

Design of Structured Multivariable Controllers for Irrigation Canals

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Abstract—Irrigation canals have a series structure which is generally used to design multivariable controllers based on the aggregation of decentralized monovariable controllers. The paper first reviews classical upstream and downstream control strategies, in terms of stability, performance and robustness. Second, a new structured mixed control scheme is proposed, leading to a fully multivariable controller, with guaranteed stability. This controller enables to mix the advantages of both classical control politics: it recovers the local upstream control performance with respect to unpredicted withdrawals, while at the same time ensuring that the water resource is efficiently managed, as in the downstream control case. It can be implemented in a semi-decentralized way, so that each localized controller only communicates with its closest neighbors. It provides an interesting framework for general design of multivariable controllers for irrigation canals.

I. INTRODUCTION

Irrigation canals are used to convey water from a resource to the users. They have a series structure, which is classically used to propose decentralized control structures. Many decentralized control techniques have already been proposed in research papers (see [7] and references therein, [11], [1], [13], [2] or applied in real situations [8], [10]. They usually implement the classical distant downstream control politics, which is parsimonious from the resource management point of view, but has a low performance with respect to water users. The other classical control politic is the local upstream control, which is very performing from the user point of view, but consumes a lot of water. These decentralized control schemes have the advantage to be easily tuned and implemented. Their structure also facilitates fault diagnosis and localization, leading to easy maintenance. However, each of them has limitations, linked to the fact that they use only one control action variable in each pool to control the output. Recent methods have been developed to design controllers with a specific structure but these methods are computationally demanding and may be difficult to implement for large scale systems such as irrigation canals.

The main issue at stake for irrigation canal control is to provide a design method to tune simple controllers that achieve a desired trade-off between water resource management and performance with respect to water users. Local upstream and distant downstream control can be viewed as solutions to one of the two design specifications.

The present paper shows how to extend the mixed control strategy presented for one canal pool in [4] to a multiple

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pool irrigation canal, with guaranteed stability. In this way, we are able to design a multivariable controller for an irrigation canal with any required performance level (high frequency control performed by local upstream controllers) and ensure that the average value of the resource comes from upstream (low frequency control performed by distant downstream controllers). The proposed solution encompasses both classical control politics, which are based on monovariable control design. The mixed controller can be designed and implemented in a structured semi-decentralized way, with no increase in complexity compared to classical control methods. It therefore gives an elegant answer to the design requirements.

II. MODELLING OF AN IRRIGATION CANAL

An irrigation canal can be represented as a series of pools (see figure 1). Each pool represents a portion of canal in between two hydraulic structures (gates or weirs for example). For pool i we denote u_i the control variable (discharge) at the upstream end, u_{i+1} the control variable at the downstream end, y_i the controlled variable (water depth at the downstream of pool i) and p_i the load disturbances (water offtake).

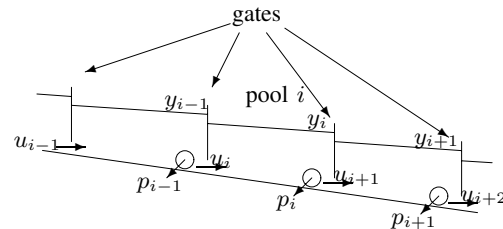


Fig. 1. Schematic longitudinal view of an irrigation canal

The dynamics of each canal pool are classically modelled with the Saint-Venant equations, which are hyperbolic non-linear partial differential equations involving the discharge $Q(x, t)$ and the water depth $Y(x, t)$ along one space dimension. Since we consider small perturbations around an equilibrium flow regime, the linearized equations lead to the following frequency domain representation:

$$y_i(s) = G_i(s)u_i(s) + \tilde{G}_i(s)(u_{i+1}(s) + p_i(s)) \quad (1)$$

where the disturbance $p_i(s)$ (corresponding to the unknown withdrawal) is supposed to act additively with the downstream discharge u_{i+1} .

Transfer functions $G_i(s)$ and $\tilde{G}_i(s)$ can either be obtained analytically in the uniform flow case, or numerically in the

general case (see [5]). A simple approximation of these transfer functions is given by the Integrator Delay model, leading to:

$$G_i(s) = \frac{e^{-\tau_i s}}{A_i s} \quad (2)$$

$$\tilde{G}_i(s) = -\frac{1}{A_i s} \quad (3)$$

with τ_i the time-delay for downstream propagation and A_i the backwater area. The delay τ_i and the integrator gain can be obtained analytically from the hydraulic parameters of the pool (see [9], [6]).

Based on equation (1), a multiple pools canal is represented by the following model:

$$y = Gu + \tilde{G}p$$

with G a bidiagonal matrix:

$$G = \begin{pmatrix} G_1(s) & \tilde{G}_1(s) & 0 & 0 & 0 & 0 \\ 0 & \ddots & \ddots & 0 & 0 & 0 \\ 0 & 0 & G_i(s) & \tilde{G}_i(s) & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & G_n(s) & \tilde{G}_n(s) \end{pmatrix},$$

and $\tilde{G} = \text{diag}(\tilde{G}_i(s))$.

Such a modelling approach implicitly assumes that irrigation canals are controlled with the discharge at the boundary of each canal pool. However, in practice, canals are controlled using hydraulic structures (gates, weirs), represented by static nonlinear equations. This problem can be bypassed by using a slave controller on each hydraulic structure that delivers a required discharge. One can then assume without any loss of generality that the control action is the discharge at each cross-structure.

III. CONTROL POLITICS FOR A CANAL POOL

There are three decentralized control politics for a canal pool: distant downstream control, local upstream control (see figure 2), and mixed control, which allows to obtain the advantages of both classical methods.

Let us recall the main points concerning performance and management of the water resource for these three control politics.

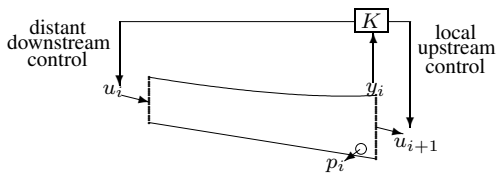


Fig. 2. Local upstream and distant downstream control of a canal pool

1) *Distant downstream control*: Distant downstream control of a canal pool consists in controlling the downstream water level y_i using the upstream control variable u_i . Let us denote K_{ii} the transfer function of the distant downstream controller. The disturbance rejection is then directly characterized by the modulus of the transfer function $\tilde{G}_i(1 +$

$G_i K_{ii})^{-1}$, which has to be small over the largest frequency bandwidth.

Transfer function $G_i(s)$ has a time-delay, which imposes a limitation on the achievable bandwidth, therefore on the performance of the controlled system. Indeed, it is well-known that a time delay limits the achievable bandwidth to about $1/\tau_i$. Unpredicted perturbations occurring at a frequency larger than $1/\tau_i$ will not be attenuated by the controller. Therefore, distant downstream control leads to a high water efficiency, since it is “demand-driven”, but has a low performance with respect to unpredicted perturbations, because of the time-delay.

2) *Local upstream control*: Local upstream control of a canal pool consists in controlling the downstream water level y_i using the downstream control variable u_{i+1} . In this case, the controller is denoted K_{i+1i} , and the disturbance rejection specification is related to the modulus of $\tilde{G}_i(1 + \tilde{G}_i K_{i+1i})^{-1}$, which has to be small over the largest frequency bandwidth. But since there is no time-delay, the achievable bandwidth is only limited by the actuators limitations.

Therefore, local upstream control leads to a high performance with respect to unpredicted perturbations, but has a low water efficiency, since all perturbations are propagated downstream without adapting the upstream discharge. Indeed, faced to a decreasing demand, the only way to maintain the downstream water level is to let the superfluous discharge go downstream, leading to an expensive water management.

3) *Mixed local upstream / distant downstream control*: In the general case, one could use both control action variables u_i and u_{i+1} to control y_i . In that case, the controller $K(s)$ is given by:

$$K(s) = \begin{pmatrix} K_{ii}(s) \\ K_{i+1i}(s) \end{pmatrix}$$

The open-loop transfer matrix is given by

$$G(s)K(s) = (G_i(s)K_{ii}(s) + \tilde{G}_i(s)K_{i+1i}(s))$$

The structure of this mixed controller corresponds to the addition of two classical controllers:

$$G(s)K(s) = \underbrace{G_i(s)K_{ii}(s)}_{\text{Distant downstream}} + \underbrace{\tilde{G}_i(s)K_{i+1i}(s)}_{\text{Local upstream}}$$

Contrarily to the classical control structures, the mixed controller leads to a real multivariable problem, where robustness is complicated to handle. The mixed controller design can be cast into the H_∞ framework in order to guarantee performance and robustness [4]. However, due to the structure of the system, one may also use simple PI controllers and evaluate robustness margins a posteriori.

a) *Mixed controller design*: We have shown that it is possible to easily design such a controller using a cascade input scheme [12]. The (rapid) upstream control u_{i+1} is used to control the output y_i :

$$u_{i+1} = K_{i+1i}(s)(r_i - y_i)$$

And the (slow) distant downstream control u_i is used to regulate u_{i+1} to a reference $r_{u_{i+1}}$ ($r_{u_{i+1}} = 0$ in steady state).

$$u_i = K_{iib}(s)(r_{u_{i+1}} - u_{i+1})$$

The controller structure can be schematized as in figure 3.

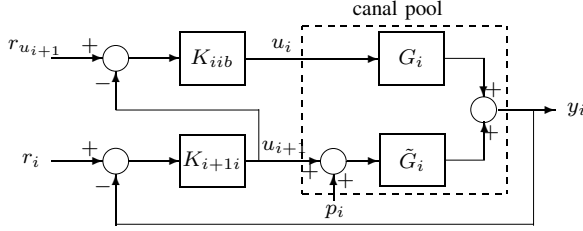


Fig. 3. Cascade architecture with two control variables (u_i, u_{i+1}) to control one output y_i

$r_{u_{i+1}}$ corresponds to the desired flow of the downstream pool. If $r_{u_{i+1}} = 0$, one obtains a multivariable controller as noted above with $K_{ii} = -K_{iib}K_{i+1i}$.

b) Robustness analysis: In the case of one pool, the input robustness margin can be evaluated using the structured singular value with respect to diagonal complex or real uncertainties. Let us denote $\mu_1 = \max_{\omega} \mu_{\Delta}(T_u(j\omega))$ and $\mu_2 = \max_{\omega} \mu_{\Delta}(S_u(j\omega))$ where S_u and T_u are respectively the input sensitivity and input complementary sensitivity functions. Then, the input complex or real gain margin δg verifies

$$\delta g \in \left(1 - \frac{1}{\mu_1}, 1 + \frac{1}{\mu_1}\right) \cup \left(\frac{\mu_2}{\mu_2 + 1}, \frac{\mu_2}{\mu_2 - 1}\right) \quad (4)$$

In the case of mixed control of a canal pool, the structured singular value corresponding to input structured complex uncertainties can be computed after tedious but straightforward manipulations, leading to:

$$\mu_{\Delta}(T_u) = |S_y|(|G_i K_{ii}| + |\tilde{G}_i K_{i+1i}|)$$

with S_y the output sensitivity function $S_y = (1 + G_i K_{ii} + \tilde{G}_i K_{i+1i})^{-1}$. Similar expression can be obtained for S_u .

In the case of efficient frequency decoupling between both designed monovariable controllers, the multivariable input gain margin is strongly related to the input gain margin of both controllers. Indeed, if $|\tilde{G}_i K_{i+1i}| \ll |G_i K_{ii}|$ in low frequencies and $|G_i K_{ii}| \ll |\tilde{G}_i K_{i+1i}|$ in high frequencies, there is ‘‘frequency decoupling’’: for low frequencies, one has $\mu_{\Delta}(T_u) \approx (1 + G_i K_{ii})^{-1} |G_i K_{ii}|$, which is the complementary sensitivity function of the distant downstream controller, and in high frequencies, $\mu_{\Delta}(T_u) \approx (1 + \tilde{G}_i K_{i+1i})^{-1} |\tilde{G}_i K_{i+1i}|$, which is the complementary sensitivity function of the local upstream controller.

IV. MULTIVARIABLE CONTROL OF A TWO POOLS IRRIGATION CANAL

Let us now consider the generalization of these control schemes to a multiple pools canal system. For simplicity,

but with no loss of generality, we focus in the following on the analysis of a two pools canal. The structure and results obtained in this specific case also apply in the general case.

In this case, the canal is represented by:

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \underbrace{\begin{bmatrix} G_1 & \tilde{G}_1 & 0 \\ 0 & G_2 & \tilde{G}_2 \end{bmatrix}}_G \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} + \underbrace{\begin{bmatrix} \tilde{G}_1 & 0 \\ 0 & \tilde{G}_2 \end{bmatrix}}_{\tilde{G}} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

and the general form of controller K is:

$$K = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \\ K_{31} & K_{32} \end{pmatrix}$$

We examine the extension of the three control politics to the case of a two-pools canal.

A. Distant downstream control of a canal with two pools

Let us study the stability and performance of a canal controlled with distant downstream controllers. In this case, the structure of the controller is given by

$$K = \begin{pmatrix} K_{11} & K_{12} \\ 0 & K_{22} \\ 0 & 0 \end{pmatrix}$$

with K_{11} and K_{22} SISO distant downstream controllers for each pool, and K_{12} an additional decoupling term.

The closed-loop system gives the relation between tracking errors e_1, e_2 and disturbances p_1, p_2 as follows:

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{11}K_{22}M_{22} \left(1 + K_F \frac{\tilde{G}_1}{\tilde{G}_1}\right) \\ 0 & M_{22} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

with $M_{11} = -\tilde{G}_1(1 + G_1 K_{11})^{-1}$, $M_{22} = -\tilde{G}_2(1 + G_2 K_{22})^{-1}$, and $K_F = K_{12} K_{22}^{-1}$ a feedforward term.

Since the closed-loop transfer matrix is upper triangular, the multivariable system naturally inherits the stability of the monovariable systems. Actually, robustness is also inherited from monovariable design, since the system is *structurally* upper triangular (the M_{21} term is null, even for model mismatch). Therefore, the multivariable system is stable and robust with respect to diagonal input uncertainties *if and only if* all monovariable systems have the same properties.

It can be shown that the perturbation caused by the second control action u_2 decreases the performance of the controlled system [3]. This interaction between pools is expressed by the non-zero off-diagonal element in matrix M . This term can be reduced with an adequate choice of $K_{12} = K_F K_{22}$.

A perfect decoupling is then achieved if:

$$K_F(s) = -\frac{\tilde{G}_1(s)}{G_1(s)}$$

Assuming that transfers \tilde{G}_1 and G_1 are given by equations (2–3), then $K_F(s) = e^{\tau_1 s}$ is a predictor, which is non causal.

In practice, hydraulic engineers use a constant gain (between 0.5 and 1). In this case, a gain of 1 should be chosen, since if $K_F(j\omega) = 1$, then $|1 - e^{-\tau_1 j\omega}| \approx 0$ for $\omega \approx 0$. However, there exists frequencies where $e^{-\tau_1 j\omega} \approx -1$ which

lead to $|1 - e^{-\tau_1 j\omega}| \approx 2$. It is therefore necessary to filter high frequencies in this feedforward decoupler.

Finally, the multivariable system appears to be directly linked in terms of performance and robustness to the mono-variable systems. However, in this case, once the distant downstream controllers are tuned, there is no way to increase the performance with respect to water users.

B. Local upstream control of a canal with two pools

Let us now study the stability and performance of a canal controlled with decentralized upstream control. In this case, the structure of the controller is given by

$$K = \begin{pmatrix} 0 & 0 \\ K_{21} & 0 \\ K_{31} & K_{32} \end{pmatrix}$$

with K_{21} and K_{32} SISO local upstream controllers for each pool, and K_{31} an additional decoupling term.

The closed-loop system gives the relation between tracking errors e_1, e_2 and disturbances p_1, p_2 as follows:

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} M_{21} & 0 \\ M_{21}K_{21} \left(\frac{G_2}{\tilde{G}_2} + K_F \right) & M_{32} \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

with $M_{21} = -\tilde{G}_1(1 + \tilde{G}_1 K_{21})^{-1}$, $M_{32} = -\tilde{G}_2(1 + \tilde{G}_2 K_{32})^{-1}$ and $K_F = K_{31} K_{21}^{-1}$ a feedforward term.

The matrix is then *structurally* lower triangular (M_{12} is null) and the results obtained in the case of distant downstream control also apply in this case: the multivariable control system is stable if and only if the monovariate systems are stable, and the robustness properties of the SISO case are recovered in the MIMO case for structured diagonal input uncertainties.

As in the distant downstream case, the interactions between pools necessarily decrease the performance of the overall closed-loop system. For an 'exact' decoupling the off-diagonal elements of matrix $M(s)$ must be zero, *i.e.*:

$$K_F(s) = -\frac{G_2(s)}{\tilde{G}_2(s)}$$

Assuming that transfers \tilde{G}_1 and G_1 are given by equations (2–3), one gets $K_F(s) = e^{-\tau_2 s}$. In this case 'exact' decoupling is theoretically possible, since the transfer function is causal. However, it is necessary to consider model uncertainties in the controller design, that will limit the performance.

C. Mixed control of a canal with two pools

We now present the new multivariable mixed control scheme, which aims at combining the advantages of both classical control politics for a multiple pools canal.

1) *Control structure*: The mixed controller is fully multivariable, and the open-loop transfer matrix is given by

$$GK = \begin{pmatrix} G_1 K_{11} + \tilde{G}_1 K_{21} & G_1 K_{12} + \tilde{G}_1 K_{22} \\ G_2 K_{21} + \tilde{G}_2 K_{31} & G_2 K_{22} + \tilde{G}_2 K_{32} \end{pmatrix}$$

which can be restated as the sum of a local upstream and distant downstream controllers:

$$GK = \underbrace{\begin{pmatrix} G_1 K_{11} & G_1 K_{12} + \tilde{G}_1 K_{22} \\ 0 & \tilde{G}_2 K_{22} \end{pmatrix}}_{\text{Distant downstream}} + \underbrace{\begin{pmatrix} \tilde{G}_1 K_{21} & 0 \\ G_2 K_{21} + \tilde{G}_2 K_{31} & \tilde{G}_2 K_{32} \end{pmatrix}}_{\text{Local upstream}}$$

By combining these two controllers, one aims at satisfying the two following objectives:

- reject low frequency perturbations using the upstream discharge;
- increase the performance with respect to water users by using the downstream discharge to reject perturbations occurring in higher frequencies.

As shown above, it is possible in the local upstream control scheme to perfectly decouple the system. Therefore, it is possible to design a controller K_{31} such that

$$G_2 K_{21} + \tilde{G}_2 K_{31} = 0$$

In this way, the open-loop control matrix $G(s)K(s)$ becomes upper triangular:

$$G(s)K(s) = \begin{pmatrix} G_1 K_{11} + \tilde{G}_1 K_{21} & G_1 K_{12} + \tilde{G}_1 K_{22} \\ 0 & G_2 K_{22} + \tilde{G}_2 K_{32} \end{pmatrix}$$

We recover an upper triangular closed-loop system, whose nominal stability and robustness are directly linked to the ones of the mixed controllers of each pool. But obviously, this is not a *structural* upper triangular matrix, and thus it is necessary to evaluate robustness against model uncertainties (M_{21} is generally not null in case of model mismatch).

This can be done a posteriori by computing the structured singular value μ with respect to dynamic uncertainties.

V. SIMULATION RESULTS

Let us now examine the controllers design and the simulation results in order to compare the performance of each control structure. The simulations are done on a test canal with two identical pools. Each pool is 3 km long, with a bed slope of 0.0001, a trapezoidal geometry, with bottom width 7 m and side slope 1.5. The average discharge is 14 m³/s. This leads to the following ID model parameters: $A_1 = A_2 = 36554 \text{ m}^2$ and $\tau_1 = \tau_2 = 707 \text{ s}$.

The PI controllers will be denoted by:

$$K(s) = K_P \left(1 + \frac{1}{T_I s} \right)$$

with K_P the proportional gain and T_I the integral time.

A. Distant downstream PI robust control of two pools

Each distant downstream PI controller is tuned in order to get a gain margin of 10 dB and a phase margin of 45°, using a classical tuning method. The controller parameters are then given by: $K_P = 24.6 \text{ m}^2/\text{s}$ and $T_I = 4800 \text{ s}$.

The simulation of an unpredicted water withdrawal of 1 m³/s in pool 1 at time $t = 100 \text{ s}$ causes a large output error of 3.5 cm, and the output is brought back to the set-point in about 3 hours, which is 15 times the time delay (results not displayed for lack of space). This time-domain performance is coherent with the gain and phase margins of the controller.

B. Local upstream PI robust control of two pools

Each local upstream PI controller is tuned in order to achieve a crossover frequency 10 times higher than the one obtained with the distant downstream controller. This leads to the following controller parameters: $K_P = -256 \text{ m}^2/\text{s}$ and $T_I = 570 \text{ s}$. In this case, there is no limitations on the control, and the gain margin is infinite, while the phase margin is equal to 65° .

Faced to the same unknown perturbation scenario, the local upstream controller performs very well (results not displayed for lack of space). The maximum error is 2 mm, and the perturbation is rejected in less than 30 minutes. The difference in time-domain performance with the distant downstream controller is striking. However, such a controller cannot be used in irrigation canal management, since it assumes that enough water is flowing in the canal to satisfy water needs. The rest is propagated downstream. Indeed, since there is no feedback on the upstream discharge u_1 , it does not adapt to the demand.

C. Mixed PI robust control of two pools

The local upstream controllers are designed as the local upstream controller in previous section. Each distant downstream controller K_{ib} is then tuned to ensure 10 dB gain margin and 60 degrees phase margin. This leads to $K_{Pb} = 0.12$ and $T_{Ib} = 170 \text{ s}$.

The mixed controller of each pool has modulus input margins of $\pm 7 \text{ dB}$. The multivariable mixed controller has modulus input margins of $\pm 4.5 \text{ dB}$, and real structured input margins computed using the structured singular value (4) of $\pm 8 \text{ dB}$. The frequency decoupling is effective, since the monovaryable margins are recovered in the multivariable case. Structured robustness with respect to typical delay mismatch is also good.

The mixed controller results on the same scenario are very similar to the one obtained with the local upstream controller, but the water resources comes from upstream, as in the distant downstream control case. There is a substitution between u_2 , u_3 and u_1 .

To illustrate the ability of the mixed control scheme to achieve control objectives for irrigation canals, let us test it for a more realistic scenario: pool 1 is supposed to be perturbed by high frequency but low amplitude water withdrawals, while pool 2 is subject to low frequency but high amplitude water withdrawals (see figure 4).

For comparison purposes, the simulation of the decentralized distant downstream controller is first depicted in figure 5. The performance in this case is limited by the time-delay, and the controller cannot efficiently reject the high frequency perturbation p_1 . However, the low frequency perturbation p_2 is slowly but efficiently rejected.

The results of the mixed controller is depicted in figure 6. The local upstream controller of pool 1 efficiently rejects the high frequency perturbation, but at the same time is able to follow the low frequency water demand of pool 2. This is visible when the demand in pool 2 changes, corresponding to a change in the downstream discharge setpoint. In terms

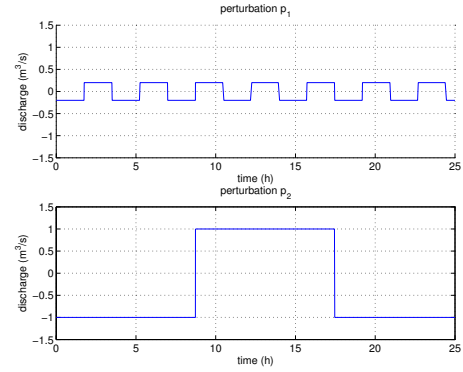


Fig. 4. Perturbations in pool 1 (high frequency low amplitude) and 2 (low frequency high amplitude).

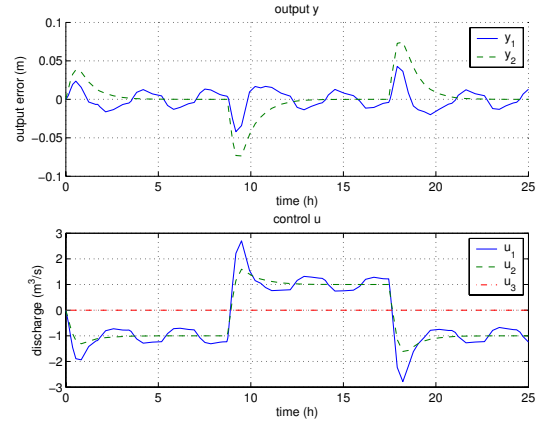


Fig. 5. Distant downstream control of a canal with two pools. Response to a high frequency perturbation in pool 1 and a low frequency perturbation in pool 2.

of water level control, the performance is more than 10 times better with the mixed controller than with the distant downstream controller. One should note that this result is obtained while guaranteeing that the perturbations occurring in low frequencies are rejected with the upstream discharge, as in the distant downstream control case.

Let us now consider the extension of the above results to the case of a multiple canal pools.

VI. EXTENSION TO A MULTIPLE POOLS CANAL

The obtained mixed controlled structure can be implemented as a fully multivariable centralized controller. But one may use its particular structure to implement it in a structured semi-decentralized fashion, which will enable easier maintenance. In this way, it is clear that each local controller only communicates with its closest neighbors located upstream and downstream. There is no need to implement other communication line.

As shown above, each control action variable u_i can be decomposed as the sum of a local upstream control u_i^{lu} and a distant downstream control u_i^{dd} :

$$u_i = u_i^{lu} + u_i^{dd}$$

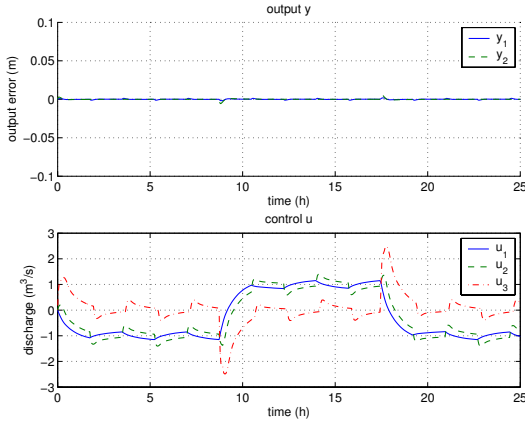


Fig. 6. Mixed local upstream distant downstream control of a canal with two pools. Response to a high frequency perturbation in pool 1 and a low frequency perturbation in pool 2.

with u_i^{lu} acting mainly in high frequencies and u_i^{dd} in low frequencies. These control actions are given by:

$$\begin{aligned} u_i^{lu} &= K_{ii-1}e_{i-1} \\ u_i^{dd} &= K_{ii}e_i \end{aligned}$$

The off-diagonal elements of the controller matrix K are then obtained by the decoupler's rules. To simplify the exposition, choosing a constant distant downstream feedforward decoupler equal to 1 leads to:

$$K_{ij}(s) = K_{jj}(s) \quad \forall i < j$$

and choosing a local upstream feedforward decoupler equal to a pure delay leads to:

$$K_{ij}(s) = e^{-(\sum_{k=i-1}^{j+1} \tau_k)s} K_{j+1j}(s) \quad \forall i > j + 1$$

Such a control scheme can be schematized as in figures 7 and 8.

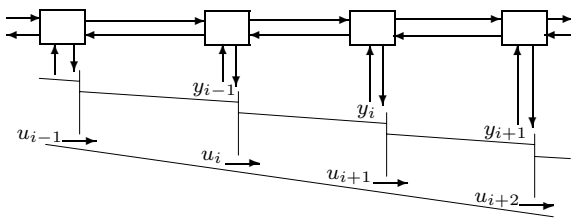


Fig. 7. General layout of the structured mixed control scheme

Such bidiagonal control structure has the advantage to be easily implemented, to enable easy fault diagnosis and fault recovery. Maintenance is also facilitated, while the controller tuning is only done locally. This structured mixed control scheme has therefore many appealing aspects.

VII. CONCLUSION

The paper has reviewed classical multivariable decentralized control politics for an irrigation canal (distant downstream control and local upstream control) in terms of

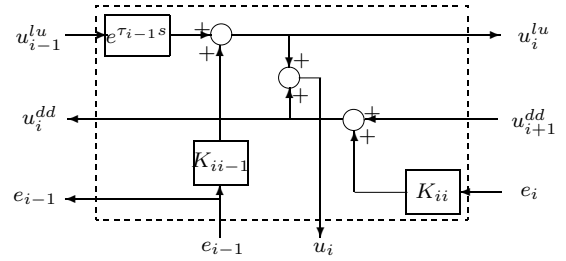


Fig. 8. Local control structure for the structured mixed control scheme

stability, robustness and performance. We have proposed a new multivariable structured control politic which combines both classical control politics, in order to trade off between resource management and performance with respect to the water user. This multivariable mixed control scheme provides an interesting framework for general design of multivariable controllers for irrigation canals. Future works will focus on the detailed theoretical study of this scheme and its practical application to various canal configurations.

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