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# Integration of Design and Control: A Robust Control Approach Using MPC

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*Abstract*- **This paper presents a method for integrating process control at the design stage. The objective function to be minimised is the sum of the capital cost, operating cost and the cost of variability in the controlled variable. An MPC strategy, designed together with the design of the process, was used to define a robust variability cost in the face of model uncertainty. Also, the robust stability and manipulated variable constraints were incorporated into the problem.**

**A multi-component distillation column was selected as a case study. The proposed integrated design methodology was compared to the traditional design methodology where the design and control steps are performed sequentially.**

**The optimization results show that the integrated design methodology leads to savings of up to 48% as compared to the traditional design methodology.**

## I. Introduction

Various attempts to integrate control considerations at the design stage have been proposed by many researchers (e.g. Luyben and Floudas, 1994; Mohideen et al., 1996; Bansal et al., 2000; Georgiadis et al., 2002; Blanco and Bandoni, 2003). The results were difficult to implement because:

i- These studies formulated the problem as multi-objective optimizations where the control cost and the design cost were accounted for by different objective functions requiring the solution of complex optimization problems. The integration of the control cost due to closed loop variability into one objective function together with capital and operating costs has not been investigated.

ii- the optimization problem has been based on a full nonlinear dynamic model of the process leading to complex and time consuming optimizations.

This paper, therefore, addresses the following issues:

1-The integration of the cost of variability in the controlled variable into one objective function together with the design cost given by the sum of capital and operating costs.

2-The development of a robust model that represents the nonlinear model by a family of linear models represented by a nominal model with model uncertainty. The worst case variability for this family of models is then quantified and its associated cost calculated. This term will be referred heretofore to as the robust variability cost.

In the traditional design approach the design of the process and the controller are performed in two separate sequential steps. Once the process has been designed, based on steady state models, a robust controller is designed to satisfy a robust-performance criteria which is related to closed loop stability and performance.

A novel aspect of the current work is that the variability cost due to imperfect control has been explicitly quantified and integrated into the objective function together with the operating and capital costs; the robust stability criteria is then used as an inequality constraint in the optimization problem. Also, by combining the robust variability cost together with capital and operating costs, it is possible to explicitly assess the effect of model uncertainty and controller design on process design.

An additional contribution of this study is related to the choice of the controller. In previous studies dealing with integration of control and design, multi-loop PI controllers were considered whereas control strategies such as Model Predictive Control (MPC), especially suited for multivariable problems, have not been addressed. Although the example discussed here consists of a single input single output (SISO) application, an MPC controller was used in the study to enable the future application of the proposed method to multivariable situations.

In the following sections, the proposed integrated design approach is introduced and applied to the design of a distillation column. The proposed approach is then compared to the traditional design approach where design and control are performed sequentially in two steps.

## II. Methodology

The integration of design and control objectives into one objective can be represented as follows:

$$\min_{d,\,u,\,c} \quad CC(d)+OC(u)+VC(c)$$
$$\text{s.t. } h(d, u, c)=0 \text{ and } g(d, u, c) \le 0$$

where CC(d) is the capital cost, OC(u) is the operating cost, VC(c) is the variability cost due to imperfect control, h(x) are equality constraints and g(x) are inequality constraints; d, u, and c represent vectors of design variables, operating variables and controller tuning parameters respectively. The calculation of the different terms in the above optimization problem is described in the following subsections. The capital and operating costs are process specific and they will be discussed later in the manuscript. On the other hand, the variability can be calculated by combining the nominal linear model plus uncertainty description with the MPC

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controller equations as explained in the following subsection.

*A. Process Variability under MPC control*

The process variability cost, VC(c) can be calculated in general form based on the representation of the nonlinear model by a nominal model with uncertainty and the MPC algorithm as follows.

The nonlinear nature of the process is captured by the so-called model uncertainty. The model uncertainty, $\ell_m$, can be described as shown in Fig 1.

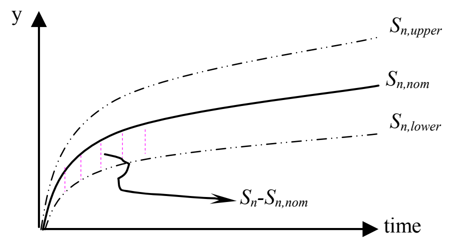


Fig 1. The uncertainy in the step response coefficients

Fig. 1 shows the step response to steps of different magnitudes or signs around a specific operating condition. The nonlinear nature of the process, resulting in different step responses, is quantified by the model uncertainty term, $\ell_m$, as follows:

$$\ell_m = \frac{S_n - S_{\mathrm{n,nom}}}{S_{\mathrm{n,nom}}} \qquad (1)$$

where $S_n$ and $S_{n,nom}$ are the actual and nominal step response coefficients respectively. The largest model uncertainty, $\ell_{m,\max}$, is the largest deviation between the nominal and upper or lower bound models shown in Fig 1 defined as follows:

$$\ell_{m,\max} = \pm\frac{S_n - S_{\mathrm{n,nom}}}{S_{\mathrm{n,nom}}}, \quad S_n = S_{n,upper} \quad or \quad S_{n,lower} \qquad (2)$$

According to the MPC algorithm, the controlled variable, **Y** can be predicted by the following equation.

$$\overline{\mathbf{Y}}(k) = \mathbf{M}_n\overline{\mathbf{Y}}(k-1) + \mathbf{S}_n\Delta\mathbf{u}(k-1) + \mathbf{S}_d\Delta\mathbf{d}(k-1) \qquad (3)$$

Assuming the measured disturbances are zero, i.e., $\mathbf{S}_d\Delta\mathbf{d}(k-1) = 0$, only unmeasured disturbances are considered. Therefore:

$$\overline{\mathbf{Y}}(k) = \mathbf{M}_n\overline{\mathbf{Y}}(k-1) + \mathbf{S}_n\Delta\mathbf{u}(k-1) \qquad (4)$$

The manipulated variable move can be computed as follows at time (k-1)

$$\Delta\mathbf{u}(k-1) = \mathbf{K}_{MPC}\boldsymbol{\varepsilon}(k) \qquad (5)$$

where the reference trajectory error vector can be determined as:

$$\boldsymbol{\varepsilon}(k+1) = \mathbf{R}(k+1) - \mathbf{M}_p\mathbf{Y}(k) - \mathbf{W}(k+1) \qquad (6)$$

From a least squares solution:

$$\mathbf{K}_{MPC} = \left(\mathbf{S}_p^T\boldsymbol{\Gamma}^T\boldsymbol{\Gamma}\mathbf{S}_p + \boldsymbol{\Lambda}^T\boldsymbol{\Lambda}\right)^{-1}\mathbf{S}_p^T\boldsymbol{\Gamma}^T\boldsymbol{\Gamma} \qquad (7)$$

where, the input weight $\boldsymbol{\Lambda} = \lambda \times \mathbf{I}_{m\times m}$, the output weight matrix is set to identity ($\boldsymbol{\Gamma}=\mathbf{I}$) for simplicity and $\mathbf{S}_p$ is the dynamic matrix depending on the first p rows of $\mathbf{S}_n$ where p is the control horizon.

Assuming the set point vector $\mathbf{R}(k+1) = 0$ without loss of generality, the error at time *k* will be given as follows:

$$\boldsymbol{\varepsilon}(k) = -\mathbf{M}_p\mathbf{Y}(k-1) - \mathbf{W}(k) \qquad (8)$$

Substituting (5) and (8) into (3) gives:

$$\overline{\mathbf{Y}}(k) = \left(\mathbf{M}_n - \mathbf{S}_n\mathbf{K}_{MPC}\mathbf{M}_p\right)\overline{\mathbf{Y}}(k-1) - \mathbf{S}_n\mathbf{K}_{MPC}\mathbf{W}(k) \qquad (9)$$

Rearranging equation (9) and after applying Z-transform, the transfer matrix between $\overline{\mathbf{Y}}(Z)$ and $\mathbf{W}(Z)$ can be obtained as follows:

$$\frac{\overline{\mathbf{Y}}(Z)}{\mathbf{W}(Z)} = -\left(\mathbf{I} - \left(\mathbf{M}_n - \mathbf{S}_n\mathbf{K}_{MPC}\mathbf{M}_p\right)Z^{-1}\right)^{-1}\mathbf{S}_n\mathbf{K}_{MPC} \qquad (10)$$

where $\mathbf{W}(Z)$ is a vector of unmeasured disturbances.

The unmeasured disturbance is assumed to be sinusoidal for the calculations. Then, the discrete frequency response can be used to compute the maximal value of the variability in **Y** after neglecting the initial transients by setting $Z = e^{j\omega T}$ where ω is the frequency of the disturbance **W**, and T is the sampling interval.

Defining the transfer functions in the transfer matrix in the right hand side of equation (10) as $G_{ij}^Y$ and recalling that $\mathbf{Y} = \overline{\mathbf{Y}} + \mathbf{W}$ then, the robust variability or the largest variability for the family of linear plants representing the nonlinear system is calculated from the frequency response as follows:

$$\overline{Y_{\max}} \le \max_{l_m}\left|1 + \sum_p G_{1j}^Y(Z = e^{j\omega T})\right|\left|\mathbf{W}\right| \quad l_m \le l_{m.\max} \qquad (11)$$

where $G_{1j}^Y$ are the transfer functions in the first row of the transfer matrix in equation (10). If the disturbance is given by a superposition of sinusoids at different frequencies, not considered in this work for brevity, a *sup* over the frequency range of interest has to be sought.

The only MPC tuning parameter used as a decision variable in the optimisation is the input weight factor, λ, defined above. This weight affects the speed of the controller moves. The other tuning parameters such as the output weight Γ, the prediction horizon p and the control horizon m were fixed, for simplicity, during the optimization (**Γ**=**I**, p=3 and m=1).

*B. Manipulated variable constraint*

Based on the MPC algorithm described in the previous section, the following transfer matrix between the manipulated variable moves and the unmeasured disturbance can be obtained:

$$\frac{\Delta \mathbf{u}(Z)}{\mathbf{W}(Z)} = \mathbf{K}_{MPC}\left[\mathbf{M}_p\left[\mathbf{I}_{n\times n} - \left[\mathbf{M}_n - \mathbf{S}_{n,nom}\mathbf{K}_{MPC}\mathbf{M}_p\right]Z^{-1}\right]^{-1}\mathbf{S}_{n,nom}\mathbf{K}_{MPC} - \mathbf{I}_{p\times p}\right] \quad (12)$$

Using as before frequency response concepts, the maximum value of the dynamic manipulated variable move, $\Delta u_{max,}$ can be calculated from the transfer matrix in equation (12). If the transfer functions in this transfer matrix are defined to be $G_{ij}^U$ then, the manipulated variable constraint in this study can be formulated as follow:

$$u_{nom} + \Delta u \le u_{\max} \quad (13)$$

where $\Delta u = \left|\sum_p G_{1j}^U (Z = e^{j\omega T})\right| \left|\mathbf{W}\right| \le u_{\max} - u_{nom}$

where $G_{1j}^U$ are the transfer functions in the first row of the transfer matrix in equation (12). Equation (13) was incorporated in the optimization problem as a constraint.

*C. Robust stability constraint*

The robust stability condition, developed by Zanovello and Budman (1999), in terms of a Structured Singular Value condition is:

$$\mu_\Delta(\mathbf{M}(j\omega)) < 1 \quad (14)$$

The matrix **M** is a function of the MPC controller and the nominal model. For details regarding the matrix **M**, the readers can refer to the paper by Zanovello and Budman (1999).

## III. CASE STUDY

In this study, a distillation column (Lee, 1993) was selected as a case study. The manipulated variable is the reflux ratio, RR and the controlled variable is the propane mole fraction in the distillate. The process was identified, for simplicity, as a first order plus dead time model as shown below:

$$y = K\left(1 - \exp\left(\frac{-(t-\theta)}{\tau_p}\right)\right) \quad (15)$$

The step response models to be used for MPC calculations can be easily obtained from (15). Steady state and dynamic simulations were performed using Aspen Plus and Aspen Dynamics to determine the model parameters i.e. process gain K and time constant $\tau_{p.}$ Since the dead time was difficult to quantify from the Aspen Dynamics simulation, it was calculated instead from a formula given by Seider *et al*. (1999)

*A. Process model*

Steady state and dynamic simulation models with different column designs were developed. Step changes of ± 10% with respect to different nominal reflux ratio (RR) values were simulated. This size of changes was found sufficient to capture the nonlinear behaviour around different RR values. The dynamic response of Equation (15) was used to obtain the dynamic matrix, $S_n$.

To simplify the optimization process, the process gain (K), process time constant ($\tau_p$) and process dead time (θ) obtained from steady state and dynamic simulations were represented by explicit correlations as functions of RR as follows:

$$K = -0.0174 \times \log(RR) + 0.0405 \quad (16)$$

$$\tau_p = 32.70 + \frac{6.79}{\ln(RR)} + \frac{7.67}{(\ln(RR))^2} - \frac{2.76}{(\ln(RR))^3} + \frac{0.28}{(\ln(RR))^4} \quad (17)$$

The dead time was calculated from Seider *et al*. (1999) as a function of RR:

$$\theta = 60 \times (0.04RR + 0.09) \quad (18)$$

*B. Model Uncertainty*

By performing step changes around different values of RR, the model uncertainty, $\ell_m$, was obtained from Equation (1) and the maximum uncertainty, $\ell_{m,\max}$, was obtained from Equation (2). Fig 2 shows the plot of the maximum model uncertainty as a function of RR. For optimization purposes the uncertainty was also fitted to an explicit function of RR:

$$\ell_{m,\max} = 11.0143 - 20.1858RR + 14.5541RR^2 - 4.9532RR^3 + 0.7913RR^4 - 0.0469RR^5 \quad (19)$$

The range of $\ell_m$ that appears in the optimization problem is $\pm\ell_{m,\max}$. The uncertainty $\ell_m$ was varied to obtain the robust process variability cost within the range of uncertainty given by $\pm\ell_{m,\max}$ as explained in equation (11) above.

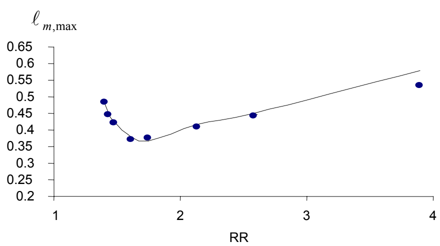


Fig. 2 The maximum model uncertainty as a function of RR

### *C. Objective function for the integrated approach*

The optimisation problem proposed for the integrated design and control design approach is the minimisation of the sum of capital, operating and variability costs as follows:

$$\min_{d,\,u,\,c} \quad CC(d) + OC(u) + VC(c) \tag{20}$$

$$\text{s.t.} \quad h(d, u, c)=0 \text{ and } g(d, u, c) \le 0$$

The capital cost, CC(d) and operating costs, OC(u) were calculated using the following correlations proposed by Luyben and Floudas (1994) for a distillation column:

$$CC = 12.3(615 + 324D^2 + 486(6 + 0.76N)D) + 245N(0.7 + 1.5D^2) \tag{21}$$

$$OC = \beta_{tax}(Q_r \times OP \times UC) \tag{22}$$

Where $Q_r$, OP and UC are reboiler heat duty, operating period and utility cost. The idea was then to express all the terms in the optimization problem above as a function of two decision variables: RR and the controller parameter $\lambda$. To accomplish this, a fixed propane target concentration of 0.783 was assumed. Then, correlations for the number of trays, *N,* the column diameter, *D,* and reboiler heat duty, $Q_r$, as a function of the reflux ratio, *RR,* necessary to obtain the target of 0.783 were obtained using Aspen Plus. Using these correlations, the capital and operating costs in equations (21) and (22) were subsequently expressed as functions of RR only.

As explained above, a sinusoidal disturbance with frequency $\omega_d$, was introduced as the disturbance in the feed composition resulting in a sinusoidal response in the distillate composition. Consequently, the concentration of propane in the distillate oscillates, due to imperfect control, around the pre-specified set point of 0.783 when under closed loop control. To quantify the cost of this variability in the controlled variable, it was assumed, following common industrial practice, that a fixed volume of product from the distillation column will be held downstream from the distillation column in an intermediate storage tank before delivery to the customer. The tank will also attenuate the variation in product composition. A large volume of material in the storage tank will naturally result in a lower variability in the exit of the storage tank but, at the same time, it will increase the inventory cost. Then, the variability cost was assumed to be equal to the inventory cost that is directly related to the volume of product stored in the intermediate storage tank .

The volume of intermediate storage was calculated at the specific frequency of the disturbance, $\omega_d$. From a simple mass balance of the intermediate storage tank,

$$V\frac{dC_{out}}{dt} = Q_{in}C_{in} - Q_{out}C_{out} \tag{23}$$

Assuming constant flow rates $Q = Q_{in} = Q_{out}$ and defining the residence time $\tau = \frac{V}{Q}$ where $V$ is the product volume, from Laplace transformation of equation (23):

$$\frac{\delta C_{out}}{\delta C_{in}} = \frac{1}{\tau s + 1} \tag{24}$$

Then converting to the frequency domain at $\omega = \omega_d$ :

$$\therefore \quad V = \sqrt{\left(\frac{\delta C_{in}}{\delta C_{out}}\right)^2 - 1} \times \frac{Q}{\omega_d} \tag{25}$$

Where equation (25) represents the volume of product in the intermediate storage tank, V, as a function of $\delta C_{in}$, the deviation of product concentration in the distillate product calculated by equation (11), $\delta C_{out}$, the maximum allowed product concentration variability to be delivered to the customer, $Q$, the product flow rate and $\omega_d$, the frequency of the disturbance. Therefore, the volume of product, V, in the intermediate storage tank depends only on $\delta C_{in}$, the propane concentration variability exiting the distillation column, because $\delta C_{out}$ and $Q$ are specified by the customer and $\omega_d$ is the frequency of the unmeasured disturbance. $\delta C_{in}$, calculated from equation (11), depends on the two decision variables i.e. the reflux ratio RR and the MPC controller tuning parameter, $\lambda$. Then, the variability cost (*VC*) is calculated by considering the price of the product held in the intermediate storage tank before retailing as follows:

$$VC = P \times V \times (A/P, i, N) \tag{26}$$

where $V$ is the volume calculated from equation (25), $P$ is the price of the product and (*A/P,i,N*) is the capital recovery factor with interest rate, *i*, and amortisation period *N.*

### *D. Optimization*

This section presents the optimization problems for the two design methodologies considered here, i.e. the proposed integrated design approach and the traditional two step design approach.

Integrated Design Methodology This method consists of only one optimisation problem as follows:

$$\begin{aligned} &\min_{RR,\lambda} \quad CC(RR) + OC(RR) + \max_{\ell_m} VC(\lambda) \\ &s.t. \quad -\ell_{m,\max} \le \ell_m \le \ell_{m,\max} \\ &\quad \Delta U_{\max} < U_{\max} - U_{nom} \\ &\quad \mu(\mathbf{M}(j\omega)) < 1 \end{aligned} \tag{27}$$

Traditional Design Methodology This method consists of two optimisation steps performed sequentially:

Step 1 Steady state optimal design

$$\min_{RR} \quad CC(RR) + OC(RR) \tag{28}$$

Step 2 Optimal controller design

$$\begin{aligned} &\min_{\beta,\lambda} \quad \beta(\lambda) \\ &s.t. \quad \Delta U_{\max} < U_{\max} - U_{nom} \\ &\quad \mu(\mathbf{M}(\beta)) < 1 \end{aligned} \tag{29}$$

where β is a scalar multiplying a pre-specified performance weight and the robust performance condition, $\mu(\mathbf{M}(\beta)) < 1$ was obtained from Morari and Zafiriou (1989).

It should be noted that an additional advantage of the proposed integrated approach is that it does not require the a priori selection of a performance weight as required in the two step approach.

*E. Results and discussion*

The optimization results for both methodologies are summarized in Tables 1 and 2 and a comparison between them is summarized in Table 3. The optimization was performed with the *fmincon* function, available in the Matlab Optimization Toolbox, that is based on a Sequential Quadratic Programming algorithm. Many different initial guesses were assumed to achieve convergence to a global optimum. Tables 1 and 2 show the results from a sensitivity-study of the optimum design with respect to the product price, *P*. The optimal solutions are presented in terms of *RR* and the MPC tuning parameter $\lambda$. The number of trays and the column diameter, although not shown for brevity, can be easily obtained from the value of *RR* for the desired propane composition. The results in Table 1 show that for the integrated approach, as the product price increases the optimum RR decreases. This result can be explained by the fact that the process gain, K, rapidly increases when *RR* decreases. Since for larger *P* the variability cost increases, a larger effect of the manipulated variable on the controlled variable, as given by *K,* is needed to reduce the variability leading to smaller optimal values of *RR.* For high product prices, i.e. P=10, 15 and 20 the optimum RR is not sensitive to changes in *P*.

TABLE 1
Integrated Method: Sensitivity of optimum design to product price

| P | RR* | λ* | CC ($/day) | OC ($/day) | VC ($/day) | TC ($/day) |
|---|---|---|---|---|---|---|
| 1 | 1.57 | 0.041 | 444.7 | 549.6 | 47.6 | 1041.9 |
| 5 | 1.51 | 0.045 | 458.3 | 542.6 | 224.6 | 1225.4 |
| 10 | 1.50 | 0.045 | 461.1 | 541.5 | 446 | 1448.6 |
| 15 | 1.50 | 0.045 | 461.1 | 541.5 | 669 | 1671.6 |
| 20 | 1.50 | 0.045 | 461.1 | 541.5 | 892 | 1894.6 |

Table 2 clearly shows that for the traditional approach the optimal *RR* is constant and only a function of P because RR is solely chosen to minimize the capital plus operating costs, i.e. it is not affected by the variability cost. The value of λ also remains constant because, for the optimal RR=1.61, a single value of λ exists to minimise the intermediate storage volume while satisfying the robust stability constraint. Also, it was found as expected that, for both methods, as the product price increases the total cost increases.

TABLE 2
Traditional Method: Sensitivity of optimum design to product price.

| P | RR* | λ* | CC ($/day) | OC ($/day) | VC ($/day) | TC ($/day) |
|---|---|---|---|---|---|---|
| 1 | 1.61 | 0.127 | 438.5 | 554.8 | 90.9 | 1084.2 |
| 5 | 1.61 | 0.127 | 438.5 | 554.8 | 454.3 | 1447.6 |
| 10 | 1.61 | 0.127 | 438.5 | 554.8 | 908.6 | 1901.9 |
| 15 | 1.61 | 0.127 | 438.5 | 554.8 | 1362.9 | 2356.2 |
| 20 | 1.61 | 0.127 | 438.5 | 554.8 | 1817.3 | 2810.6 |

Table 3 summarizes the comparison between the two methods. The design using the integrated method results in a lower cost than the design using the traditional method; savings can be up to 48 % depending on the product price. The savings are due to the fact that the design using the integrated method allows for a more aggressive tuning of the controller.

TABLE 3
Comparison of traditional and integrated methods

| P | Traditional Approach ($/day) | Integrated Approach ($/day) | Savings ($/day) | Savings (%) |
|---|---|---|---|---|
| 1 | 1084.2 | 1041.9 | 42.3 | 4.1 |
| 5 | 1447.6 | 1225.4 | 222.2 | 18.1 |
| 10 | 1901.9 | 1448.6 | 453.3 | 31.3 |
| 15 | 2356.2 | 1671.6 | 684.6 | 40.9 |
| 20 | 2810.6 | 1894.6 | 916 | 48.4 |

Fig 3 shows the controlled process variability using both the integrated and traditional methodologies for P=20. This clearly shows that the magnitude of the process variability corresponding to the integrated method is much smaller than the one resulting from the traditional method.

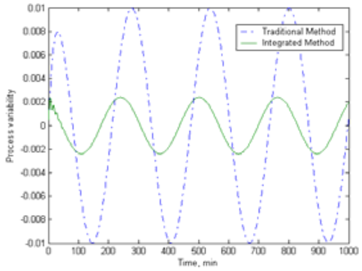


Fig 3. The simulation of process variability using integrated and traditional methods

The reason that the variability is so large with the traditional method is because for the design obtained in Step 1 of the traditional method, the best controller tuning constant, λ, could not reduce the variability any further without violating the robust stability constraint.

To show the importance of closed loop control the value of λ was set to 1000 in the integrated method. This has the effect of freezing the MPC controller at the optimal nominal value of the manipulated variable that is equivalent to open loop operation. Compared to the case with smaller optimal λ i.e. with control, the optimization resulted in a similar column design but required a 20 times larger storage tank to attenuate the product variability to the level specified by the customer. This result simply quantifies the benefit of closed loop control as opposed to attempting to reduce variability by using the tank without control.

## IV. CONCLUSIONS

A new design methodology which integrates the cost of variability in the controlled variables with the capital and operating costs into one objective function is proposed. The nonlinear process was modelled by a family of linear models represented by a nominal FOPDT model and model uncertainty to account for nonlinearity with respect to the nominal models around different values of reflux ratio. An MPC controller tuned for robustness in the presence of model uncertainty was used. The proposed integrated design methodology was compared with the traditional design methodology where the process design and control design are performed sequentially.

The proposed integrated design approach resulted in lower costs because the design using the integrated method allowed for a more aggressive tuning of the controller without violating the robust stability constraint. The differences between the integrated method and the traditional one become more significant as the price of the product increases.

The integrated method offers the additional benefit that a performance weight is not necessary since the variability cost is explicitly considered in the objective function.

## REFERENCES


[1] Bansal V., Perkins J. D., Pistikopoulos E. N., Ross R. and Schijndel J. M. G. (2000). Simultaneous design and control optimization under uncertainty. Computers and Chemical Engineering, 24, 261-266.

[2] Blanco A. M. and Bandoni J. A. (2003). Interaction between process design and operability of chemical processes: an eigenvalue optimization approach. *Computers and Chemical Engineering, 27,* 1291-1301.

[3] Georgiadis M. C., Schenk M. and Pistikopoulos E. N. (2002). The interactions of design, control and operability in reactive distillation systems. *Computers and Chemical Engineering, **26**,* 735-746.

[4] Lee L. P. (1993). *Nonlinear Process control.* London: Spriger-Verlag.

[5] Luyben M. L. and Floudas C. A. (1994). Analyzing the interaction of design and control- 1: A Multi objective framework and application to binary distillation synthesis. *Computers and Chemical Engineering, **18**,* 933-969.

[6] Mohideen M. J., Perkins J. D. and Pistikopoulos E. N. (1996). Optimal Design of Dynamic Systems under Uncertainty. *AIChE Journal, **42**,* 2251-2272.

*[7]* Morari M. and Zafiriou E. (1989). *Robust Process Control*. New Jersey: Prentice Hall.

[8] Seider, W. D., Seader, J. D. and Lewin, D. R. (1999). *Process Design Principle.* New York: Wiley.

[9] Zanovello, R. and Budman, H. M.(1999). Model predictive control with soft constraints with application to lime kiln control, *Computers and Chemical Engineering*, **23**, 791-806.


**Nomenclature**

c controller tuning parameters
$C$ concentration
d design variables
$\mathbf{G}^Y$ transfer matrix between disturbance to controlled variable
$\mathbf{G}^U$ transfer matrix between disturbance to manipulated variable
$\mathbf{K}_{MPC}$ MPC controller gain matrix
$K$ process gain=final value of the step response model
$l_m$ multiplicative model uncertainty
$l_{m,max}$ bound on the multiplicative uncertainty
**M** Interconnection matrix to test robust stability using μ
$\mathbf{M_n}$ shift matrix with dimension nxn
$\mathbf{M_p}$ shift matrix with dimension pxn
$P$ price of the product
$Q$ volumetric flow
RR reflux ratio
$\mathbf{R}(k)$ set point vector
$\mathbf{S}_n$ step response model vector
$\mathbf{S}_{n,nom}$ step response around nominal operating point
$u_{nom}$ value of the manipulated variable at the nominal operating point.
$u_{max}$ bound on the manipulated variable
$V$ volume
**W** vector of future unmeasured disturbances
$\overline{\mathbf{Y}}(k)$ output prediction vector.

**Symbols**

$\mathbf{\Delta u}$ manipulated moves vector calculated by the MPC controller
$\Delta d$ change in the measured disturbance.
ε prediction error vector used for MPC calculations
ω frequency
Γ controlled variable weight in MPC objective function
Λ manipulated variable weight in MPC objective function
μ Structured Singular Value
$\theta$ dead time between manipulated to controlled variables
$\tau_p$ process time constant
$\delta C$ deviation in concentration with respect to the nominal value.
$\beta$ scalar weight multiplying performance weight
$\lambda$ diagonal elements of the weight matrix $\Lambda$ (assumed equal)