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A Finite Time Functional Observer For Linear Systems

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Abstract— **This paper presents a finite time functional observer for linear systems, i.e., an observer which estimates directly linear functions of the state of a linear system in a predefined finite time. The finite time functional observer is demonstrated on an academic and on a practical example.**

I. INTRODUCTION

State estimation has gained constantly high interest in the literature since its advent [6], [7]. Clearly, this is because state estimation is of fundamental importance with a wide range of applications ranging from standard control applications, like output feedback design, fault detection, to applications in computer vision, digital image processing, and even cryptography [11]. In some applications it is sufficient to estimate directly linear functions of the state instead of the full state. One well-known example for that is the state feedback design for linear systems in which it is sufficient to estimate directly the state feedback $u = kx$ in order to stabilize a linear system [4]. The design of observers which estimate linear functions of the state, in short functional observers, were first studied in [8]. Later on, a couple of other approaches have been developed to design linear functional observers, see e.g. [2], [4], [5], [10], [12], [13]. All these approaches share one common property, namely, the estimation error converges to zero in an asymptotically fashion. For various reasons like, e.g. good performance in control, a finite time convergence of the estimation error is often desirable.

In this paper, a functional observer is proposed which converges in finite time. The proposed finite time functional observer combines the functional observer [2] with the finite time observer [3]. In particular, the functional observer [2] is used to obtain a linear estimation error dynamic so that the observer structure of the finite time observer [3] can be applied. The proposed observer merges the advantages of the individual observers [2], [3], namely, finite time convergence and low-order observer dimension.

The remainder of the paper is organized as follows: In Section II, the functional observer presented in [2] is summarized which lays the foundation for the proposed finite time functional observer presented in Section III. In Section IV, two examples are given. The first example shows the finite time convergence property of the observer. In the second example the observer is applied to stabilize an active magnetic bearing. Section V concludes with a summary.

II. FUNCTIONAL OBSERVER

In this section, the functional observer developed in [2] is summarized because the observer lays the foundation of the finite time functional observer presented in Section III. For a more detailed discussion on the theory of functional observers, see, e.g. [2] and the references quoted therein. Consider the linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \;\; x(t_0) = x_0, \;\; t \geq t_0 \\ y(t) &= Cx(t) \\ z(t) &= Lx(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ the input, $y \in \mathbb{R}^q$ is the measurable output and $z \in \mathbb{R}^r$ with $r \leq n$ are linear functions of the state which have to be estimated. Furthermore, the matrices C and L have full column rank, i.e., rank $C = q$ and rank $L = r$. The problem of designing a functional observer for the system (1) is to design a reduced order observer

$$\begin{aligned} \dot{w}(t) &= Nw(t) + Jy(t) + Hu(t), \;\; w(t_0) = w_0, \;\; t \geq t_0 \\ \hat{z}(t) &= w(t) + Ey(t), \end{aligned} \tag{2}$$

where N, J, H, and E are constant matrices of appropriate dimensions, such that $\hat{z} \in \mathbb{R}^r$ is an asymptotic estimate of z. With $e = z - \hat{z} = Lx - \hat{z} = (L - EC)x - w$, the estimation error dynamic of (1), (2) becomes

$$\begin{aligned} \dot{e}(t) &= Ne(t) + [(L - EC)A - N(L - EC) - JC]\, x(t) \\ &+ [(L - EC)B - H]\, u(t). \end{aligned} \tag{3}$$

Of course, the estimation error dynamic (3) is in general not asymptotically stable. The next theorem summarizes how the matrices N, J, H, and E have to be chosen such that the estimation error dynamic (3) is asymptotically stable for all initial conditions $x(t_0), w(t_0)$, and $u(t)$:

Theorem 1: [2] The linear functional $\hat{z}(t)$ of (2) is an asymptotic estimate of the linear functional $z(t)$ of (1) for any $x(t_0)$, $w(t_0)$, and $u(t)$ if and only if

$$0 = (L - EC)A - N(L - EC) - JC \tag{4}$$
$$H = (L - EC)B, \tag{5}$$

where N is a Hurwitz matrix.

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Note, due to Theorem 1 the estimation error dynamic (3) becomes $\dot{e}(t) = Ne(t)$ in which N is a Hurwitz matrix. Hence, the design of the functional observer (2) is reduced to solve the equations (4), (5). As mentioned in [2], [5], the equation (4), which is a Sylvester equation, is difficult to solve. In [2], necessary and sufficient conditions for the existence and stability of the functional observer (2) are given which are summarized in the next theorem:

Theorem 2: [2] The necessary and sufficient conditions for the existence and stability of the functional observer (2) for the system (1) are:

(i)

$$\text{rank}\begin{bmatrix} LA \\ CA \\ C \\ L \end{bmatrix} = \text{rank}\begin{bmatrix} CA \\ C \\ L \end{bmatrix},$$

(ii)

$$\text{rank}\begin{bmatrix} sL - LA \\ CA \\ C \end{bmatrix} = \text{rank}\begin{bmatrix} CA \\ C \\ L \end{bmatrix}, \forall\, s \in \mathbb{C},\ \Re(s) \geq 0.$$

For a more detailed discussion of Theorem 1 and Theorem 2, see [2]. The design of the matrices N, J, H, and E of the functional observer is given in [2]. First the matrices

$$\begin{aligned} F &= LAL^+ - LAQ\begin{bmatrix} CAQ \\ CQ \end{bmatrix}^+ \begin{bmatrix} CAL^+ \\ CL^+ \end{bmatrix}, \\ G &= \left[I - \begin{bmatrix} CAQ \\ CQ \end{bmatrix}\begin{bmatrix} CAQ \\ CQ \end{bmatrix}^+ \right]\begin{bmatrix} CAL^+ \\ CL^+ \end{bmatrix} \end{aligned} \tag{6}$$

are calculated whereas, e.g., L^+ is the Moore-Pensrose generalized inverse of the matrix L, i.e., $L^+ := (L^T L)^{-1} L^T$. Furthermore, the matrix Q is defined as $Q := I - L^+ L$. Then the matrix N is calculated by any pole placement procedure for the pair (F, G), i.e.,

$$N = F - ZG, \tag{7}$$

where Z is the matrix obtained from the pole placement of the pair (F, G). Note that the pole placement is possible since the pair (F, G) is detectable due to the condition (ii) of Theorem 2. Using the matrix N, one obtains the matrices J and E from the equation

$$\begin{aligned} \begin{bmatrix} E & J - NE \end{bmatrix} &= LAQ\begin{bmatrix} CAQ \\ CQ \end{bmatrix}^+ \\ &+ Z\left[I - \begin{bmatrix} CAQ \\ CQ \end{bmatrix}\begin{bmatrix} CAQ \\ CQ \end{bmatrix}^+ \right]. \end{aligned} \tag{8}$$

Finally, the matrix H is obtained by

$$H = (L - EC)B. \tag{9}$$

The functional observer (2) of this section paves the road to the finite time functional observer in the next section.

III. Finite Time Functional Observer

In this section, a finite time functional observer for the system (1) is presented. Before stating and proofing the main result, the structure of the finite time functional observer is motivated. Note that the structure is based on the observers developed in [3], [2]. Consider two functional observers (2) for the system (1)

$$\begin{aligned} \dot{m}_i(t) &= N_i m_i(t) + J_i y(t) + H_i u(t),\ m_i(t_0) = m_{i0} \\ q_i(t) &= m_i(t) + E_i y(t), \quad i = 1, 2. \end{aligned} \tag{10}$$

with $t \geq t_0$. Using $e_i = z_i - \hat{z}_i,\ i = 1, 2$, the estimation error dynamics are

$$\dot{e}_i(t) = Ne_i(t), \quad i = 1, 2. \tag{11}$$

Now consider q_1, q_2 at two time instances t and $t - D$

$$\begin{aligned} q_1(t) &= z(t) + e_1(t) \\ q_1(t - D) &= z(t - D) + e_1(t - D) \\ q_2(t) &= z(t) + e_2(t) \\ q_2(t - D) &= z(t - D) + e_2(t - D), \end{aligned} \tag{12}$$

where D is a suitable chosen time delay. The $4r$ equations in (12) have $6r$ unknowns $z(t), z(t - D)$, $e_1(t), e_2(t)$, $e_1(t - D), e_1(t - D)$ and the equations are therefore underdetermined. Hence, it seems that nothing is gained by using two functional observers (2). However by considering the estimation error dynamics (11), one obtains the additional relationships

$$e_i(t - D) = e^{-N_i D} e_i(t), \quad i = 1, 2. \tag{13}$$

Now, the estimate $\hat{z}$ can be uniquely determined from the equations (12), (13). By writing the two functional observers (10) in a compact form

$$\begin{aligned} \dot{m}(t) &= Mm(t) + Ny(t) + Vu(t) \\ q(t) &= m(t) + Ey(t) \end{aligned} \tag{14}$$

and by solving the equations (12), (13), one obtains the finite time functional observer:

$$\begin{aligned} \dot{m}(t) &= Mm(t) + Ny(t) + Vu(t),\ m(t_0) = m_0, t \geq t_0 \\ q(t) &= m(t) + Wy(t) \\ \hat{z}(t) &= K\left[q(t) - e^{MD} q(t - D) \right], \end{aligned} \tag{15}$$

where the matrices M, N, V, W, and K and the vectors m and q are defined as:

$$\begin{aligned} &M := \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix},\ N := \begin{bmatrix} J_1 \\ J_2 \end{bmatrix},\ V := \begin{bmatrix} H_1 \\ H_2 \end{bmatrix}, \\ &T := \begin{bmatrix} I_{r,r} \\ I_{r,r} \end{bmatrix},\ W := \begin{bmatrix} E_1 \\ E_2 \end{bmatrix},\ m := \begin{bmatrix} m_1 \\ m_2 \end{bmatrix},\quad q := \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \\ &K := \begin{bmatrix} I_{r,r} & 0_{r,r} \end{bmatrix}\begin{bmatrix} T & e^{MD} T \end{bmatrix}^{-1}. \end{aligned} \tag{16}$$

As it can be seen from (15), the initial condition of the observer must be also defined for $t \in [t_0 - D, t_0]$. Without loss of generality, the initial condition is chosen to be $q(t) = T\hat{z}(t_0)\ \forall t \in [t_0 - D, t_0]$. Now, the next theorem states conditions for the convergence of the finite functional observer (15) in finite time D.

Theorem 3: Suppose that the conditions in Theorem 1 and Theorem 2 are satisfied so that a functional observer (2) for the system (1) exists. Furthermore suppose that the matrix M is Hurwitz and the inverse of the matrix $\begin{bmatrix} T & e^{MD}T \end{bmatrix}$ exists. Then the equations (15) define a finite time functional observer for the system (1) in the sense that the estimation error $e = z - \hat{z}$ converges to zero in finite time D and stays bounded during the convergence interval $[t_0, t_0 + D]$.

Proof 1: The proof has three parts and goes almost along the lines of the proofs presented in [3], [9]. First the existence of the functional observer (2) is considered. Second it is shown that the estimate $\hat{z}(t)$ stays bounded during the convergence time $t_0 \leq t \leq t_0 + D$. Then it is shown that the observer converges in finite time D, i.e., $\hat{z}(t) = z(t)\ \forall t \geq t_0 + D$.

1. Existence of functional observers (2): Functional observers (2) exist due to the assumptions of Theorem 3.

2. Boundedness: With equation (4) and (5) it follows

$$\begin{aligned}\frac{d}{dt}(q - Tz) &= \frac{d}{dt}(q - TLx) \\ &= Mw + Ny + Vu - (TL - WC)(Ax + Bu) \\ &= M(q - Tz) \\ &+ \underbrace{[MTL - MWC + NC + WCA - TLA]}_{0} x \\ &+ \underbrace{[V + WCB - TLB]}_{0} u \\ &= M(q - Tz).\end{aligned}$$

Hence, for the time interval $t_0 \leq t \leq t_0 + D$ with $q(t_0) = T\hat{z}(t_0)$, one obtains

$$\begin{aligned} q(t) &= Tz(t) + e^{M(t-t_0)}[q(t_0) - Tz(t_0)] \\ &= Tz(t) + e^{M(t-t_0)}T[\hat{z}(t_0) - z(t_0)]. \end{aligned} \tag{17}$$

Therefore, one derives from equation (15) with $q(t - D) = T\hat{z}(t_0)$ in the time interval $t_0 \leq t \leq t_0 + D$

$$\begin{aligned}\hat{z}(t) &= KTz(t) + Ke^{M(t-t_0)}T[\hat{z}(t_0) - z(t_0)] \\ &\quad - Ke^{MD}T\hat{z}(t_0).\end{aligned}$$

Since $KT = I_{r,r}$ and $Ke^{MD}T = 0_{r,r}$ the estimate $\hat{z}(t)$ can be rewritten as

$$\hat{z}(t) = z(t) + Ke^{M(t-t_0)}T[\hat{z}(t_0) - z(t_0)].$$

This shows that $\hat{z}(t)$ and $e(t)$ are bounded $\forall t \in [t_0, t_0 + D]$.

3. Finite Time Convergence: For $t \geq t_0 + D$ one has from equation (17)

$$\begin{aligned} q(t - D) &= Tz(t - D) \\ &\quad + Ke^{M(t-D-t_0)}T[\hat{z}(t_0) - z(t_0)] \end{aligned} \tag{18}$$

Using equation (17) and equation (18), one obtains

$$\begin{aligned}\hat{z}(t) &= K[q(t) - e^{Md}q(t - d)] \\ &= \underbrace{KT}_{I_{r,r}} z(t) + Ke^{M(t-t_0)}T[\hat{z}(t_0) - z(t_0)] \\ &- \underbrace{Ke^{MD}T}_{0_{r,r}} z(t - D) \\ &- \underbrace{Ke^{MD}e^{M(t-D-t_0)}}_{e^{M(t-t_0)}} T[\hat{z}(t_0) - z(t_0)]. \\ &= z(t)\ \ \forall t \geq t_0 + D,\end{aligned}$$

which proofs the finite time convergence of the observer (15). This completes the proof. ■

Note that Theorem 3 is based on three assumptions. The first assumption is that the functional observer (2) for the system (1) exists. This assumption can be checked via Theorem 2. The other two assumptions, i.e., M is Hurwitz and the inverse of the matrix $\begin{bmatrix} T & e^{MD}T \end{bmatrix}$ exists, are not very restrictive. If Theorem 2 is satisfied, then the matrix M can be chosen such that it has only negative eigenvalues. Before given two examples in the next section, this section concludes with some remarks about the finite time functional observer.

Remark 1: [3] If the matrix M is chosen such that

$$Re(\lambda_i(N_2)) < \sigma < Re(\lambda_j(N_1)) < 0, \quad \forall i, j = 1, 2, ...r$$

for some $\sigma < 0$, then $\det \begin{bmatrix} T & e^{Md}T \end{bmatrix} \neq 0$ for almost all $D \in \mathbb{R}^+$. Of course, this can be easily fulfilled with any pole placement procedure if Theorem 2 of the functional observer is satisfied. This ensures the existence of the matrix K and thus the existence of the whole estimator for almost any choice of the convergence time D.

Remark 2: [9] Theorem 3 states that the estimate $\hat{z}$ remains bounded during the convergence interval D. However, nothing more can be said about the estimate during this time interval. Hence, the estimate during this time is typically meaningless and should not be used for applications.

Remark 3: [9] The convergence time for the functional observer can be made arbitrarily small independant of the initial condition of the system (1). Hence it is possible to recover the performance of some state feedback controllers as shown in Section IV.

IV. Example and Application

In this section, two examples of the proposed finite time functional observer are given. The first example shows the finite time convergence property of the finite time functional observer. In the second example, the observer is applied to stabilize an active magnetic bearing and to recover the performance of a state feedback.

A. Example

Consider the system (1) with

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

$$u(t) = \sin(t).$$

The task is to estimate the functional z while having the information of the output y and the input u. Note that the functional z depends also on the non-measured state x_3. The finite time functional observer is designed as follows: First two functional observers (2) are designed as described in Section II. To satisfy the assumptions stated in Theorem 3, the matrices $N_i, \ i = 1, 2$ are chosen as $N_1 = -1$ and $N_2 = -10$. Then, the convergence time D is chosen e.g. as $D = 0.3$. Furthermore, $q(t-D)$ is chosen to be $q(t-D) = m(0) + Wy(0) = [0\ 0]^T + WC[1\ 1\ -5]^T = [0\ \ 9]^T \quad \forall t \in [0, 0.3]$. Hence, the finite time functional observer is given as

$$\begin{aligned} \dot{m}(t) &= Mm(t) + Ny(t) + Vu(t), \ \ m(0) = 0, \ \ t \geq 0 \\ q(t) &= m(t) + Wy(t) \\ \hat{z}(t) &= K\left[q(t) - e^{MD}q(t-D)\right], \end{aligned}$$

with

$$M = \begin{bmatrix} -1 & 0 \\ 0 & -10 \end{bmatrix}, \ N = \begin{bmatrix} -0.6 & 0.6 \\ -16.8 & -46.2 \end{bmatrix},$$

$$e^{MD} = \begin{bmatrix} 0.7048 & 0 \\ 0 & 0.0498 \end{bmatrix}, \ W = \begin{bmatrix} 0.4 & -0.4 \\ 3.1 & 5.9 \end{bmatrix},$$

$$K = \begin{bmatrix} -0.072 \\ 1.072 \end{bmatrix}^T, \ V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Fig. 1 shows the trajectory z and the trajectory of the estimate $\hat{z}$. Fig. 1 shows nicely that the observer converge in the predefined finite time $D = 0.3$. Furthermore, the estimate $\hat{z}$ is large but bounded during the convergence time interval $[0, 0.3]$. This behavior underpins Remark 3 that the estimation is meaningless during the convergence time $[0, 0.3]$ and it should not be used in an application.

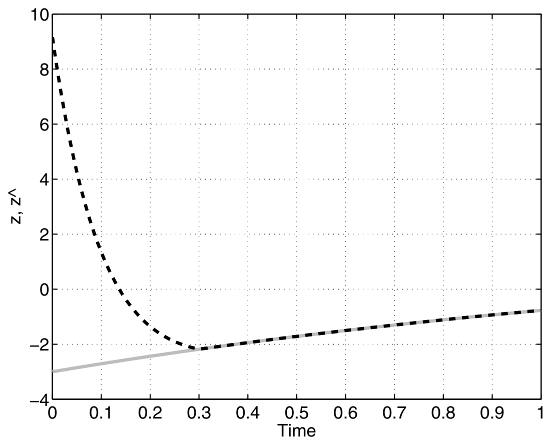

Fig. 1. Trajectory z (gray) of the system and trajectory $\hat{z}$ (black) of the finite time functional observer with an initial condition $x(0) = [1\ 1\ -5]$ and $m(0) = [0\ 0]^T$.

B. Application

Recovering the performance of a state feedback with an output feedback has many advantages in control engineering and control theory. In this section it is shown that the performance of a state feedback $u = Lx$ can be recovered via the finite time functional observer with $z = Lx$, i.e., the finite time functional observer is used to estimate directly the state feedback $u = Lx$. The performance recovery is possible because the finite time functional observer can converge in an arbitrarily short time D. However, the peaking of the estimate during the convergence time D can destroy the performance recovery. One approach to recover the performance of a state feedback despite the peaking is shown in Fig. 2. During the convergence time D of the observer the control input is set to zero, i.e., $u(t) = 0 \ \ \forall t \in [t_0, t_0 + D]$. After the convergence time D, the control input is the estimate of the finite time functional observer, i.e., $u(t) = \hat{z}(t) \quad \forall t \geq t_0 + D$. In the following, this approach is applied to stabilize a voltage-controlled active magnetic bearing system.

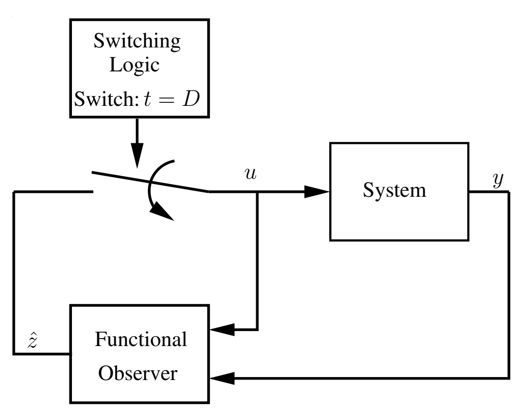

Fig. 2. Feedback structure for performance recovery.

Active magnetic bearings (AMB) are an alternative for mechanical bearings, especially for high-speed rotating devices, because they eliminate the need of lubricants and seals which accompany mechanical bearings. Therefore, the AMB brings up many advantages over the mechanical bearings such as lower power loss and longer life span. Fig. 3 shows a voltage-controlled 3-pole active magnetic bearing. The three poles of the AMB are arranged in a radially symmetric “Y” structure to produce uniform force distribution. The two coil currents i_1 and i_2 are generated by two independent variable input voltages v_1 and v_2. At steady state, there exists a permanent voltage v_1 for the bias current i_1 to support the rotor weight and no permanent voltage v_2 for the current i_2. The objective of the feedback design is to stabilize the rotor at the bearing center of the AMB. Since the full state is not measurable, the finite time functional observer is applied in conjunction with a state feedback to stabilize the AMB and to recover the performance of the state feedback.

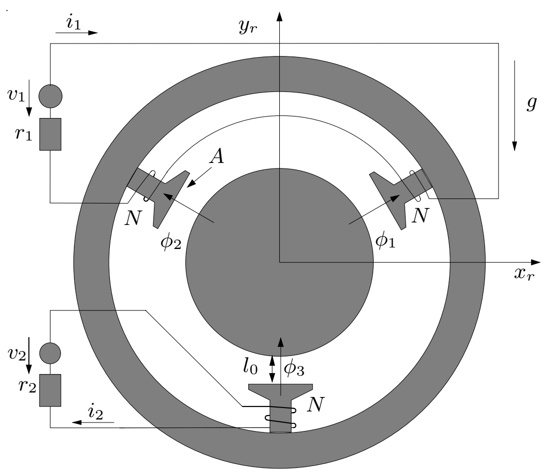

Fig. 3. Active Magnetic Bearing.

A	Pole face area
N	Number of coil turns
g	Gravitational acceleration
i_1, i_2	Coil current
l_0	Nominal air gap
m	Rotor mass
r_1, r_2	Resistances of coils
v_1, v_2	Input voltages
x_r, y_r	Position of the rotor
ϕ_1, ϕ_2, ϕ_3	Magnetic flux
μ	Magnetic permeability of the air

The AMB system is taken form [1]. After a coordinate transformation, which normalizes the variables and shifts the desired equilibrium point to the origin and a linearization around it, the dynamics of the AMB system can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \quad x(0) = x_0 \\ y(t) &= Cx(t), \end{aligned}$$

where

$$A = \begin{bmatrix} 0 & \frac{v_0 t_0}{l_0} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4C\Phi_0^2 t_0}{3mv_0} \\ 0 & 0 & 0 & \frac{v_0 t_0}{l_0} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{4C\Phi_0^2 t_0}{3mv_0} & 0 \\ 0 & 0 & \frac{r_1 l_0 t_0}{4C} & 0 & \frac{-r_1 l_0 t_0}{2C} & 0 \\ \frac{3r_2 l_0 t_0}{4C} & 0 & 0 & 0 & 0 & \frac{-3r_2 l_0 t_0}{2C} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{\sqrt{3}u_0 t_0}{4C\Phi_0} & 0 \\ 0 & \frac{3u_0 t_0}{4C\Phi_0} \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

and $x = [x_1\ x_2\ x_3\ x_4\ x_5\ x_6]^T$. x_1 represents the position of the AMB system in x_r direction, x_2 the velocity in x_r direction, x_3 the position in y_r direction, x_4 the velocity in y_r direction, x_5 and x_6 are flux related quantities of the AMB system. Furthermore, $l_0 = 3 \cdot 10^{-4} m$, $v_0 = 0.05 \frac{m}{s}$, $u_0 = 1V$, $t_0 = 0.01s$, and ϕ_0 are positive reference values to obtain dimensionless state variables in the output feedback design. To locally stabilize the AMB, a finite time functional observer (15) for the AMB system with

$$L = \begin{bmatrix} -0.98 & -2.61 & 8.25 & 31.62 & -4.06 \\ 4.44 & 17.084 & -0.40 & -1.07 & 54.34 \end{bmatrix}$$

is designed, where Lx is a state feedback which locally stabilizes the AMB system. Hence, the finite time functional observer estimates directly the state feedback $u = Lx$. Fig. 4 to Fig. 6 show the initial response of the closed-loop state trajectories x_1 and x_3 of the state feedback (solid) and the output feedback (dashed) with the initial condition $x_0 = [1\ 0\ -0.5\ 0\ 0\ 0]^T$ and the different convergence times $D = 0.5$, $D = 0.1$, and $D = 0.05$. One can nicely see that the performance of the state feedback can be recovered with the finite time functional observer if the convergence time D is sufficiently small.

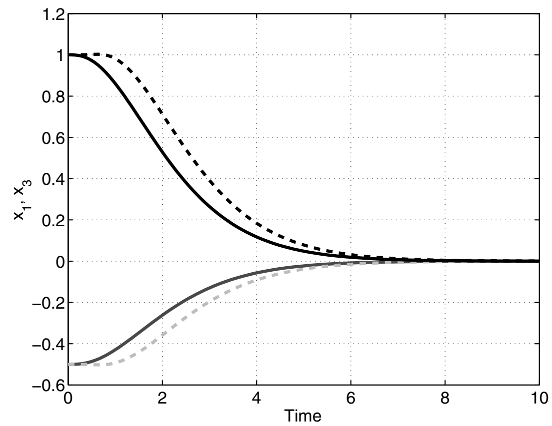

Fig. 4. Closed-loop state trajectories x_1 (black) and x_3 (gray). The solid curves correspond to the state feedback and the dashed curves correspond to the output feedback. Convergence time: $D = 0.5$.

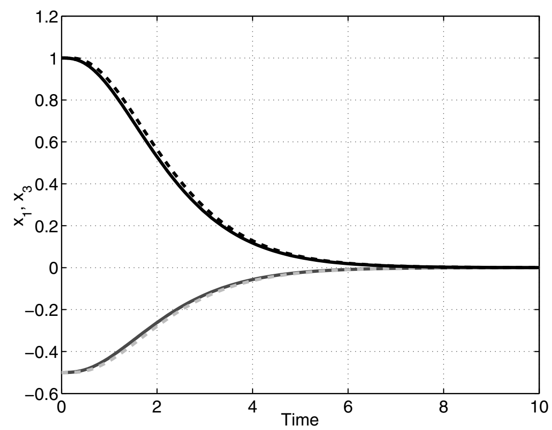

Fig. 5. Closed-loop state trajectories x_1 (black) and x_3 (gray). The solid curves correspond to the state feedback and the dashed curves correspond to the output feedback. Convergence time: $D = 0.1$.

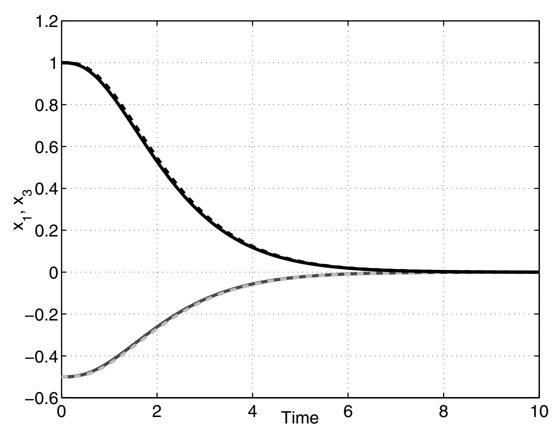

Fig. 6. Closed-loop state trajectories x_1 (black) and x_3 (gray). The solid curves correspond to the state feedback and the dashed curves correspond to the output feedback. Convergence time: $D = 0.05$.

V. Conclusion and Outlook

In this paper a finite time functional observer for linear systems has been presented, i.e., an observer which estimates directly linear functions of the state of a linear system in a predefined finite time. The structure of the finite time functional observer is based on the observers developed in [3], [2]. The proposed finite time functional observer merges the advantages of the individual observers [2], [3], namely, finite time convergence and low-order observer dimension. The finite time functional observer has been demonstrated on two examples. The first example shows the finite time convergence property while in the second example the finite time functional observer has been applied to stabilize an active magnetic bearing system. Furthermore, it has been shown that the performance of a state feedback can be recovered via the finite time functional observer. Future work involves to use the structure of the finite time observer [3] to design unknown input observers and observers for descriptor systems which converge in finite time.

References

[1] S.L. Chen and C.T. Chen. Exact linearization of a voltage-controlled 3-pole active magnetic bearing system. *IEEE Transactions on Control Systems Technology*, 10:618–625, 2002.

[2] M. Darouach. Existence and design of functional observers for linear systems. *IEEE Transactions on Automatic Control*, 45:940–943, 2000.

[3] R. Engel and G. Kreisselmeier. A continous-time observer which converges in finite time. *IEEE Transactions on Automatic Control*, 47:1202–1204, 2002.

[4] T.E. Fortmann and D. Williamson. Design of low-order observers for linear feedback control laws. *IEEE Transactions on Automatic Control*, 17:301–308, 1972.

[5] M. Hou, A.C. Pugh, and P.C. Müller. Disturbance decoupled functional observers. *IEEE Transactions on Automatic Control*, 44:382–386, 1992.

[6] R.E. Kalman and R.S. Bucy. Observing the state of a linear system. *ASME Journal of Basic Engineering*, 83D:95–108, 1961.

[7] D.G. Luenberger. Observing the state of a linear system. *IEEE Transactions on Military Electronics*, 8:74–80, 1964.

[8] D.G. Luenberger. Observers for multivariable systems. *IEEE Transactions on Automatic Control*, 11:190–197, 1966.

[9] P. Menold. *Finite time and asymptotic time state estimation for linear and nonlinear systems*. PhD thesis, University of Stuttgart, Institute for Systems Theory in Engineering, 2004.

[10] P. Murdoch. Observer design for a linear functional of the state vector. *IEEE Transactions on Automatic Control*, 18:308–310, 1973.

[11] H. Nijmeijer and T. Fossen. *New trends in nonlinear observer design*. Springer, 1999.

[12] J.R. Roman and T.E. Bullock. Design of minimal order stable observers for linear functions of the state via realization theory. *IEEE Transactions on Automatic Control*, 20:613–622, 1975.

[13] D.W. Russell and T.E. Bullock. A frequency domain approach to minimal-order observer design for several linear functions of the state. *IEEE Transactions on Automatic Control*, 22:600–604, 1977.