquad (4) \end{aligned}$$

$$\begin{aligned} F_\theta L_1 &= \ddot{\theta}(m_1L_1^2 + m_2L_2^2 + m_3L_3^2) \\ &\quad + \frac{1}{2}\dot{\gamma}^2\sin 2\theta(m_1L_1^2 + m_3L_3^2) \\ &\quad + \frac{1}{2}m_2L_2^2\dot{\gamma}^2\sin 2(\theta+180) \\ &\quad + g\cos\theta(m_1L_1 + m_3L_3) \\ &\quad + m_2gL_2\cos(\theta+180) \\ &\quad + k_1\dot{\theta} \qquad (5) \end{aligned}$$

A state space representation for these dynamic equations can be found by defining the state as $x = [x_1 \ x_2 \ x_3 \ x_4]^T = [\gamma \ \dot{\gamma} \ \theta \ \dot{\theta}]^T$ together with the input vector $u = [F_\gamma \ F_\theta]^T$. This state space representation is expressed as

$$\dot{x} = f(x) + g(x)u \qquad (6)$$

where

$$f(x) = [x_2 \ f_2(x) \ x_4 \ f_4(x)]^T$$

$$g(x) = \begin{bmatrix} 0 & 0 \\ L_1/a(x_3) & 0 \\ 0 & 0 \\ 0 & L_1/b \end{bmatrix}$$

with

$$
\begin{aligned}
f_2(x) &= \frac{1}{a(x_3)}[[(m_1L_1^2 + m_3L_3^2)\sin(2x_3) \\
&\quad +m_2L_2^2 \sin 2(x_3 + 180)]x_4x_2 \\
&\quad -k_2x_2]
\end{aligned}
$$

$$
\begin{aligned}
f_4(x) &= \frac{1}{b}[\frac{1}{2}[-(m_1L_1^2 + m_3L_3^2)x_2^2 \sin(2x_3) \\
&\quad -m_2L_2^2x_2^2 \sin 2(x_3 + 180)] \\
&\quad -g\cos x_3(m_1L_1 + m_3L_3) \\
&\quad -m_2gL_2\cos(x_3 + 180) - k_1x_2]
\end{aligned}
$$

The scalar function $a(x_3)$ and the coefficient b are given by

$$
\begin{aligned}
a(x_3) &= (m_1L_1^2 + m_3L_3^2)(\cos^2 x_3) \\
&\quad +m_2L_2^2 \cos^2(x_3 + 180) \\
b &= m_1L_1^2 + m_2L_2^2 + m_3L_3^2
\end{aligned}
$$

This state representation was used for the control design described in the next section.

B. Force to Current Relationship

The interaction of the currents applied in the fixed coils with the moving coil through forces was experimentally obtained. For simplicity, a linear relationship between the currents applied and the forces generated was assumed. Such a consideration is also made in [2] where the variation of the mutual inductances between the fixed coils and the moving coil is used. The linear relationship is expressed as

$$
\begin{aligned}
F_\theta &= G_{\theta 1}i_1I + G_{\theta 2}i_2I + G_{\theta 3}i_3I \\
&\quad +G_{\theta 4}i_4I + G_{\theta 5}i_5I + G_{\theta 6}i_6I \\
F_\gamma &= G_{\gamma 1}i_1I + G_{\gamma 2}i_2I + G_{\gamma 3}i_3I \\
&\quad +G_{\gamma 4}i_4I + G_{\gamma 5}i_5I + G_{\gamma 6}i_6I \qquad (7)
\end{aligned}
$$

where i_1, i_2,...,i_6 are the currents passing through each fixed coil and I is a constant current passing through the moving coil. $G_{\theta 1}$, $G_{\theta 2}$,...,$G_{\theta 6}$ are the the *levitation coefficients* associated to the levitation force F_θ while $G_{\gamma 1}$, $G_{\gamma 2}$,...,$G_{\gamma 6}$ are the the *propulsion coefficients* associated to the propulsion force F_γ. These coefficients were experimentally obtained by measuring the levitation and propulsion forces for different sets of current values at the fixed coils. The force F_γ was measured with the aid of a dynamometer placed at the extreme of the bar, where the moving coil was held. Then, the fixed coils were energized in a range form 0 to 0.5 $amperes$ keeping the current through the moving coil with a constant value of 0.5 $amperes$. For the measurement of the force F_θ the moving coil (also energized with a fixed current of 0.5 $amperes$) was placed above a set of 3 fixed coils that were energized so that levitation is produced. Once the moving coil is at an steady state position, the currents through the fixed coils were changed within the same range of 0 to 0.5 $amperes$ measuring, at the same time, the angular dispalcement θ. These measurements were then substituted into equation (5) to obtain the values of F_θ.

The linear relationship (7) can be written in a matrix format as

$$F = IGi \qquad (8)$$

where $F = [F_\theta \; F_\gamma]^T$, $i = [i_1 \; i_2 \cdots i_6]^T$ and

$$
G = \begin{bmatrix} G_{\gamma 1} & G_{\theta 1} \\ G_{\gamma 2} & G_{\theta 2} \\ G_{\gamma 3} & G_{\theta 3} \\ G_{\gamma 4} & G_{\theta 4} \\ G_{\gamma 5} & G_{\theta 5} \\ G_{\gamma 6} & G_{\theta 6} \end{bmatrix}^T \qquad (9)
$$

A numerical method, given by the software package MATLAB, was used to obtain the pseudoinverse of current vector in such a way that an approximated value of the matrix G, G_{ap}, was computed. This is

$$G_{ap} = I^{-1}Fi^T(ii^T)^{-1} \qquad (10)$$

where the measured values of the forces F_θ and F_γ together with the values of the currents that generate them was used. Thus

$$
G_{ap} = \begin{bmatrix} 0.0134237 & 0.2960997 \\ 0.0135476 & 0.2988329 \\ 0.0134547 & 0.2967830 \\ 0.0133101 & 0.2935942 \\ 0.0132688 & 0.2926832 \\ 0.0132482 & 0.2922276 \end{bmatrix}^T \qquad (11)
$$

Since the control scheme described in the next section computes the values of the forces F_γ and F_θ needed to have propulsion and levitation of the moving coil, the linear relationship (8) is used to obtain the corresponding values of the currents through the fixed coils.

IV. CONTROL DESIGN

In this section the design of a control scheme control that allowed to regulate the angular displacements γ and θ of the magnetic system is presented. The sliding mode technique was chosen for the design because of its applicability to general nonlinear systems and its ability to cope with system uncertainties such as modelling errors and disturbances (see, for example, [8], [9] and the references therein). Since it is desired to track given angular positions for the state variables

x_1 and x_3, the following switching vector function was proposed:

$$\sigma(e_1, e_2) = [\sigma_1(e_1)\ \sigma_2(e_2)]^T \tag{12}$$

with

$$\begin{aligned} \sigma_1(e_1) &= -k_{p1}e_1 - k_{d1}\dot{e}_1 \\ \sigma_2(e_2) &= -k_{p2}e_2 - k_{d2}\dot{e}_2 \end{aligned}$$

where $e_1 = x_1 - x_{d1}$ and $e_2 = x_3 - x_{d3}$ with x_{d1} and x_{d3} being reference signals. k_{p1}, k_{p2}, k_{d1} and k_{d2} are constant real coefficients. In accordance to the *equivalent control method* [3], when the system motion (6) is restricted to the switching surface defined by $\sigma(e_1, e_2) = 0$, the *equivalent control*, u_{eq}, needed to acomplish it is

$$u_{eq} = \begin{bmatrix} \frac{a(x_3)}{L_1 k_{d1}}(-k_{d1}f_2(x) - k_{p1}(x_2 - \dot{x}_{d1}) + k_{d1}\ddot{x}_{d1}) \\ \frac{b}{L_1 k_{d2}}(-k_{d2}f_4(x) - k_{p2}(x_4 - \dot{x}_{d3}) + k_{d2}\ddot{x}_{d3}) \end{bmatrix} \tag{13}$$

In order to complete the control design it is set [3]

$$u = u_{eq} + u_N \tag{14}$$

where u_N acts when $\sigma(e_1, e_2) \neq 0$. In this work u_N was selected as

$$u_N = \begin{bmatrix} \frac{a(x_3)}{L_1 k_{d1}}\kappa_1 sign(\sigma_1) \\ \frac{b}{L_1 k_{d2}}\kappa_2 sign(\sigma_2) \end{bmatrix} \tag{15}$$

where κ_1 and κ_2 are positive real constants. This selection assures the existence of *sliding mode* since the attractivity condition $\sigma^T\dot{\sigma} < 0$ is satisfied.

In Fig. 3 a block diagram is shown for the closed-loop propulsion and levitation control of the electromagnetic system where the block corresponding to the force current relationship described in section III (block FI) appears.

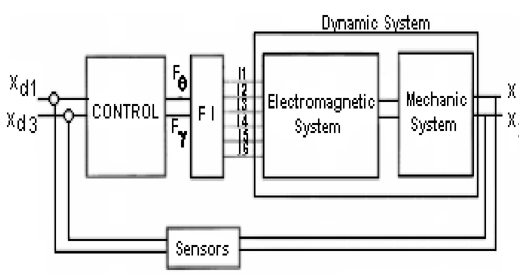

Fig. 3. Block diagram of the closed-loop propulsion and levitation control.

V. EXPERIMENTAL RESULTS

Some experiments were carried out on the magnetic system setup using the control (14) and the force to current relationship (8). The parameters of the system equations were measured directly from the setup and have the following values: $m_1 = 0.83\ Kg$, $m_2 = 4.63\ Kg$, $m_3 = 0.39\ Kg$, $L_1 = 0.72\ m$, $L_2 = 0.15\ m$, $L_3 = 0.29\ m$, $g = 9.81\ m/s^2$, $k_1 = 0.01979\ N - m/rad$, $k_2 = 0.01127\ N - m/rad$. The coefficients used in the control law were $k_{p1} = 80$, $k_{d1} = 5$, $k_{p2} = 100$, $k_{d2} = 7$, $\kappa_1 = 50$ and $\kappa_2 = 220$. Both reference signals were selected as step functions. The sampling period used was $0.001\ s$. Figures 4 and 5 show the dynamic behaviour of the angular displacements γ (rotation) and θ (levitation) when the moving coil was initially set at a position $\theta(0) = -0.016\ rad$. In order to first achieve levitation, the reference signals x_{d1} and x_{d3} were set to $0.0\ rad$ for approximately $1\ s$. Once levitation was obtained, the reference signal x_{d1} was set to $0.15\ rad$. Besides, two disturbance forces were applied to the moving coil, while it was moving, at $t = 3\ s$ and $t = 5.5\ s$. On may notice that the dynamics of γ and θ are coupled and the control scheme is acting on both of them all the time. It can also be noticed that the displacement γ oscilates close to the desired reference with almost no influence of the disturbance forces. Also, the displacement θ can be controlled to its desired reference, compensating the disturbance that is first applied; the second force disturbance can not be adequately compensated due, mainly, to its magnitude and the restrictions of the actuators (power circuits). Research and experimental work is being carried out in order to eliminate these restrictions.

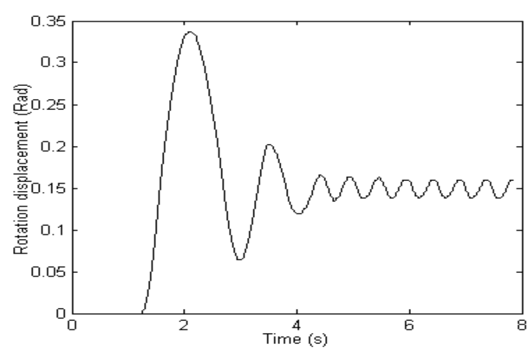

Fig. 4. Dynamic behaviour of γ (rotation).

VI. CONCLUSION

A control scheme design has been presented for a two degrees of freedom magnetic levitation system. The electronic sensors and the power circuits needed to drive the system

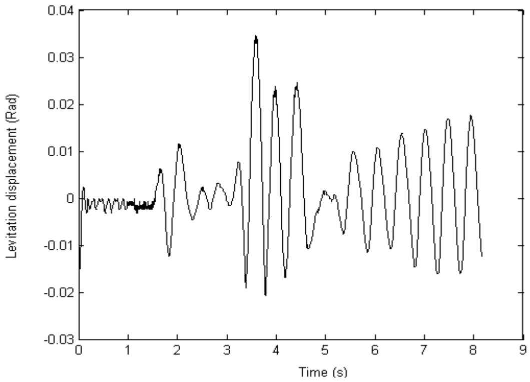

Fig. 5. Dynamic behaviour of θ (levitation).

and control it using a PC were described. A dynamic model of the system that includes the force to current relationship, which are basic for the design and implementation of the control scheme, is also given. The approach proposed for the control design is based on the sliding mode technique for the purpose of having reference tracking of the system rotation and levitation displacement angles. The proposed strategy was successfully applied to the magnetic system setup showing a good performance. Besides, external force disturbances could be compensated so that the moving coil could be kept within the desired reference values. The closed-loop system performance is only limited because of the physical restrictions of the actuators linked to the system.

VII. ACKNOWLEDGMENTS

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