Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

On Active Magnetic Bearing Control with Input Saturation

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Abstract

We study the low-bias stabilization of active magnetic bearings (AMBs) subject to voltage saturation based on a recently proposed model for the AMB switching mode of operation. Using a forwarding-like approach, we construct a stabilizing controller of arbitrarily small amplitude and a control-Lyapunov function for the AMB dynamics. We illustrate our construction using a numerical example.

1 Introduction

Active magnetic bearings (AMBs) are being employed in a variety of rotating machines (e.g., compressors, milling spindles, and flywheels) in place of conventional mechanical bearings. Due to the non-contact nature of the magnetic bearings and rotor, AMBs have the unique ability to suspend loads with no friction, operate rotors at higher speeds, and operate in applications where the use of lubricants is prohibited. Since AMBs can be actively controlled, they offer other potential advantages over mechanical bearings, viz., the elimination of vibration through active damping, the adjustment of the stiffness of the suspended load, and the automatic balancing of rotors. AMBs are normally operated with all electromagnets active at all times. An alternative mode of operation is to activate only one electromagnet along each direction at any given time. Although posing more difficulty to the control design, this switching operation prevents opposing electromagnets from producing counteracting forces, and thus helps reduce power consumption.

Typically, an AMB is operated by introducing a sufficiently high, fixed magnetic flux in each electromagnet, which is referred to as the *bias flux*. The bias value is normally set to a fraction of the saturation flux of the electromagnet. This procedure facilitates the design of the AMB 'control' flux, which is superimposed on the bias flux. Specifically, this conservative practice allows the system to be modeled by a controllable linear system, thus, enabling the use of linear control design techniques; see for example [1, 5, 7]. Although the bias flux facilitates the control synthesis, it increases electric power losses in the AMB system, causing rotor heating and affecting the machine efficiency. While lowering or eliminating the bias flux is desirable in order to minimize power losses, it enhances the AMB system nonlinearities and may lead to a control saturation[1] or singularity. Due to these conflicting objectives, the design of AMB controllers with reduced power loss is a challenging problem.

In this paper, we consider the problem of low-bias[2] control of AMBs operating in the switching mode with the constraint that the input voltages are amplitude limited. This problem was previously addressed in [11, 12] using the nested saturation design method of Teel [9, 10]. In addition, an optimal solution was sought to the power-loss minimization problem under voltage saturation in [2]. (A comprehensive literature review of low-, asymptotic-zero-, and zero-bias AMB controllers *without* voltage saturation can be found in [8, 13].) Here, we pursue a different approach to stabilizing the AMB model of [12]. Namely, we use a *forwarding-like* method [6] to design a control law of arbitrarily small amplitude that renders the AMB system globally asymptotically stable (GAS) to the origin. The main differences between our approach and [12] are: (i) we allow the use of different saturation functions (both 'hard' and 'soft'), thus providing more flexibility in the control implementation, and (ii) we provide an explicit construction for a 'global' control Lyapunov function (CLF) for the system. This contrasts with the result of [12], which relies on the standard 'hard' saturation and whose Lyapunov-like function is only an 'asymptotic' CLF for the system, i.e., a CLF only when the system operates in a certain region of the state space.

The rest of this paper is organized as follows. In Section 2, we introduce the AMB model and motivate our stabilization problem. In Section 3, we prove a lemma that constructs a stabilizing controller of arbitrarily small amplitude and a CLF for a three-dimensional chain of integrators. In Section 4, we use our lemma to con-

This work was supported in part by NSF grant DMS-0424011.
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[1] Saturation may arise due to limits on the amplitude of the output voltage of the power amplifier driving the electromagnets.

[2] By *low bias*, we mean an AMB control law where the bias can be set to an arbitrarily small positive constant without affecting the system stability.

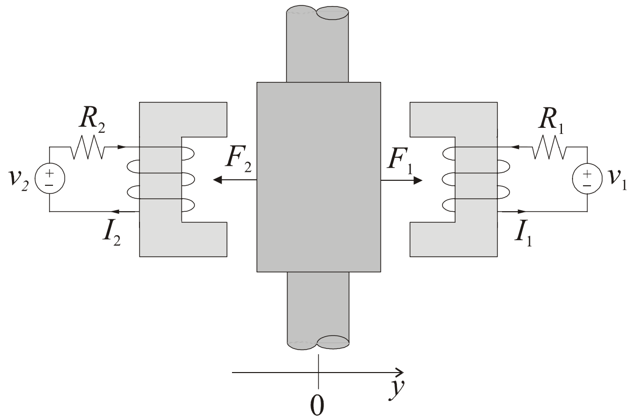

Figure 1: Schematic of the AMB system.

struct a CLF and corresponding stabilizing feedback for the AMB model. We provide a numerical example in Section 5, and we close in Section 6 with a summary of our work.

2 AMB Model and Problem Statement

The original nonlinear electromechanical model of the one degree-of-freedom AMB system shown in Figure 1 can be subdivided into the mechanical subsystem dynamics, the magnetic force equation, and the electrical subsystem dynamics. The mechanical subsystem is governed by

$$m\ddot{y} = \sum_{i=1}^{2} F_i(\Phi_i), \tag{1}$$

where m is the rotor mass, $y \in \mathbb{R}$ represents the position of the rotor center, $\Phi_i \in \mathbb{R}$ is the magnetic flux in the ith electromagnet, $F_i(\Phi_i) \in \mathbb{R}$ denotes the force produced by the ith electromagnet, given by [14]

$$F_i = \frac{(-1)^{i+1}\Phi_i^2}{\mu_0 A}, \quad i = 1, 2, \tag{2}$$

μ_0 is the permeability of air, and A is the cross-sectional area of the electromagnet. The electrical subsystem is governed by the equations [14]

$$N\dot{\Phi}_i + R_i I_i = v_i, \quad i = 1, 2, \tag{3}$$

where N denotes the number of coil turns in the electromagnet, R_i is the resistance of the ith electromagnet coil, $v_i \in \mathbb{R}$ is the input control voltage of the ith electromagnet, $I_i \in \mathbb{R}$ is the current in the ith electromagnet which is related to the flux according to [14]

$$I_i = \frac{2\left(g_0 + (-1)^i y\right)\Phi_i}{\mu_0 AN}, \; i = 1, 2, \tag{4}$$

and g_0 is the nominal air gap.

In this paper, we use the new form proposed in [12, 13] for the AMB dynamics in the switching mode of operation. We only outline the model derivation here, and refer the reader to [13] for the details and justification. Consider that the flux is given by

$$\Phi_i = \Phi_0 + \phi_i, \quad i = 1, 2 \tag{5}$$

where $\Phi_0 > 0$ is the constant *bias* flux and ϕ_i is the *control* flux. Let the *generalized* control flux be defined as

$$\phi = \phi_1 - \phi_2, \tag{6}$$

and consider the voltage switching strategy

$$\begin{array}{llll} v_1 = v, & v_2 = 0 & \text{if} & \phi \geq 0 \\ v_1 = 0, & v_2 = -v & \text{if} & \phi < 0 \end{array} \tag{7}$$

where v is the *generalized* control voltage. Based on (5)-(7), the AMB model (1)-(3) has the equivalent form

$$\ddot{y} = \frac{1}{m\mu_0 A}\left(2\bar{\Phi}_0\phi + \phi|\phi|\right), \; \dot{\phi} = \frac{v}{N}, \tag{8}$$

where $\bar{\Phi}_0 = \Phi_0 + \min\{\phi_1(0), \phi_2(0)\}$ and the coil resistance was neglected for simplicity.

Now, assume that the input voltages to the original AMB model are amplitude limited, i.e., $|v_i| \leq v_{\max}$, $i = 1, 2$ where $v_{\max}$ is the known limit. Defining the states $x_1 = y$, $x_2 = \dot{y}$, and $x_3 = \phi$ and the change of input $v = Nu$, the above model becomes

$$\dot{x}_1 = x_2, \;\; \dot{x}_2 = \beta_0 x_3 + \beta_1 x_3 |x_3|, \;\; \dot{x}_3 = u, \tag{9}$$

where $\beta_0 = 2\bar{\Phi}_0/(m\mu_0 A)$ and $\beta_1 = 1/(m\mu_0 A)$, with the input constraint $|u| \leq v_{\max}/N$.

Our goal in this work is to design a feedback control law $u(x)$, satisfying the above saturation constraint, such that $x = 0$ is GAS where $x = (x_1, x_2, x_3)^\top$. The GAS property means there is a continuously differentiable (C^1) function $V : \mathbb{R}^3 \to [0, \infty)$ that is radially unbounded and zero only at the origin, and for which the derivative $\dot{V}$ along all trajectories of the system in closed-loop with the controller $u(x)$ is negative definite. A function V satisfying these requirements for some feedback $u(x)$ is called a CLF for (9) [4]. See [13] for a discussion on the relation between the global asymptotic stability of x and the stability of the original AMB states.

3 Preliminary Result

In the following lemma, we use a forwarding-like approach based on [6] to construct a CLF and corresponding stabilizing state feedback of arbitrarily small amplitude for a three-dimensional chain of integrators. We let σ denote the standard saturation projecting $\mathbb{R}$ onto

$[-1,+1]$, i.e., $\sigma(s)=s$ when $|s|\le 1$ and $\sigma(s)=\mathrm{sign}(s)$ otherwise. However, see Remark 1 below for a version of our construction where the hard saturation σ is replaced by a smoother function.

Lemma 1 Consider the chain of integrators

$$\dot{\xi}_1=\xi_2,\ \ \dot{\xi}_2=\xi_3,\ \ \dot{\xi}_3=\mu, \tag{10}$$

where $\mu\in\mathbb{R}$ is the control input. Let $c_1,c_2>0$ be given and

$$\delta\le\min\left\{\frac{4}{3},c_1\right\}. \tag{11}$$

Define the change of variables

$$z_1=\delta\left(c_1\xi_1+\xi_2\right)+\frac{\delta}{c_1}\xi_3,\ \ z_2=c_1\xi_2+\xi_3,\ \ z_3=\xi_3 \tag{12}$$

and the functions

$$\begin{aligned} U(z_2,z_3) &= \frac{1}{2}z_3^2+\int_0^{z_2}\sigma(s)ds && (13)\\ g(z_1,z_2,z_3) &= \frac{\delta}{c_1}\sigma(z_1)+4\left(1+U(z_2,z_3)\right)\\ &\quad\times\left(z_3+\sigma(z_2)\right)-\frac{\delta}{2}\left(z_2+z_3\right). && (14)\end{aligned}$$

Then the function

$$\begin{aligned} V(\xi_1,\xi_2,\xi_3) &= 4U(z_2,z_3)+2U^2(z_2,z_3)\\ &\quad -\frac{\delta}{2}z_2z_3+\int_0^{z_1}\sigma(s)ds \qquad (15)\end{aligned}$$

is a CLF for (10) since its derivative along the trajectories of system (10) in closed loop with the control

$$\mu(\xi_1,\xi_2,\xi_3)=-c_1\sigma(z_2)-c_2\sigma(g(z_1,z_2,z_3)) \tag{16}$$

is negative definite. In particular, the bounded control (16) renders (10) GAS to the origin.

Remark 1 The saturation function σ from Lemma 1 is Lipschitz continuous but not continuously differentiable. We chose this hard saturation for simplicity of analysis. However, one can prove a similar result with smoother saturation functions. For example, if one instead chooses the $\mathcal{C}^1$ function

$$\sigma(s)=\begin{cases} s \text{ if } |s|\le 1\\ \mathrm{sign}(s)\left[1+\dfrac{1}{\pi}\tan^{-1}\left(\pi\left(|s|-1\right)\right)\right] \text{ if } |s|>1, \end{cases} \tag{17}$$

then the only change in the statement of Lemma 1 is to the right-hand side of the inequality (11). The proof for this saturation is similar to the proof we give below.

Remark 2 Condition (11) in Lemma 1 allows cases where c_1 is arbitrarily large and δ is as small as desired, as well as cases where $\max\{c_1,c_2,\delta\}$ is arbitrarily small, including the case where $c_1=c_2=\delta\in(0,4/3]$.

Proof: We use (12) and $\mu=-c_1\sigma(z_2)+\alpha$, where α is the new control input, to transform (10) into

$$\begin{aligned} \dot{z}_1 &= \delta\left(z_2-\sigma(z_2)\right)+\frac{\delta}{c_1}\alpha\\ \dot{z}_2 &= c_1z_3-c_1\sigma(z_2)+\alpha\\ \dot{z}_3 &= -c_1\sigma(z_2)+\alpha. \end{aligned} \tag{18}$$

We first perform a Lyapunov-type analysis on the (z_2,z_3)-subsystem using the function

$$V_1(z_2,z_3)=4U(z_2,z_3)+2U^2(z_2,z_3)-\frac{\delta}{2}z_2z_3, \tag{19}$$

where $U(z_2,z_3)$ was defined in (13). By separately considering the cases where $|z_2|>1$ and $|z_2|\le 1$, one can easily verify that (19) is positive-definite and radially unbounded for $\delta<2$. In fact, if $|z_2|\le 1$, then

$$\begin{aligned} V_1(z_2,z_3) &\ge 4U(z_2,z_3)-|z_2z_3|\ge 2z_3^2+2z_2^2-|z_2z_3|\\ &\ge \frac{3}{2}\left(z_2^2+z_3^2\right). \end{aligned} \tag{20}$$

On the other hand, if $|z_2|>1$, then $\int_0^{z_2}\sigma(s)ds=1/2+(|z_2|-1)\ge|z_2|/2$, so

$$\begin{aligned} V_1(z_2,z_3) &\ge 4\left(\frac{1}{2}z_3^2+\frac{1}{2}|z_2|\right)+2\left(\frac{1}{2}z_3^2+\frac{1}{2}|z_2|\right)^2\\ &\quad -|z_2z_3|\\ &\ge 2z_3^2+2|z_2|+\frac{1}{2}z_2^2-|z_2z_3|\ge\frac{3}{2}z_3^2\\ &\quad +2|z_2|. \end{aligned} \tag{21}$$

The time derivative of (19) along the trajectories of the (z_2,z_3)-subsystem is given by

$$\begin{aligned} \dot{V}_1 &= 4\left(1+U(z_2,z_3)\right)\left(-c_1\sigma^2(z_2)+\left(z_3+\sigma(z_2)\right)\alpha\right)\\ &\quad -\frac{\delta}{2}\left(c_1z_3-c_1\sigma(z_2)+\alpha\right)z_3\\ &\quad -\frac{\delta}{2}z_2\left(-c_1\sigma(z_2)+\alpha\right)\\ &= -4c_1\left(1+U(z_2,z_3)\right)\sigma^2(z_2)-\frac{\delta c_1}{2}z_3^2\\ &\quad +\frac{\delta c_1}{2}\sigma(z_2)z_3+\frac{\delta c_1}{2}z_2\sigma(z_2)\\ &\quad +\left[4\left(1+U(z_2,z_3)\right)\left(z_3+\sigma(z_2)\right)\right.\\ &\quad \left.-\frac{\delta}{2}z_3-\frac{\delta}{2}z_2\right]\alpha. \end{aligned} \tag{22}$$

Since $|\sigma(z_2)|\le|z_2|$ and $|z_2\sigma(z_2)|=z_2\sigma(z_2)$ give

$$-\frac{1}{4}z_3^2+\frac{1}{2}\sigma(z_2)z_3\le\frac{1}{4}\sigma^2(z_2)\le\frac{1}{4}z_2\sigma(z_2), \tag{23}$$

(22) yields

$$\begin{aligned}\dot V_1 \quad \le \quad & -4c_1\left(1+U(z_2,z_3)\right)\sigma^2(z_2)-\frac{\delta c_1}{4}z_3^2\\ & +\frac{3}{4}\delta c_1 z_2\sigma(z_2)\\ & +\left[4\left(1+U(z_2,z_3)\right)(z_3+\sigma(z_2))\right.\\ & \left.-\tfrac{\delta}{2}z_3-\tfrac{\delta}{2}z_2\right]\alpha. \qquad (24)\end{aligned}$$

We now distinguish between two cases:

1. $|z_2|\le 1$. Then $\sigma(z_2)=z_2$, $z_2\sigma(z_2)=\sigma^2(z_2)$, and since $\delta\le 4/3$, we get

$$\begin{aligned}\dot V_1 \le & -3c_1\left(1+U(z_2,z_3)\right)\sigma^2(z_2)-\frac{\delta c_1}{4}z_3^2\\ & +\left[4\left(1+U(z_2,z_3)\right)(z_3+\sigma(z_2))\right.\\ & \left.-\tfrac{\delta}{2}z_3-\tfrac{\delta}{2}z_2\right]\alpha. \qquad (25)\end{aligned}$$

2. $|z_2|>1$. Then $\sigma(z_2)=\mathrm{sign}(z_2)$, $z_2\sigma(z_2)=|z_2|$, and

$$\begin{aligned}&\left(1+U(z_2,z_3)\right)\sigma^2(z_2) \ \ge\ 1+\textstyle\int_0^{z_2}\sigma(s)ds\\ &\ge 1+\textstyle\int_0^{\mathrm{sign}(z_2)} sds+\int_{\mathrm{sign}(z_2)}^{z_2}\sigma(s)ds\\ &= 1+\textstyle\int_0^{\mathrm{sign}(z_2)} sds+\mathrm{sign}(z_2)\int_{\mathrm{sign}(z_2)}^{z_2}ds\\ &=\frac{1}{2}+|z_2|.\end{aligned} \qquad (26)$$

Since $\delta\le 4/3$, applying (26) to (24) again yields (25).

Thus (25) holds for all z_2.

Finally, we perform a Lyapunov-type analysis on the whole system (18) using the positive definite radially unbounded function V from (15). Since $V(z_1,z_2,z_3)=V_1(z_2,z_3)+\int_0^{z_1}\sigma(s)ds$, where $V_1(z_2,z_3)$ was defined in (19), it follows from (25) that the time derivative of (15) along the system trajectories satisfies

$$\begin{aligned}\dot V \quad \le \quad & -3c_1\left(1+U(z_2,z_3)\right)\sigma^2(z_2)-\frac{\delta c_1}{4}z_3^2\\ & +\delta|z_2-\sigma(z_2)|\\ & +\left[\frac{\delta}{c_1}\sigma(z_1)+4\left(1+U(z_2,z_3)\right)(z_3+\sigma(z_2))\right.\\ & \left.-\frac{\delta}{2}z_3-\frac{\delta}{2}z_2\right]\alpha. \qquad (27)\end{aligned}$$

We again distinguish between two cases:

1. $|z_2|\le 1$. Then $z_2-\sigma(z_2)=0$, so (27) gives

$$\begin{aligned}\dot V \le & -2c_1\left(1+U(z_2,z_3)\right)\sigma^2(z_2)-\frac{\delta c_1}{4}z_3^2\\ & +\left[\frac{\delta}{c_1}\sigma(z_1)+4\left(1+U(z_2,z_3)\right)\right.\\ & \left.\times(z_3+\sigma(z_2))-\frac{\delta}{2}z_3-\frac{\delta}{2}z_2\right]\alpha. \qquad (28)\end{aligned}$$

2. $|z_2|>1$. Then (26) holds and since $|z_2-\sigma(z_2)|\le |z_2|$ and $\delta\le c_1$, we again get (28).

Thus, (28) holds for all z_2. Using $\alpha = -c_2\sigma(g(z_1,z_2,z_3))$ and (14), (28) becomes

$$\begin{aligned}\dot V \quad \le \quad & -2c_1\left(1+U(z_2,z_3)\right)\sigma^2(z_2)-\frac{\delta c_1}{4}z_3^2\\ & -c_2\sigma(g(z_1,z_2,z_3))g(z_1,z_2,z_3)\\ \le \quad & -2c_1\sigma^2(z_2)-\frac{\delta c_1}{4}z_3^2\\ & -c_2\sigma^2(g(z_1,z_2,z_3)) \qquad (29)\end{aligned}$$

by separately considering the cases $|g|\ge 1$ and $|g|<1$, i.e., $\dot V$ is negative definite. One can easily verify that the origin is a unique equilibrium point of (18) in closed loop with $\alpha=-c_2\sigma(g(z_1,z_2,z_3))$. Thus, $(z_1,z_2,z_3)=0$ is GAS [3]. It follows from (12) that $(\xi_1,\xi_2,\xi_3)=0$ is GAS, and that (15) is a CLF for (10). ■

4 Main Result

We are now ready to state our main result on the global asymptotic stabilization of (9) with control saturation.

Theorem 2 The control law

$$u(x)=\frac{\mu(x_1,x_2,X_3)}{\beta_0+2\beta_1|x_3|}, \qquad (30)$$

where $\mu(\cdot)$ was defined in (16) and

$$X_3=\beta_0x_3+\beta_1x_3|x_3|, \qquad (31)$$

in closed loop with (9) ensures $x=0$ is GAS, and has arbitrarily small amplitude. Moreover, with the choice (15), the function $x\mapsto V(x_1,x_2,X_3)$ is a CLF for (9).

Proof: We use the change of variable (31) and the change of input (30) to transform (9) into the chain of integrators

$$\dot x_1=x_2,\quad \dot x_2=X_3,\quad \dot X_3=\mu. \qquad (32)$$

After setting μ to (16) with $\xi_1=x_1$, $\xi_2=x_2$, and $\xi_3=X_3$, we can invoke Lemma 1 to conclude $(x_1,x_2,X_3)=0$ is GAS for (32). It then follows from (31) that $x=0$ is GAS for the system (9) in closed loop with the feedback (30)-(31). The fact that $x\mapsto V(x_1,x_2,X_3)$ is a CLF for the AMB dynamics follows from the proof of Lemma 1. ■

Remark 3 Notice that the control law (30) has arbitrarily small amplitude since along the closed-loop trajectories,

$$|u(x(t))|\le\frac{|\mu(x(t))|}{\beta_0}\le\frac{c_1+c_2}{\beta_0}\quad \forall t\ge 0. \qquad (33)$$

Thus, one can choose c_1, c_2 such that

$$c_1 + c_2 \leq \frac{\beta_0 v_{\max}}{N} \tag{34}$$

to ensure the AMB voltages satisfy the voltage constraint $|v_i(t)| \leq v_{\max}$, $i = 1, 2\ \forall t \geq 0$. From (34), one can also see that the control gains can be adjusted to accommodate sufficiently small bias levels due to the direct dependency of β_0 on the bias flux Φ_0.

Remark 4 It is worth comparing our construction from Theorem 2 with the AMB controller from Section VI of [12], where the stabilizing controller took the form of nested hard saturations. In [12], the proof that the control stabilizes the AMB system is based on the method of [9], which uses the hardness of the saturation σ in an essential way. The argument shows that the nested hard saturations cause the AMB dynamics (9) to assume a simplified form after sufficiently large time. Then one shows that this new system of 'asymptotic' equations is GAS using a Lyapunov-like analysis. Unfortunately, the Lyapunov function used in the analysis of [12] is not a CLF for the AMB dynamics (9). On the other hand, our construction is not restricted to the standard hard saturation as explained in Remark 1, and thus can lead to smoother stabilizing controllers. Also, the Lyapunov function from Theorem 2 is negative definite along the closed-loop AMB trajectories with the feedback (30) *for all time*, and thus is a CLF for the AMB dynamics (9).

5 Numerical Example

To illustrate the proposed control law, we simulated the AMB model (1)-(3) with the following parameters

$$\begin{array}{lll} m = 4.5 \text{ kg} & A = 98.12 \text{ mm}^2 & N = 244 \\ g_0 = 0.3 \text{ mm} & \mu_0 = 4\pi \times 10^{-10} \text{ H/mm} & \\ R_1 = R_2 = 0\ \Omega & \Phi_0 = 2 \times 10^{-6} \text{ Wb} & v_{\max} = 10 \text{ V}, \end{array}$$

and with the initial conditions set to $y(0) = 0.2$ mm, $\dot{y}(0) = 0.0$ mm/s, $\Phi_1(0) = 45 \times 10^{-6}$ Wb, and $\Phi_2(0) = 55 \times 10^{-6}$ Wb. The control parameters c_1, c_2 were selected according to (34) as $c_1 = 3.9$ and $c_2 = 3$. Since (11) is only a sufficient (conservative) condition for stability, the control parameter δ was chosen as $\delta = 17 > \min\left\{\frac{4}{3}, c_1\right\}$ to obtain a faster system response. The results for the system states y, $\dot{y}$, and ϕ_i ($i = 1, 2$), and for the control voltages v_i ($i = 1, 2$) are shown in Figure 2.

Despite several attempts, we were unable to simulate a case where the control voltages saturate for $t > 0$. The reason for this can be understood by considering (30). Even if we set $\Phi_1(0) = \Phi_2(0)$ to saturate the control voltages at $t = 0$ (recall $x_3 = \Phi_1 - \Phi_2$), the denominator

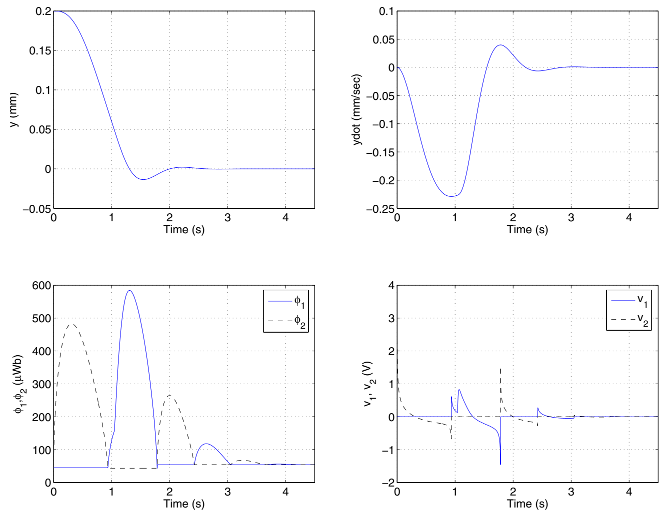

Figure 2: State trajectories and control voltages.

of (30) increases for $t > 0$ since likely $x_3(t) \neq 0$, and avoids the saturation.

6 Conclusion

We constructed a stabilizing controller and control-Lyapunov function for low-bias active magnetic bearings (AMBs) with voltage saturation. The AMB dynamics used in the construction is borrowed from a recently-developed model that allows one electromagnet to create a net force at any given time. Our construction is based on a new forwarding-like approach to stabilizing a three-dimensional chain of integrators with input saturation. The proposed AMB control law has arbitrarily small amplitude and the flexibility of being implemented with different saturation-like functions.

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