Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Stable Cooperative Surveillance

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***Abstract*— We consider a cooperative surveillance problem for a group of autonomous air vehicles (AAVs) that periodically receives information on suspected locations of targets from a satellite and then must cooperate to decide which AAV should search for each target. This cooperation must be performed in spite of imperfect inter-vehicle communications, less than full communication connectivity between vehicles, uncertainty in target locations, and imperfect vehicle search sensors. We represent the state of the search progress with a "search map," and use an invariant set to model the set of states where there is no useful information on target locations. We show that the invariant set is exponentially stable for a class of cooperative surveillance strategies. Next, we show via simulations the impact of imperfect communications, imperfect vehicle search sensors, uncertainty in search locations, and pop-up suspected locations on performance.**

I. Introduction

Groups of possibly many AAVs of different types, connected via a communication network to implement a "vehicle network," are technologically feasible and hold the potential to greatly expand operational capabilities at a lower cost. Cooperative control for navigation of such vehicle groups involves coordinating the activities of several agents so they work together to complete tasks in order to achieve a common goal. Here, we study how to perform a cooperative AAV surveillance task that involves two components: (i) search for stationary targets located in a region and (ii) reacting to "pop up" suspected target locations that are provided by a satellite (or other high-flying platform).

There is a significant amount of current research activity focused on cooperative control of AAVs and some research directions in this field are identified in [1]. Solutions to general cooperative control problems can be obtained via solutions to vehicle route planning (VRP) problems [2]. Additional VRP-related work focusing on cooperative search and coordinated sequencing of tasks is in [3]-[6]. Using such approaches, significant mission performance benefits can be realized via cooperation in some situations, most notably when there is not a high level of uncertainty.

One challenge in cooperative control problems is to overcome the effects of uncertainty so that benefits of cooperation can still be realized. When uncertainty dominates the system, cooperative strategies will not be able to achieve the high level of coordination achieved in many of the above-mentioned studies that assume perfect communications. The most that can be hoped for is to achieve *some* benefit from cooperation. Along these lines, recent work considering imperfect communications is found in [7]-[11]. Here, we continue along the lines of such work, but with a focus on cooperative surveillance when there are a variety of information flow constraints.

Of all the current work in cooperative control, the most closely related to this study are the "persistent area denial (PAD)" [12], search theory [13], [14], and the "map-based approaches" in [15]-[18].

The main contributions of this paper are (i) the use of search-theoretic ROR maps [14] for the coordination of the search efforts of multiple AAVs, (ii) a novel characterization of cooperation objectives as stability properties of an invariant set, and (iii) the use of standard Lyapunov stability-theoretic methods for verification of cooperative control systems. In Section II the cooperative surveillance problem is formulated and modeled. The stability analysis is in Section III. Simulations are in Section IV.

II. Cooperative Surveillance Problem Formulation

Suppose that there is a group of AAVs that performs surveillance of a given area and can use information from a satellite to pursue suspected locations in that area. Assume that the set of AAVs is given a number of targets to search for with their respective likely locations. AAVs must work together autonomously in order to try to maximize the probability of finding targets in the environment with minimal AAV effort.

A. Discrete Event System Model

The cooperative surveillance problem is modeled as a nonlinear discrete time asynchronous dynamic system [19] with the model $G = (\mathcal{X},\, \mathcal{E},\, f_e,\, g,\, E_v)$. Here, $\mathcal{X}$ is the set of states. The set of events is denoted by $\mathcal{E}$. State transitions are defined by $f_e : \mathcal{X} \longrightarrow \mathcal{X}, e \in \mathcal{E}$. The enable function, g, defines the occurrence of an event e by $g : \mathcal{X} \longrightarrow \mathcal{P}(\mathcal{E}) - \{\emptyset\}$ being $\mathcal{P}(\mathcal{E})$ the power set of $\mathcal{E}$. Note that f_e is defined when $e \in g$. Let $k \geq 0$ represent the time indices associated with both the states $x(k) \in \mathcal{X}$ and the enabled events $e(k) \in g$.

The authors would like to thank AFRL/VA and AFOSR for financial support of the OSU Collaborative Center of Control Science (CCCS) where this research was performed (Grant F33615-01-2-3154). The authors would like to thank Andrew Sparks from AFRL for assistance in formulating the problem and Yong Liu for some initial collaboration on this problem.
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Let E_v denote a set of "valid" event trajectories. Next, we define these for the cooperative surveillance problem.

B. Vehicle Model

Suppose that the m^{th} AAV flies at a constant altitude and obeys a continuous time kinematic model given by the Dubin's car [20], $\dot{x}_1^m(t) = v\cos\theta^m(t), \dot{x}_2^m(t) = v\sin\theta^m(t), \dot{\theta}^m(t) = \omega_{max}u^m(t)$ where x_1^m is its horizontal position, x_2^m is its vertical position, v is its forward (constant) velocity, θ^m is its heading direction, ω_{max} is its maximum angular velocity, and $-1 \leq u^m \leq 1$ is the steering input. Hence, $u^m = +1(-1)$ stands for the sharpest possible turn to the right (respectively, left). The minimum turn radius for the vehicle is $R = \frac{v}{\omega_{max}}$. Rather than allow $u^m(t)$ to be arbitrary, vehicles will either travel on the minimum turn radius or on straight lines that are the optimal paths from a vehicle's current location and orientation to a desired location and orientation [21].

C. Search Environment and Targets

The search environment is assumed to be rectangular with length L and width W. Divide the edge with length L into $r \in \mathbb{Z}^+$ segments and the edge with length W into $s \in \mathbb{Z}^+$ segments ($\mathbb{Z}^+$ is the set of the positive integers). This decomposes the search area into rs cells, each with a size of $\frac{LW}{rs}$. Thus, there are $N_Q = rs$ discrete cells to search, and these cells are numbered so that the discretized search space is $Q = \{1, \ldots, N_Q\}$. It is assumed that all the AAVs know how the cells are numbered and hence know N_Q. It is also assumed that there are N_V AAVs that search for targets and let $V = \{1, \ldots, N_V\}$ denote the set of vehicles.

Assume that there are N_D distinct valid stationary targets that the AAVs are searching for and let $D = \{1, \ldots, N_D\}$ denote the set of targets. Suppose that the size of each target is considerably smaller than the size of each cell. Also, there could be more than one target located in one cell; however, it is assumed that all targets inside the same cell are separated in such a way that they can be detected by the sensors of the AAVs if, for example, they take enough "looks" at the cell.

AAV sensors return information about the targets. Assume that the sensor of AAV m has a rectangular "footprint" with depth $d_{f_1}^m = N_\ell^m \frac{L}{r}$, width $d_{f_2}^m = N_w^m \frac{W}{s}$, $N_\ell^m, N_w^m \in \mathbb{Z}^+$ with $N_\ell^m < r, N_w^m < s$, and the distance from AAV m to the center of the footprint is denoted by d_s^m for all $m \in V$. Whenever a AAV is going to search a cell, it will approach this cell at an angle in the set $\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$. Due to sensor inaccuracies, it is assumed that searching a cell once may not result in finding a target even if it is there. Multiple searches of the same cell may be needed to find a target. In order to avoid detection by an adversary, AAVs turn their sensors on only when they have reached the desired orientation and location with respect to the center of the cell where the target is suspected to be located. Once a AAV takes a snapshot of the cell to be searched, the AAV will turn its sensor off until a new cell of interest is reached again. When targets are placed on the boundaries of the cells, it is assumed that when a cell is searched the search includes the left and lower boundaries of the cell, but it does not include the two endpoints lying opposite to where the boundaries join. This guarantees that the same object cannot be found by searching two different cells.

D. The Rate of Return (ROR) Map

Let $p_j^m(q)$ be the AAV $m \in V$ a priori probability that target $j \in D$ is in cell $q \in Q$. These a priori probabilities must be specified by information gathering sources (e.g., the satellite or other high-flying platform). For simplicity, we assume here that $p_j^m(q) = p_j(q)$ for all $m \in V$ so all AAVs get the same information.

Let $\ell^m(q,k) \in \{0, 1, \ldots\}$ be the number of looks (passes when a "snapshot" is taken) performed in cell $q \in Q$ by AAV $m \in V$ by time index k, which is a measure of the amount of search effort dedicated to cell $q \in Q$ [14]. The AAVs share $\ell^m(q,k)$ for all $m \in V$, via inter-vehicle send/receive communications that may have arbitrary but finite delays. These delays should not be thought of as arising only from delays on, for instance, communication network links, but also from occlusions and sensing/communication range constraints, or temporary loss of a communication link between vehicles. The maximum delay between AAV m' and AAV m is known and equal to $B^{m'm} \in \mathbb{Z}^+$. The unknown but bounded delay is modeled by a random choice of $\tau_k^{m'm}, 1 \leq \tau_k^{m'm} \leq B^{m'm}$ (with no assumption on the underlying statistics). If $m = m'$, then $\tau_k^{m'm} = 0$ for all k since a vehicle knows its own information. Each AAV $m \in V$ stores the values of $\ell^{m'}(q, k - \tau_k^{m'm})$ (i.e., the most recent information update that it has received from AAV $m' \in V$) if the received $\ell^{m'}(\cdot)$ is greater than the current value held by AAV m. Otherwise, AAV m discards $\ell^{m'}(\cdot)$ and keeps the old one (this event could arise due to the random nature of the inter-vehicle communication that could result in message misordering of the $\ell^{m'}(\cdot)$ values sent by AAV m'). Each AAV $m \in V$ in general only has up to date information on $\ell^m(q,k)$ (its own number of looks), not on the number of looks of other AAVs $m' \neq m$ that are out of date by $k - \tau_k^{m'm}$. AAV $m \in V$ uses the latest information it has to form $\hat{\ell}^m(q,k) = \sum_{m'=1}^{N_V} \ell^{m'}(q, k - \tau_k^{m'm})$ its estimate of the total number of looks taken by all N_V AAVs.

Let $L^m \subset V$ be a set that contains the AAVs $m', m' \neq m$ that have looked at cells $q \in Q$, possibly multiple times denoted by $n^m(\ell^{m'}(q,k)) \geq 1$, between the time when AAV m decides to perform an additional look at target j in cell q and the time when AAV m actually receives the new looks. Let $Q^m \subset Q$ be the set held by AAV m, which represents the cells that have been visited by AAV $m' \in L^m \subset V$. Let C_m denote the set of cells covered by the sensor footprint of AAV m. Define the set of "local" events for each AAV m as $\left\{e_1^{m,L^m,Q^m}, e_2^{m,C_m}, e_3^{m,L^m,Q^m,C_m}\right\}$, where e_1^{m,L^m,Q^m} = "Reception of looks at $q \in Q^m$ from $m' \in L^m$," e_2^{m,C_m} = "AAV m sensor on for cells $q \in C_m$," e_3^{m,L^m,Q^m,C_m} = "e_1^{m,L^m,Q^m} and e_2^{m,C_m} occur simultaneously" so that the set of events of the system such that $e(k) \in g$ is any subset of

$$\mathcal{E} = \mathcal{P}\left(\bigcup_{m=1}^{N_V} \left\{ \quad e_1^{m,\, L^m,\, Q^m},\ e_2^{m,\, C_m}, \right.\right.$$
$$\left.\left. e_3^{m,\, L^m,\, Q^m,\, C_m} \right\}\right) - \{\emptyset\} \qquad (1)$$

that contains all the elements of the right side of Equation (1) except that only one of the three types of local events can be in any event $e \in \mathcal{E}$. Thus, the enable function g allows the occurrence of a set of possible events at the same time.

Let $a_{j,q}^m > 0$ be a parameter that is proportional to the sensor capabilities of AAV m for target j in cell q. Let $\pi_{j,q}^m \in (0, 1]$ be the probability that AAV m detects target j in cell q on a single look, given that target j is in cell q. Assume that AAV m knows each $\pi_{j,q}^{m'}$ and $a_{j,q}^{m'}$ from AAV $m', m' \neq m, m \in V$. Let the detection function [13], [14], $b_j^m(\hat{\ell}^m(q,k))$, be the conditional probability of detecting target j in cell q by AAV m, given that target j is in cell q. If each cell look has an independent probability of finding the target, then the detection function is $b_j^m(\hat{\ell}^m(q,k)) = 1 - \prod_{m'=1}^{N_V}(1-\pi_{j,q}^{m'})^{a_{j,q}^{m'}\ell^{m'}(q,k-\tau_k^{m'm})}$. Since $\hat{\ell}^m(q,k)$ is an estimate, $b_j^m(\hat{\ell}^m(q,k))$ should be thought of as an estimate of the probability $j \in D$ was found. AAV $m \in V$ will use this estimate to decide where to search.

The cost for searching for target j with $\ell^m(q,k)$ looks is defined as $c_j^m(\ell^m(q,k)) = c\,\ell^m(q,k)$ where $c > 0$ is the cost of a single search. Note that the cost of one look at target j in cell q is given by $\gamma_j^m(\ell^m(q,k)+1) = c_j^m(\ell^m(q,k)+1) - c_j^m(\ell^m(q,k)) = c$. In our simulations we will incorporate the cost of moving to a location as part of the decision making; however, in our theory c is independent of distance. It is a future direction to incorporate distance into c for the theory.

Let $\hat{\beta}_j^m(\hat{\ell}^m(q,k)+1)$ be the m^{th} AAV's probability of failing to detect target j on the first $\hat{\ell}^m(q,k)$ looks in cell q and succeeding on the $\hat{\ell}^m(q,k)^{th}+1$ look (where 1 in the expression $\hat{\ell}^m(q,k)+1$ is the look that AAV m is considering whether to take), given that target j is in cell q and assuming that no other AAVs look at cell q before AAV m does. The variable $\hat{\beta}_j^m(\hat{\ell}^m(q,k)+1)$ is

$$\hat{\beta}_j^m(\hat{\ell}^m(q,k)+1) = \hat{b}_j^m(\hat{\ell}^m(q,k)+1) - b_j^m(\hat{\ell}^m(q,k)) \quad (2)$$

where $\hat{b}_j^m(\hat{\ell}^m(q,k)+1)$ is the estimated detection function computed by time index k when AAV m will perform an additional look in cell q. It is an estimated value since there exists the possibility that new number of looks by AAVs $m' \neq m$ could be taken before AAV m takes the look at cell q, which would change the expression shown in Equation (2).

The rate of return (ROR) [14] in cell q for target j and AAV m can be defined in order to determine the benefit that AAV m obtains at time index k when an additional look will be performed to find target j in cell q. Hence, $\hat{\rho}_j^m(\hat{\ell}^m(q,k)+1) = \frac{p_j(q)\hat{\beta}_j^m(\hat{\ell}^m(q,k)+1)}{\gamma_j^m(\ell^m(q,k)+1)}$ which gives the ratio of the increase in probability to the increase in cost when $\hat{\ell}^m(q,k)$ looks for target j have been performed in cell q and an additional look will be executed by AAV m in cell q. Clearly AAV m would like to place search effort in cells where this quantity is the largest.

E. Detection of Targets

Note that when AAV m finds target j, this AAV makes $p_j(q) = 0$ and communicates this information to the satellite, which broadcasts to all the AAVs the target that has been found. Hence, all the AAVs make their respective $p_j(q) = 0$ as well. By making $p_j(q) = 0$, the AAV that found target j along with the ones that received this information are forcing their ROR maps to be equal to zero. This means that this target is no longer of interest to these AAVs. Below, in our stability analysis we will assume either that targets are not found, or that if one is then this corresponds to a state perturbation and hence is viewed as restarting the process.

F. Percentage Rate of Return (PROR) Map

Next, we introduce what we call the "percentage rate of return" (PROR) map $\overline{\rho}_j^m(\hat{\ell}^m(q,k)+1)$, which is the ratio of the percentage change in probability to the percentage change in cost when $\hat{\ell}^m(q,k)$ looks for target j have been performed by AAV m in cell q and an additional look will be executed by AAV m in cell q. Hence, $\hat{\overline{\rho}}_j^m(\hat{\ell}^m(q,k)+1) = \frac{p_j(q)\hat{\beta}_j^m(\hat{\ell}^m(q,k)+1)c_j^m(\ell^m(q,k))}{b_j^m(\hat{\ell}^m(q,k))\gamma_j^m(\ell^m(q,k)+1)}$. Note that there are differences between the PROR and the ROR map cases. However, as in the ROR case, AAV m would like to place search effort in cells where $\hat{\overline{\rho}}_j^m(\hat{\ell}^m(q,k)+1)$ is the largest.

G. Cooperative Surveillance Strategies

The ROR maps will be used in the strategies; however, the PROR maps can also be used by replacing each variable $\hat{\rho}_j^m(\cdot)$ by $\hat{\overline{\rho}}_j^m(\cdot)$. Next, one choice for search strategies is discussed. This strategy commands a AAV to search for the target in the cell with the highest ROR map that is obtained using both its own number of looks and the possibly outdated number of looks received from other AAVs through communications. In particular, the search decision for AAV $m \in V$ at time index k is $q^*(m,k) \in Q$ which defines the cell it will search in next. Here, the strategy "go to the cell where a single target is most likely to be present" is chosen as

$$q^*(m,k) = \arg\max_{j \in D,\ q \in Q}\left\{\hat{\rho}_j^m\left(\hat{\ell}^m(q,k)+1\right)\right\} \qquad (3)$$

and once AAV m reaches the location where cell $q^*(m,k)$ is, it searches for all target types in it (break ties arbitrarily or via going to the closest cell). If AAV m receives a new look value for cell q from any AAV $m' \neq m$ while AAV m is heading to cell q, this could lead AAV m to cell $q' \neq q$ next if the ROR map of cell q is not the maximizer at the reception time of the new information. Thus, AAV decision reconsiderations about the current maximum benefit are allowed in this framework.

A policy "go look in any cell such that *any* target is most likely to be present" can be defined as follows

$$q^*(m,k) = \arg\max_{q \in Q} \left\{ \sum_{j \in D} \left(\hat{\rho}_j^m \left(\hat{\ell}^m(q,k) + 1 \right) \right) \right\} \qquad (4)$$

where ties are broken arbitrarily or by going to the closest cell.

Note that both policies only rely on information at time index k that is locally available at AAV m; hence, they can be implemented in a distributed fashion. Also, both policies are computationally simple.

H. Distributed ROR Map Updates

The cooperative AAV surveillance problem can be viewed as being composed of N_V subsystems and AAV $m \in V$ is associated with the m^{th} subsystem. Let the state of AAV m be $x^m(k) \in {\mathbb{R}^+}^{N_D N_Q}$ where $x^m(k) = \left[\rho_1^m(\hat{\ell}^m(1,k)), \ldots, \rho_{N_D}^m(\hat{\ell}^m(N_Q,k))\right]^\top$ is the vector that contains all the ROR map values that AAV m holds by time index k. Let $x(k) = [x^1(k)^\top, \ldots, x^{N_V}(k)^\top]^\top = \mathcal{X} \in {\mathbb{R}^+}^{N_V N_D N_Q}$ be the states of the system at time index k.

Let the sequence $\{x(k-1)\} \in \mathcal{X}^{\mathbb{N}}$ denote the state trajectory such that $x(k) = f_{e(k)}(x(k-1))$ for some $e(k) \in g(x(k))$ for all $k \geq 0$. Define the sequence $\{e(k)\} \in \mathcal{E}^{\mathbb{N}}$ as an event trajectory such that there exists a state trajectory, $\{x(k-1)\} \in \mathcal{X}^{\mathbb{N}}$, where $e(k) \in g(x(k))$ for every k. The set of all event trajectories is denoted by $E \subset \mathcal{E}^{\mathbb{N}}$. Since each AAV m stores $\ell^{m'}(q, k - \tau_k^{m'm})$ from any AAV $m' \neq m$ when it is greater than the current value that AAV m holds, the set $E_v \subset E$ can account for these events and it prunes the set E.

An ROR map will be updated at anytime when an event $e(k) \in g(x(k))$ occurs using the state transition function f_e. Thus, there are three ways of updating the ROR maps for each AAV corresponding to the three event types. An ROR map update is performed by AAV m when it is not visiting a cell of interest and receives one or more number of looks from AAV $m' \neq m$. For event $e_1^{m,\, L^m(q,k),\, Q^m(k)} \in e(k) \in g(x(k))$, components of $x^m(k)$ with $q \in Q^m(k)$ have $\rho_j^m(\hat{\ell}^m(q,k)) = \rho_j^m(\hat{\ell}^m(q,k-1))h(\ell^m(q,k))$ where $h(\ell^m(q,k)) = \prod_{m'' \in L^m(q,k)} (1 - \pi_{j,q}^{m''})^{a_{j,q}^{m''} n^m(\ell^{m''}(q,k))}$. The occurrence of event $e_2^{m,\, C_m(k)} \in e(k) \in g(x(k))$ forces AAV m to update $\rho_j^m(\hat{\ell}^m(q,k))$ so that the components of $x^m(k)$ with $q \in C_m(k)$ have $\rho_j^m(\hat{\ell}^m(q,k)) = \rho_j^m(\hat{\ell}^m(q,k-1))(1-\pi_{j,q}^m)^{a_{j,q}^m}$. When an event $e_3^{m,\, L^m(q,k),\, Q^m(k),\, C_m(k)} \in e(k) \in g(x(k))$ occurs, the components of $x^m(k)$ with $q \in C_m(k) \bigcap Q^m(k)$ have $\rho_j^m(\hat{\ell}^m(q,k)) = \rho_j^m(\hat{\ell}^m(q,k-1))(1 - \pi_{j,q}^m)^{a_{j,q}^m} h(\ell^m(q,k))$, the components with $q \in (C_m(k) - Q^m(k))$ have $\rho_j^m(\hat{\ell}^m(q,k)) = \rho_j^m(\hat{\ell}^m(q,k-1))(1 - \pi_{j,q}^m)^{a_{j,q}^m}$, while the components of $x^m(k)$ with $q \in (Q^m(k) - C_m(k))$ have $\rho_j^m(\hat{\ell}^m(q,k)) = \rho_j^m(\hat{\ell}^m(q,k-1))h(\ell^m(q,k))$. If the PROR approach is used, then map updates can also be derived as above.

III. Stability Analysis

Let $H = \{1, \ldots, N_V N_D N_Q\}$. If z is a vector, then $(z)_i$ denotes the i^{th} component of this vector. Let $\mathcal{X}_\varepsilon = \{x(k) \in \mathcal{X} : 0 \leq (x(k))_i \leq \varepsilon, \text{ for all } i \in H\}, \ \varepsilon > 0$ be the set that holds the AAVs' ROR map values that are less than or equal to ε at time index k. Typically, ε is chosen to be small so that, since the search cost c is fixed, the set $\mathcal{X}_\varepsilon$ characterizes when the probability that further looks of any cell are unlikely to find a target. Hence, when the state is in $\mathcal{X}_\varepsilon$ the group of AAVs has "used up" the information about likely target locations that was provided initially by the satellite. Initially, the AAVs by themselves are assumed to have no target location information (so the state is in $\mathcal{X}_\varepsilon$) and then at $k = 0$ the satellite provides $p_j(q)$ and this corresponds to a perturbation from the $\mathcal{X}_\varepsilon$ set. Let $d(x(k), \mathcal{X}_\varepsilon) : \mathcal{X} \to \mathbb{R}^+$ denote a metric on $\mathbb{R}^+$

$$d(x(k), \mathcal{X}_\varepsilon) = \inf\{\max_i\{|(x(k))_i - (x')_i|\}\} \qquad (5)$$

for all $i \in H, x' \in \mathcal{X}_\varepsilon$. We are interested in showing that the cooperative surveillance strategy will drive perturbations from $\mathcal{X}_\varepsilon$ back into $\mathcal{X}_\varepsilon$. To do that, we first introduce a lemma that shows that every component $(x(k))_i$ will get closer to the set $\mathcal{X}_\varepsilon$ several time indices after k. The derivation of these time indices after k depends on temporal (i.e., delays) and spatial (i.e., the position of all the AAVs at time index k, and the density of the suspected target locations) characteristics of the cooperative surveillance problem. Finally, we use the result obtained in the lemma to show that $\mathcal{X}_\varepsilon$ is exponentially stable.

Lemma 3.1: Let $N_q^{m'}$ be the number of times that AAV m' visits cell q between time indices k and $k + C(N_q^{m'})$. Let q^ℓ be the last cell visited by AAV m at time index $k + C(N_q^{m'})$. If we define a Lyapunov function $V(k) = \max_i\{|(x(k))_i - \varepsilon|\}$ and N_V AAVs use any cooperative surveillance strategy that satisfies Equation (3), then there exists a function

$$\begin{aligned} C(N_q^{m'}) \;=\; & N_Q N_V - N_V + 1 + \sum_{m' \neq m} \left[\sum_{q=1}^{N_Q} N_q^{m'} \right. \\ & \left. + \sum_{q \neq q^\ell} N_q^{m'}(N_V - 1) + N_{q^\ell}^{m'}(N_V - 2) \right] \end{aligned} \qquad (6)$$

that guarantees that $V(k + C(N_q^{m'})) < V(k)$.

Remark 1 A lower bound for $C(N_q^{m'})$ could be obtained in Equation (6) for the special case where every AAV visits every cell once (i.e., $N_q^{m'} = 1$ for all $m' \in V, q \in Q$) and the delays are so large so that Equation (6) is now $C(1) = N_Q N_V$.

Remark 2 A similar result to Remark 1 can be obtained for the case when the delays are small and when every AAV m visits a different group of cells between k and $k + C(N_q^{m'})$ such that $Q_c^m \bigcap Q_c^{m'} = \{\emptyset\}$ for all $m \neq m'$ and $\bigcup_{m=1}^{N_V} Q_c^m = Q$. Assume that $N_q^m = 1$, and that every AAV $m' \neq m$ receives this information. Using Equation (6) for this particular case, $C(1) = N_V N_Q$. The benefit of sharing information is shown in this particular scenario since every AAV does not have to visit every cell, which minimizes AAV's fuel expenditure.

Theorem 3.2: If N_V AAVs use any cooperative surveillance strategy that satisfies Equation (3), then the set $\mathcal{X}_\varepsilon$ is invariant and exponentially stable in the large w.r.t. E_v.

For $c_1 = c_2 = 1$, $0 < \sigma < 1$, and $0 < c_3 = \max\{\overline{\pi}^{\underline{a}}, 1 - \max\{(1-\underline{\pi})^{\underline{a}}, \sigma\}\} < c_2$, Equations (7) and (8) are satisfied

$$c_1\, d(x(k), \mathcal{X}_\varepsilon) \le V(k) \le c_2\, d(x(k), \mathcal{X}_\varepsilon) \qquad (7)$$

$$V(k + C(N_q^{m'})) - V(k) \le -c_3 d(x(k), \mathcal{X}_\varepsilon) \qquad (8)$$

for all $x(0) \in \mathcal{X}$, $k \ge 0$ with $0 < c_3/c_2 < 1$ for some specified $C(N_q^{m'}) > 0$.

Remark 3 The ROR map values will enter faster into the invariant set when the probability of detecting targets in cells on one single look is close to 1 (i.e., $\underline{\pi} = \overline{\pi} \approx 1$) and the AAVs' group possesses high quality sensors (i.e., $\underline{a} \gg 1$) for all $m \in V, j \in D, q \in Q$ (see the variables in c_3).

Remark 4 An analogous stability analysis can be carried out for the strategy shown in Equation (4). Moreover, an analysis can be conducted for showing the stability of $\mathcal{X}_\varepsilon$ when PROR maps are used.

Remark 5 A "noncooperative" case can be defined if the AAVs do not communicate with each other so they do not share the number of looks. It can be shown that the performance of the cooperative strategies can be no worse than the noncooperative ones under some special cases.

IV. Simulations

Here, two simulation are shown in this section. Travel distances between cells are considered for each AAV at each decision time. Let $x^q = [x_1^q, x_2^q]^\top$ denote the coordinates in the (x_1^q, x_2^q) plane of the center of the q^{th} cell. Let $d_s(x_m(k), x^q) > 0$ be the minimum distance that AAV m must travel at index k from its current location and orientation $x_m(k) = [x_1^m(k), x_2^m(k), \theta^m(k)]^\top$ to take a snapshot at cell q. Notice that since all AAVs are moving all the time, it is not possible to have $d_s(x_m(k), x^q) = 0$ due to if an AAV needs to take two consecutive snapshots of the same cell, then this particular AAV will travel a distance, different from zero, determined by the path generated by Dubin's car. The cooperative strategy defined in Equation (3) is modified for the nonplanning case as $q^*(m,k) = \arg\max_{j\in D,\ q\in Q}\left\{\hat{\rho}_j^m\left(\hat{\ell}^m(q,k)+1\right) + \frac{1}{d_s(x_m(k),x^q)}\right\}$.

Consider, for the cooperative strategy with planning, that AAV m plans to visit a sequence of cells next at index k. The planning cooperative surveillance strategy for AAV m is made over the ROR maps. Suppose now that the AAVs share the cells included in the planning horizon along with the number of looks for all $m \in V$, via inter-vehicle communications with arbitrary but finite delays. Thus, each AAV m first flattens out each ROR map associated with each cell contained in the projection length of each AAV $m' \neq m$, and then it further flattens out, except in the first projection, each ROR map covered by the sensor footprint in its previous projection length before making any decision about the cell to be visited next.

Let $L = 10000$ meters, $W = 10000$ meters, $r = s = 100$, $N_V = 3$, $v = 150$ meters/sec, $R = 800$ meters, $d_{f_1}^m = d_{f_2}^m = 500$ meters, $d_s^m = 400$ meters, $N_D = 4$ (i.e., four suspected areas in the environment, see Figure 1), $\pi_{j,q}^{m'} = 0.8$, $a_{j,q}^{m'} = 0.9$ (e.g., imperfect sensors), for all $m \in V, j \in D, q \in Q$, the sampling time $T_s = 0.1$ sec., and the simulation length of 600 sec. All these above parameters are used in the simulations unless otherwise stated.

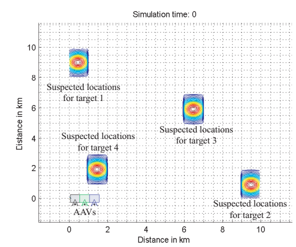
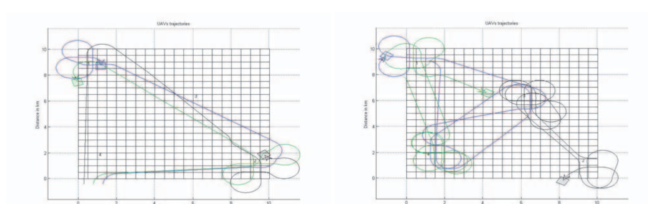


Fig. 1. Initial positions and orientations of AAVs and suspected locations of targets. AAVs are labeled 1 up to 3 from left to right.

A. Performance in the Presence of Pop-Up Suspected Locations

The behavior of the cooperative strategies with planning is shown in this section when some suspected locations pop-up when the group of AAVs have already begun the mission. The same parameters mentioned above are used in this simulation except the following: The suspected area of targets 1 and 2 are available to all the AAVs at the beginning of the simulation, while the suspected areas of targets 3 and 4 pop up at time 230 sec. Moreover, the horizon length for each AAV is 3 and the communication delays, $1 \le \tau_k^{m'm} \le 3$ sec., are fixed randomly between AAVs during the mission. The trajectories of the AAVs are shown in Figure 2. Note that the group of AAVs focus their effort in the suspected locations of targets 1 and 2 since the beginning of the mission and up to the 230 sec. (see Figure 2(a)). Thus, the suspected locations of targets 3 and 4 pop-up 230 sec. after the mission has begun and the group of AAVs focus on those new areas and finally make some visits to the location where targets 1 and 2 might be (see Figure2(b)).



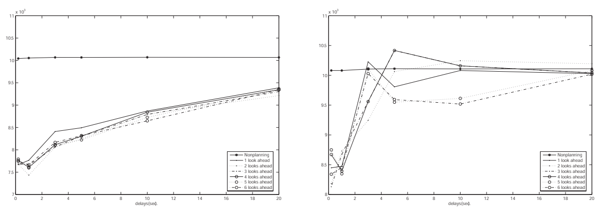
(a) Mission performance without targets 3 and 4 (b) Mission performance with targets 3 and 4

Fig. 2. Performance of cooperative surveillance strategies in the presence of pop-up suspected locations.

B. Influences of Communication Delays and Plan Horizons

We focus here on the AAVs' performance when both nonplanning and planning strategies are used for the cooperative surveillance in a mission. We would like to obtain conditions under which it is best to plan over the maps and when it is best not to do it in the presence of communications delays. We run a Monte Carlo simulation with the following values: the maximum delay in a range $B^{m'm} \in \{0.2, 1, 3, 5, 10, 20\}$ sec. for each $m, m', m \neq m'$, and a projection lengths range of $P \in \{1, 3, 5\}$. Each delay-projection length case consists of 35 simulations where the communication delays are randomly generated in each of these simulations such that $1 \leq \tau_k^{m'm} \leq B^{m'm}$ for all $m \neq m'$. The number of simulations for each delay-projection length combination was chosen such that the standard deviation of the performance measures did not change significantly and settled to a relatively small constant value beyond 35 simulations.

We use the metric defined in Equation (5) as a way to evaluate performance. We compute the average of the average of the metric at each delay-projection length case with $\varepsilon = 10^{-4}$. We also compute the maximum of the average of this quantity over the entire simulation run. Figure 3(a) shows the performance of both the nonplanning and planning case for the average of the average of the metric. Notice that the



(a) Average of average of metric (b) Maximum of average of metric

Fig. 3. Performance measures of the nonplanning and planning case.

performance of the nonplanning case stays almost constant for the range of communication delays considered in the simulation. For the planning case, the performance decreases for small delays, except for the "one look ahead" case, due to the fact that most of the time the AAVs make decisions at the same time, which results in a worst performance for the bounded delay equals to 0.2 second. Then, all the performances increase almost steadily for larger delays than 1 second. It is important to highlight that it is worthwhile to consider large projection lengths for small communication delays. However, when the communication delays are large then the value of planning ahead over the maps is not useful (e.g., note, for instance, how the performance for all the projection lengths have almost the same value for a delay of 20 seconds). Moreover, if we run the simulation for larger delays, then the performance of the planning case will eventually be equal to the nonplanning case. Figure 3(b) shows the performance for both the nonplanning and planning case when the maximum of the average of the metric is used as a performance measure. Notice that it is better to use a nonplanning strategy for the AAVs instead of some planning strategies for delays greater than 3 seconds. However, all the planning strategies are superior than the nonplanning one for delays less than 3 seconds. Note that when the delay is equal to 20 seconds all the performances are about the same so it is better to use a nonplanning strategy for large delays.

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