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RST-Controller Design for Sinewave References by Means of an Auxiliary Diophantine Equation

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Abstract—**The RST-design of discrete-time controllers is a very handsome method, that brings the solution almost automatically, once the discrete model of the process is known and a model transfer function for the closed loop has been chosen. The cancellation of steady state errors in response to reference signals has been solved a few years ago for polynomial reference signals of any order by the introduction of an auxiliary Diophantine equation. This method has now been extended to sinusoidal references as well, even to a combination of references of both types.**

I. Introduction

Among the numerous design methods of a discrete controller for linear SISO systems, the RST design is a very elegant pole placement method, based on the resolution of a Diophantine equation [1]-[4]. In these early works, the authors have dealt with the rejection of low-frequency disturbances of polynomial type of any order, and ensured for the closed loop a static gain of unity, thus taking care only of constant reference inputs (step functions). The error-free tracking of higher order inputs, such as ramp-type reference functions, has been solved later by the resolution of a second, or *auxiliary,* Diophantine equation [5]. For low frequency sinewave reference signals, an approximation by first order polynomials has yielded a fairly good steady state response in an application of this newer design to PWM inverters [7], but no solution existed until now for sinewave inputs of a frequency approaching the system's bandwidth.

It is the purpose of this paper to extend the design method presented in [5], [6], to sinewave references, in order to ensure in that case also a fully vanishing steady state error between reference and controlled output.

In the second section, the design steps will be described. Two applications will then be presented in the following two sections: a purely academic example, which had already been used in [5], and the PWM inverter dealt with in [7] and used in an industrial application. A discussion of the properties and limits of this new method is then covered in section IV, and section V will conclude our work.

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II. Problem Formulation and Design Steps

Let us assume that a discrete-time plant, or a continuous-time plant sampled at a given period T_s, with control signal $u(k)$, measured output signal $y(k)$ and measurement and load disturbances $e(k)$ and $v(k)$ respectively is described in terms of z-transforms by

$$Y(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}U(z^{-1}) + \frac{B(z^{-1})}{A(z^{-1})}V(z^{-1}) + E(z^{-1}) \qquad (1)$$

As apparent in Fig. 1, an RST-controller, consisting of the polynomials $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$, provides the control law $u(k)$ with the following z-transform:

$$U(z^{-1}) = \frac{T(z^{-1})}{S(z^{-1})}Y_r(z^{-1}) - \frac{R(z^{-1})}{S(z^{-1})}Y(z^{-1}), \qquad (2)$$

where $y_r(k)$ is the reference signal.

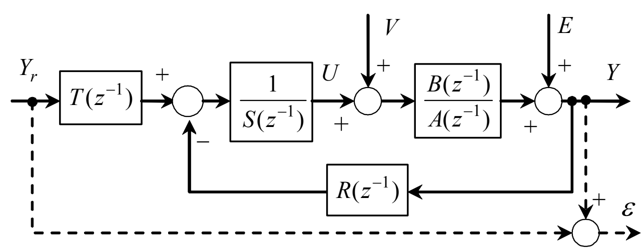

Fig. 1. Discrete-time system controlled by an RST-regulator.

The first step of the usual pole placement design consists then in choosing arbitrarily a desired closed-loop transfer function or *model*,

$$\frac{Y(z^{-1})}{Y_r(z^{-1})} = F_m(z^{-1}) = \frac{B_m(z^{-1})}{A_m(z^{-1})}, \qquad (3)$$

to which the actual input-output closed loop transfer function Y/Y_r, resulting from (1) and (2),

$$\frac{Y}{Y_r} = \frac{BT}{AS+BR}, \qquad (4)$$

is to be made equal. In this expression, as will be done in most of the followings, the dependence of z^{-1} has been

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omitted, for the sake of simplicity. In the previous works, such as in [1], $F_m(z^{-1})$ was chosen entirely arbitrarily in order to impose the desired closed loop poles, with the only constraint that its numerator had to contain as a factor the plant pure delay and its "uncompensable" zeros (zeros lying outside of the unit circle) and zeros which one decides not to compensate for practical reasons, all regrouped in the polynomial $B^-(z^{-1})$ factor of $B(z^{-1})$:

$$B_m = B^- B_m' . \tag{5}$$

With the choice of expressing the plant transfer function in negative powers of z, its pure delay is automatically embedded in B^-. A constant factor was also introduced so as to make $F_m(1) = 1$ in order to ensure unity gain in steady state. This determines completely the polynomial T, since

$$T = A_0 B_m' , \tag{6}$$

where the filter polynomial A_0 is often set equal to one [6].

Steady state errors of higher orders, e.g. in response to a ramp reference, were not cancelled by that design, unless the two polynomials $R(z^{-1})$ and $T(z^{-1})$ were chosen to be the same, which is the case only for the series controller embedded in the more general RST structure.

A. Solution for polynomial reference signals

To remedy that situation, a different way of determining the polynomial $T(z^{-1})$ has been proposed [5], [6], which in turn imposes $B_m(z^{-1})$. Let us recall briefly the main steps of this design, since they will also be at the root of the new development presented in this paper.

The partial z-transfer function from the reference Y_r to the true error signal $\varepsilon(z^{-1}) = Y_r(z^{-1}) - Y(z^{-1})$ between reference and measured output is given, according to (3), by:

$$\frac{\mathcal{E}(z^{-1})}{Y_r(z^{-1})} = 1 - \frac{Y}{Y_r} = 1 - F_m = \frac{A_m - B_m}{A_m} . \tag{7}$$

In order to cancel steady-state errors in response to polynomial references of the form $y_r(t) = t^m$, having thus a z-transform

$$Y_r(z) = \frac{Y_{r1}}{(1 - z^{-1})^{m+1}} , \tag{8}$$

where Y_{r1} is some polynomial in z^{-1}, it is necessary and sufficient that $(1-z^{-1})^{m+1}$ divides $A_m - B_m$, according to the final limit theorem of the z-transform, or that

$$A_m - B_m = (1 - z^{-1})^{m+1} L(z^{-1}) , \tag{9}$$

where $L(z^{-1})$ is some unknown polynomial, to be determined. With (5) it is straightforward that both $L(z^{-1})$ and $B_m'(z^{-1})$ become now solutions to the equation

$$(1 - z^{-1})^{m+1} L + B^- B_m' = A_m , \tag{10}$$

which has been designated as *auxiliary Diophantine equation* in [5], the primary Diophantine equation being the one that yields the polynomials R and S. The case of the DC input-output unity gain, corresponding to $m = 0$, is embedded in this derivation, since it results from equation (9) that, for $m \geq 0$, $A_m(1) = B_m(1)$. Once $B_m'(z^{-1})$ is determined by the resolution of (10), $T(z^{-1})$ is given by (6).

B. Solution for sinewave reference signals

The case of sinewave reference signals is somewhat different. Since the final limit theorem does not apply here, consider instead the frequency response of the error $\mathcal{E}$ versus sinusoidal inputs. Its magnitude is given from (7) by

$$\left| \frac{\mathcal{E}(z^{-1})}{Y_r(z^{-1})} \right|_{z = e^{j\omega T_s}} = \left| \frac{A_m - B_m}{A_m} \right|_{z = e^{j\omega T_s}}$$

where T_s is the period at which the plant has been sampled.

In order to cancel the error at a given angular frequency ω_0, it is necessary to introduce a transmission zero into this transfer function at that frequency. This can be done by letting $(1 - e^{j\omega_0 T_s} z^{-1})(1 - e^{-j\omega_0 T_s} z^{-1})$ divide $A_m - B_m$, the complex conjugate pair of zeros being chosen so as to ensure a product polynomial with pure real coefficients. Equation (9) is then replaced by

$$\begin{aligned} A_m - B_m &= (1 - e^{j\omega_0 T_s} z^{-1})(1 - e^{-j\omega_0 T_s} z^{-1}) L(z^{-1}) \\ &= (1 - 2\cos\omega_0 T_s \cdot z^{-1} + z^{-2}) L(z^{-1}) \end{aligned} \quad , \tag{11}$$

and the auxiliary Diophantine equation which must be solved becomes here

$$(1 - 2\cos\omega_0 T_s \cdot z^{-1} + z^{-2})\, L + B^- B_m' = A_m \tag{12}$$

instead of (10). It is worthwhile to note that the cancellation can apply simultaneously to more than one sinewave reference. If for instance the reference is the sum of two sinewaves, of frequencies ω_1 and ω_2, the resulting input-output error will be fully cancelled in steady state if two factors are inserted in the auxiliary Diophantine, one for each frequency:

$$\begin{aligned} &(1 - 2\cos\omega_1 T_s \cdot z^{-1} + z^{-2})(1 - 2\cos\omega_2 T_s \cdot z^{-1} + z^{-2})\, L \\ &\qquad + B^- B_m' = A_m \end{aligned} \tag{13}$$

Likewise, a combination of one or more sinusoidal and polynomial reference signals is being taken into account by simply introducing the corresponding factors in the auxiliary Diophantine equation, as e.g. in the case of a sinewave of frequency ω_0 added to a polynomial reference of order m:

$$(1 - 2\cos\omega_0 T_s \cdot z^{-1} + z^{-2})(1 - z^{-1})^{m+1} L + B^- B_m' = A_m . \tag{14}$$

III. Academic Example

The very simple academic example which was already used in [5] will illustrate the method. Assume a plant is given by the following discrete model, which results from the sampling with zero-order hold of some continuous-time plant at a period of $T_s = 0.1$ s:

$$G(z) = \frac{2z^{-1}(1+2z^{-1})}{(1-z^{-1})(1-0.3\,z^{-1})}\,.$$

According to the comments of section II, no zero of the plant will be cancelled here, so that $B^{-} = 2z^{-1}(1+2z^{-1})$. Assume that this plant is to be controlled so that the closed loop has a second order type response with a damping factor of 0.8 and a bandwidth of 10 rad/s, thus a discrete characteristic polynomial

$$A_m(z^{-1}) = 1 - 0.7417\,z^{-1} + 0.2020\,z^{-2}\,,$$

and so that it has zero steady-state error in response to a sinusoidal reference signal with $\omega_0 = 7$ rad/s. The primary Diophantine equation gives

$$R(z^{-1}) = 0.1031 - 0.0264\,z^{-1} \quad ; \quad S(z^{-1}) = 1 + 0.3521\,z^{-1}$$

and the auxiliary Diophantine equation (12), in this case

$$(1-1.530\,z^{-1}+z^{-2})L + 2z^{-1}(1+2z^{-1})B_m' = 1-0.7417\,z^{-1}+0.2020\,z^{-2}\,,$$

gives then $T(z^{-1}) = B_m'(z^{-1}) = 0.0944 - 0.1473\,z^{-1}$.

A simulation of the time response of the closed loop is shown in Fig. 2. As can be seen, the error vanishes completely at the sampling times in steady state.

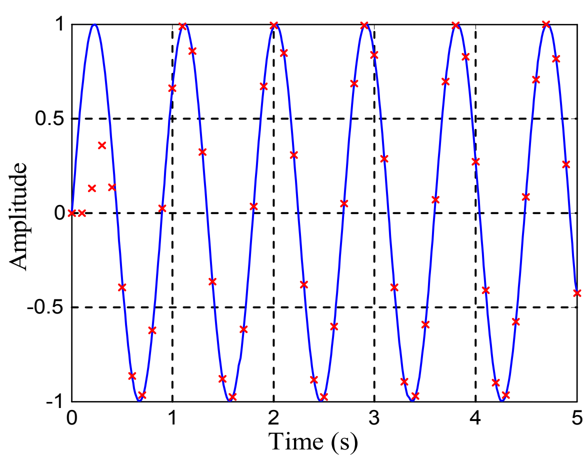

Fig. 2. Response of the closed loop to a sinewave reference (full line); the crosses indicate the sampled output.

Fig. 3 shows the time response obtained for a reference signal obtained by the sum of two sinewaves, with frequencies $\omega_1 = 7$ rad/s and $\omega_2 = 5$ rad/s, and amplitudes of 1 and 2 arbitrary units, and a ramp with a slope of 2 arbitrary units per second, where the polynomial T has been determined according to a combination of (13) and (14).

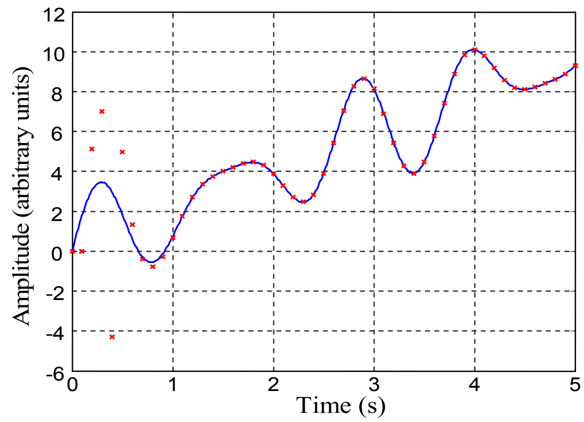

Fig. 3. Response of the closed loop to a superposition of three reference signals: two sinewaves and a ramp.

IV. Application to a PWM Inverter

The plant is here a pulse width modulation (PWM) controlled power inverter resulting from an industrial project, a simplified model of which is represented in Fig. 4 independently of the load. The digital controller has the purpose to let the output voltage of the inverter track a sinusoidal reference signal as perfectly as possible, in spite of load variations, and to maintain the current in the semiconductors under a given limit.

A. Model description

A discrete control model has been developed, which takes into account the pure time delay due to the conversion and computation times. The PWM is here of the constant frequency type, and the pure time delay induced by the digital control can represent a fraction of the period T_s of the switching device or even reach a full period.

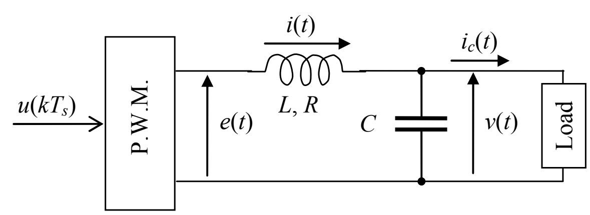

Fig. 4. Open-loop controlled plant. The discrete-time controller is sampled at a period T_s.

It is easy to prove that the constant frequency PWM is equivalent to a "classical" control problem by means of a DAC which has a scaling factor (control by average value) [7]. By choosing the current $i(t)$ through the input inductance and the voltage $v(t)$ across the capacitor as state variables and by modelling the load current $i_c(t)$ as a disturbance, the continuous-time plant of Fig. 4 has the following state space representation:

$$\mathbf{x}(t)^{\mathrm{T}} = [i(t) \quad v(t)]$$

$$\dot{\mathbf{x}}(t) = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} e(t) + \begin{pmatrix} 0 \\ -\frac{i_c(t)}{C} \end{pmatrix} \qquad (15)$$

For a two-level PWM control, the signal $e(t)$ consists of a sequence of pulses of amplitude $+E$ or $-E$, whose width at time kT_s can be controlled. The PWM signal (at fixed frequency) is generated numerically by using a programmable timer. Thus, the discrete control signal corresponds to the desired impulse width.

To derive a discrete-time model of the plant, we have thus chosen to represent the control signal $u(k)$ at time $t = kT_s$ as the pulse width variation around a reference signal having a duty cycle of $50\,\%$. This choice makes $u(k)$ symmetrical, as illustrated in Fig. 5:

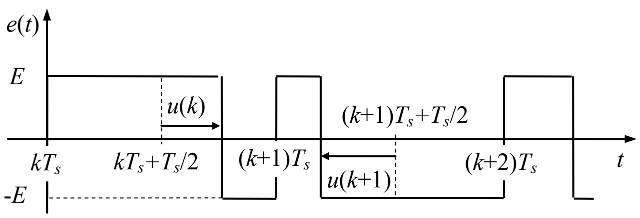

Fig. 5. Time diagram for a two-level PWM control. The control signal $u(k)$ varies between $-T_s/2$ and $+T_s/2$.

By integrating (15) over one sampling period, one can show that the effect of the control signal on the plant during that time is the same as if a constant average value of $e_m = (2E/T_s)u(k)$ had been applied during that period. The plant can thus be described by an approximate model,

$$\mathbf{x}(t)^{\mathrm{T}} = [i(t) \quad v(t)]$$

$$\dot{\mathbf{x}}(t) = \begin{pmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} \frac{1}{L} \\ 0 \end{pmatrix} e_m(t) + \begin{pmatrix} 0 \\ -\frac{i_c(t)}{C} \end{pmatrix}$$

$$= \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,e_m(t) + \mathbf{G}\,i_c(t) \qquad (16)$$

The operations of conversion and computation introduce a time delay before the control signal is output. In the case of a sampling which is synchronous with the switching clock of the PWM, let us assume that this pure time delay t_c amounts to some fraction of one full sampling period (Fig. 6). An analysis of the stability margins indicates that this delay must be introduced into the model, for the expected performance to be obtained.

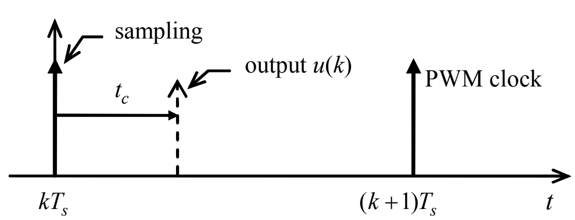

Fig. 6. Delay due to the conversion and computation times.

The integration interval of equation (16) from time $t = kT_s$ to $t = (k+1)T_s$ must be separated in two parts, since during the same control period two different signals $e_m(t)$ are applied:

$$e_m(t) = \begin{cases} e_m(k-1)\,; & 0 \le t < t_c \\ e_m(k) \quad ; & t_c \le t < T_s \end{cases}$$

According to [1], the discrete time model obtained by integrating (16), at vanishing disturbance, between two sample times is

$$\mathbf{x}(k+1) = e^{\mathbf{A}T_s}\mathbf{x}(k) + e^{\mathbf{A}(T_s-t_c)}\int_0^{t_c} e^{\mathbf{A}\tau}d\tau\,\mathbf{B}\,e_m(k-1) + \\ + \int_0^{T_s-t_c} e^{\mathbf{A}\tau}d\tau\,\mathbf{B}\,e_m(k)$$

By use of an augmented state vector,

$$\mathbf{x}_a(k) = \left[\mathbf{x}^{\mathrm{T}}(k) \quad e_m(k-1)\right]^{\mathrm{T}} = \left[i(k) \quad v(k) \quad e_m(k-1)\right]^{\mathrm{T}}$$

the state equation which describes the behaviour of the system becomes :

$$\mathbf{x}_a(k+1) = \mathbf{\Phi}\,\mathbf{x}_a(k) + \mathbf{\Gamma}\,e_m(k)\,, \qquad (17)$$

where

$$\mathbf{\Phi} = \left[\begin{array}{c|c} e^{\mathbf{A}T_s} & e^{\mathbf{A}(T_s-t_c)}\int_0^{t_c} e^{\mathbf{A}\tau}d\tau\,\mathbf{B} \\ \hline 0 & 0 \end{array}\right], \mathbf{\Gamma} = \left[\begin{array}{c} \int_0^{T_s-t_c} e^{\mathbf{A}\tau}d\tau\,\mathbf{B} \\ \hline 1 \end{array}\right] \qquad (18)$$

If one neglects some minor nonlinearities, such as the voltage drop due to the dead-time in the solid state switches, the simulation diagram is then given by the block diagram of Fig. 7, where the zero-order hold B_0 represents a digital to analog converter:

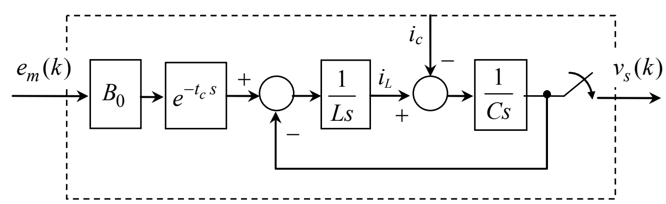

Fig. 7. Control system block diagram.

B. Controller design

The controller used here adopts an RST structure according to Fig. 1, and has been designed as described in section II. The control model is the discrete transfer function $B(z^{-1})/A(z^{-1})$ of the control system derived directly from (17) and (18):

$$G(z) = \frac{B(z^{-1})}{A(z^{-1})} = (0 \;\; 1 \;\; 0)(z\mathbf{I} - \mathbf{\Phi})^{-1}\mathbf{\Gamma}$$

$$= \frac{z^{-1}(b_1 + b_2\,z^{-1} + b_3\,z^{-2})}{1 - 2\cos(\omega_r T_s)\,z^{-1} + z^{-2}}, \quad \text{with} \;\; \omega_r = \frac{1}{\sqrt{LC}}$$

The coefficients b_1, b_2 and b_3 depend on the values of the output filter inductance and capacitance, but also on the pure time delay. Note however that, for the two extreme situations, the following values hold:

$$\text{at } t_c = 0 \ : b_1 = b_2 = (1-\cos\omega_r T_s);\ b_3 = 0$$
$$\text{at } t_c = T_s : b_1 = 0\ ;\ b_2 = b_3 = (1-\cos\omega_r T_s)$$

The denominator of the closed loop transfer function, $AS+BR$ as given by (4), is formally identified with the desired closed-loop characteristic polynomial, here a second order polynomial characterized by an undamped natural frequency $\omega_n = 1000$ rad/s and a damping ratio $\zeta = 0.7$:

$$A_m = 1-2e^{-\zeta\omega_n T_s}\cos(\omega_n T_s\sqrt{1-\zeta^2})\ z^{-1} + e^{-2\zeta\omega_n T_s} z^{-2}$$
$$= 1-1.9117\,z^{-1} + 0.9154\,z^{-2} \quad .$$

In order to reject the disturbance resulting from the load current i_c, an integral action is added to the S polynomial, and the final values of $R(z^{-1})$ and $S(z^{-1})$ are determined by the resolution of the primary Diophantine equation, as in [7].

At the difference however to our previous work, the polynomial $T(z^{-1})$ is determined here in two ways: by solving the auxiliary Diophantine equation first with a polynomial of first order on its left side as in (10), then by solving it so as to cancel exactly the static error resulting from a sinusoidal reference as in (12), here of frequency $\omega_0 = 2\pi \times 50 = 314$ rad/s.

C. Simulation results

Fig. 8 shows the error signal $\varepsilon(t)$ versus time in the case of a closed loop bandwidth of $\omega_n = 1000$ rad/s obtained by simulation, the inverter remaining unloaded. The reference signal is a sinewave of 325 V peak, at 50 Hz. The PWM clock, or sampling frequency, is $F_s = 1/T_s = 16$ kHz and a fractional delay of 20 µs is taken into account in the model, resulting in a discrete plant transfer function

$$G(z) = \frac{0.02526z^{-1} + 0.07785z^{-2} + 0.005613z^{-3}}{1-1.891z^{-1}+z^{-2}} = \frac{B(z^{-1})}{A(z^{-1})}$$

The choice here is not to compensate any pole nor zero of $G(z)$, which results in $B^{-}(z^{-1}) = B(z^{-1})$.

The polynomial T is determined here according to our first solution, thus by the Diophantine equation (10) with $m = 1$, resulting in $T = 0.8405 - 0.8061\,z^{-1}$.

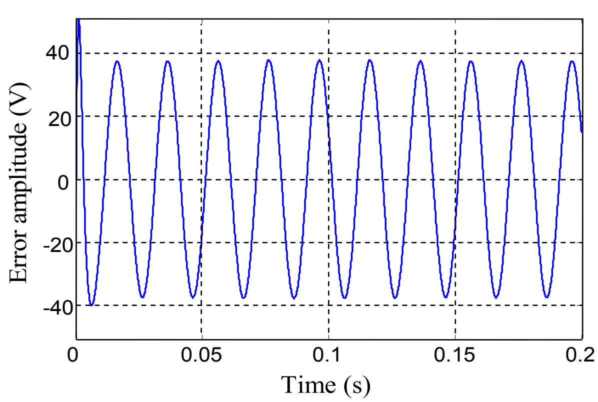

Fig. 8. Error signal versus time in the case of a polynomial factor in the auxiliary Diophantine equation.

Fig. 9 represents the error signal obtained from our simulation with the same ω_n and the same reference input, but this time with the use of the second type of auxiliary Diophantine equation, as given by (12) with $\omega_0 = 314$ rad/s:

$$(1-1.9996\,z^{-1}+z^{-2})\,L + B^{-}B'_m = A_m\,.$$

The resulting polynomial T is in this case:

$$T(z^{-1}) = 0.8338 - 0.8033\,z^{-1}\,.$$

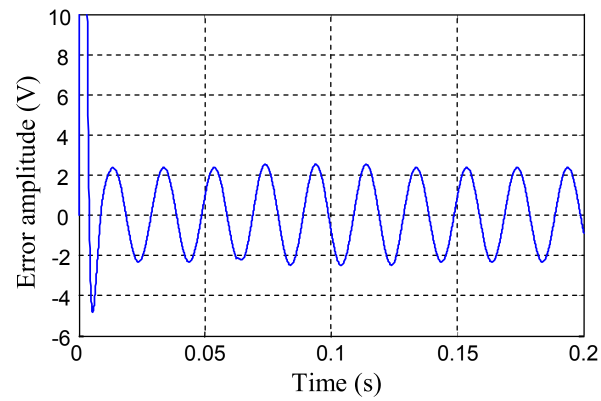

Fig. 9. Error signal versus time in the case of a sinusoidal factor in the auxiliary Diophantine equation.

It easily seen that the error is reduced by almost a factor of 20 in the second case.

D. Discussion and numerical results

As suggested in [8], it is also possible to add a term of oscillatory type to the polynomial $S(z^{-1})$, in order to cancel the steady-state error with respect to sinusoidal references. A main difference between this solution and the one presented here is that the inclusion of such a term inside the loop will generally result in a degradation of the stability margins, whereas in our solution the compensation occurs outside the loop and will thus have no effect on the stability of the closed loop. As a matter of fact, in the worst-case situation, corresponding to a pure time delay of one sampling period, the solution of adding an oscillatory term to the polynomial $S(z^{-1})$ did not enable us to obtain an RST regulator guaranteeing stability in the case of the complete process.

Table I summarizes and compares the performances obtained, in terms of stability margins and amplitude of error in response to a sinusoidal reference at a frequency of 50 Hz, for the various strategies of RST regulator design.

The following observations resulting from the tests carried out are worth being pointed out:

- For "classical" RST regulator, to ignore the specificity of the sinusoidal reference results in an important error amplitude, ranging from 38 V to 185 V according to the bandwidth of the closed loop.
- The RST regulator determined with a polynomial term in the auxiliary Diophantine equation limits the error amplitude in a way inversely proportional to the increase of the bandwidth (from 40 V to 7 V), but accompanied

by an important decrease of the stability margins.

- For the RST regulator determined with a sinusoidal term in the auxiliary Diophantine equation, the amplitude of the error remains small and depends little on the increase of the bandwidth.

TABLE I
RST-CONTROLLER PERFORMANCE FOR VARIOUS DESIGNS

Design #	Closed-loop bandwith (rad/s)	Phase margin (deg.)	Gain margin (dB)	Delay margin (samples)	Peak error (V)
1	1000	53	5	1.61	185
2	1000	53	5	1.61	40
3	1000	53	5	1.61	3.6
4	3000	31	5	0.86	68
5	3000	31	5	0.86	7
6	3000	31	5	0.86	2.6
7	6000	14	2.6	0.3	38
8	6000	14	2.6	0.3	7
9	6000	14	2.6	0.3	2.7

(1, 4, 7) Classical RST design.
(2, 5, 8) RST controller in the case of a polynomial factor in the auxiliary Diophantine equation.
(3, 6, 9) RST controller in the case of a sinusoidal factor in the auxiliary Diophantine equation.

To summarize, the addition of a sinusoidal term in the auxiliary Diophantine equation makes it possible to lower the closed loop bandwidth while ensuring a small error with respect to sinusoidal references and sufficient stability margins.

Finally, it is interesting to mention that the residual error, which is present in the case of a synthesis with a sinusoidal term (cases 3, 6 and 9 of Table I), is mainly due to the anti-aliasing filter placed in the simulation diagram before sampling of the continuous-time measurements.

E. Comparison with PID regulator

Fig. 10 shows the temporal evolutions of the output voltages obtained in the case of an RST and a PID regulator with comparable cut off frequencies and a linear load. An important amplitude error takes place in the second case, reaching almost 30% of the amplitude of the reference signal, associated with a large phase shift, which can be harmful in case of a coupling to the distribution network.

The use of a nonlinear load, which consumes an impulse current, leads to a comparable difference in terms of tracking error, whereas the rate of harmonic distortion is roughly the same with the two types of regulators.

Lastly, one can notice that adjusting a PID regulator is relatively difficult within the framework of this application, because of the desired performance and the time delay.

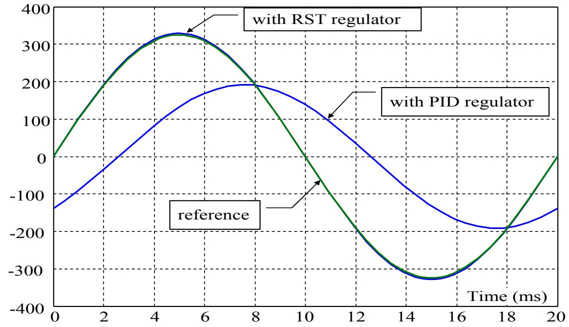

Fig. 10. Output voltage versus time with RST and PID regulators.

V. CONCLUSION

In this paper a methodology of synthesis of an RST regulator, enabling to achieve steady-state error cancellation with respect to harmonic references, is carried out. At the difference to a more traditional approach, which consists in adding an oscillatory term in the direct chain, thus inside of the control loop, it is based on the resolution of an auxiliary Diophantine equation. This makes it possible to express the problem in the form of a polynomial equation, the solution of which yields the polynomial T of the regulator.

This approach leads to better stability margins and thus allows the synthesis of a regulator, even when robustness is difficult to ensure due to a bad conditioning of the control model.

Lastly, this approach can be also used in the case of reference signals comprising harmonic signals of different frequencies as for example for the control of power line active compensators [9].

The interest of the proposed control strategy was shown with the regulation of an average power inverter, which has been investigated in the framework of an industrial project, and numerical examples have proved its efficiency and robustness.

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