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Estimation of the power coefficient in a wind conversion system

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Abstract— In this paper, we present the design of an observer/estimator for the estimation of the power coefficient in a wind energy conversion system (WECS) based on a separately excited dc generator. It is shown that the estimator is capable of supplying accurate estimates of the power coefficient, and that it can handle measurement noise and provide satisfactory convergence qualities. A further advantage of the observer design presented is that it can be easily extended to other WECS where different types of generators are employed.

Keywords: WECS, DC generator, observer, power coefficient

I. INTRODUCTION

Over the last decade wind energy is the fastest growing energy technology in the world. More recently focus has shifted considerably in terms of application to electricity generation. This shift is due to the fact that wind represents a sustainable, economically viable and moreover clean renewable energy source for the future with minimal environmental impact [1].

The UK's Government has extended it's commitment to producing 10% of its electricity requirements using renewable energy sources by 2010 to 15% by 2015 [2]. As Europe's windiest country the Tyndall study shows that 34% of this generation must come from wind energy. Further commitments to the Kyoto Protocol to cut green house gas emissions and more specifically reduce 20% of CO_2 emissions by 2010 provide an additional impetus to maximise electrical power produced by wind [3].

A wind energy conversion system (WECS) consists of a turbine, used to extract power from the wind, a transmission to gear up the rotational speed of the turbine shaft, a mechanical to electrical converter (usually an induction generator), and a controller to control the overall system behaviour.

The expression for the amount of power $P_m(u)$ a wind turbine is capable of producing is given by:

$$P_m(u) = \frac{1}{2} C_p(\lambda, \beta) \rho \pi R^2 u^3 \qquad (1)$$

where u is the wind speed (ms^{-1}), ρ is the air density (kgm^{-3}), R is the rotor radius (m), and C_p is the power coefficient of the wind turbine. The tip-speed ratio is defined as:

$$\lambda = \frac{R\omega}{u} \qquad (2)$$

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where ω is the angular speed of the turbine rotor (rads^{-1}). The power coefficient C_p is the most important parameter for the WECS controller design, especially in the case of power regulation. It represents the turbine efficiency, and is defined as the fraction of wind energy extracted by the turbine of the total energy that would have flowed through the area swept by the rotor blades if the turbine had not been there [4]. In general, C_p is a non-linear function of the tip speed ratio λ and therefore changes with wind velocity. It is calculated from the turbine design, the pitch angle β and it is sensitive to dirt etc. on the blade surface [5]. In practice, the C_p is difficult to obtain and is different for every turbine type. In many instances, it is provided by the manufacturer documentation which are used in many control schemes as look-up tables to generate optimal target power references [6], [1]. Various models are used to describe C_p in the literature (see e.g.. [7], [8]). However, these models are only approximate and in many cases there is no valid justification for their use. Consequently, there is a real incentive to estimate C_p on-line without assuming any particular model for its structure. To the best of the authors' knowledge, there is no published work done in this area. From this point of view, the work to be presented has a significant contribution.

In this paper, we present the design of a proportional observer/estimator for the estimation of the power coefficient in a WECS where a separately excited dc generator is employed. It is shown that the estimator is capable of supplying accurate estimates of the power coefficient. In addition, it is demonstrated that the estimator can handle measurement noise and provide satisfactory convergence qualities if the gain of the observer is not excessively high. A further advantage of the observer design presented is that it can be easily extended to other WECS where different fixed and/or variable speed generators are employed (such as the cage induction generators, wound-field or permanent magnet synchronous generators, doubly-fed slip ring induction generators etc.). The C_p estimate can then be employed for designing various control schemes. These observer-based algorithms will have the advantage of being flexible and applicable to different turbines; thus making the controller as independent of the turbine parameters as possible.

An outline of the paper is as follows: In the next section, we present the modelling aspects of both the dc generator and the WECS. We then show how reduced order models of the dc generator and the turbine can be obtained using output injection. In Section III, these reduced order models are used to design the power coefficient estimator. In section IV, simulations are carried out in order to demonstrate the performance of the estimator. Finally some important conclusions from the results presented are drawn, and the potential directions for future work given.

II. SYSTEM MODELLING

A. DC Generator Modelling

The model of the separately excited dc generator can be subdivided into its electrical dynamics and its mechanical dynamics[9], [10], [11]. The corresponding equivalent circuit is shown in Fig. 1. The electrical dynamics describe the field winding and the armature winding and can be represented by the following differential equations:

$$\frac{di_f}{dt} = \frac{V_f}{L_f} - \frac{R_f i_f}{L_f} \quad (3)$$

$$\frac{di_a}{dt} = \frac{K_1 i_f \omega_e}{L_a} - \frac{V_a}{L_a} - \frac{R_a i_a}{L_a} \quad (4)$$

where V_f, R_f, and L_f are the field winding voltage, resistance and inductance respectively; V_a, R_a, and L_a are the armature winding voltage, resistance and inductance respectively, ω_e is the rotational speed of the generator (rad.s^{-1}), and K_1 is the induced emf constant.

The mechanical dynamic equation of the dc generator is of the well-known form:

$$\frac{d\omega_e}{dt} = \frac{T_p}{J_e} - \frac{T_e}{J_e} - \frac{B_e\omega_e}{J_e} \quad (5)$$

where B_e is the coefficient of viscous friction (N.m), $T_e = K_1 i_f i_a$ is the electromagnetic torque (N.m), T_p is the drive torque (N.m), J_e is the inertia of the dc generator (kg.m^2).

Additionally, the electromagnetic torque T_e can be written as:

$$T_e = \frac{e i_a}{\omega_e} \quad (6)$$

where e is the induced emf which can be written as:

$$e = K_1 i_f \omega_e. \quad (7)$$

B. WECS Modelling

A typical WECS configuration is shown in Fig. 2. The power from the wind drives the turbine with a torque T_m and consequently the rotor of the wind turbine rotates at an angular speed ω. The transmission output torque T_p then drives the generator, which produces an electromagnetic torque T_e at a rotational speed ω_e. Note that the turbine and generator speeds are not the same due to the use of the gearbox.

The mechanical dynamics of the WECS and the dc generator can be described by the following set of equations [12]:

$$T_m - T = J_m\dot{\omega} + B_m\omega \quad (8)$$

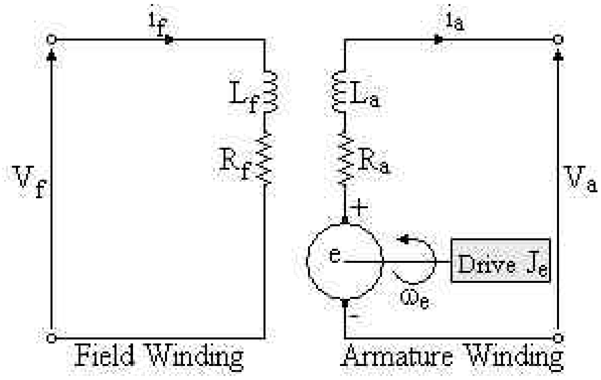

Fig. 1. Separately excited dc generator equivalent circuit

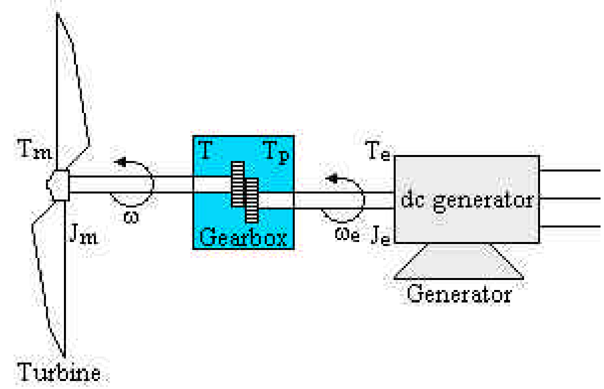

Fig. 2. Wind energy conversion scheme schematic

$$T_p - T_e = J_e\dot{\omega}_e + B_e\omega_e \quad (9)$$

$$T_p\omega_e = T\omega \quad (10)$$

where B_m and B_e are the frictional constants of the turbine and the generator respectively, T_m, T_e, T, T_p are the rotor torques at the turbine end, generator end, before and after the gear box, J_m, J_e the moment of inertia of the turbine and the generator respectively, and ω, ω_e are the rotational speed of the rotor at turbine end and generator end respectively. Note that no torsion related losses are considered here to simplify the modelling. Furthermore, the transmission is assumed ideal i.e. lossless.

The transmission gear ratio is defined as:

$$\gamma = \frac{\omega_e}{\omega} \quad (11)$$

Using (10), (11), we can combine (8) and (9) to show the mechanical equation of the WECS:

$$\begin{aligned} J\dot{\omega} + B\omega &= T_m - \gamma T_e \\ J\dot{\omega} + B\omega &= \frac{P_m}{\omega} - \gamma\frac{P_e}{\omega_e} \end{aligned} \quad (12)$$

where:

$$J = J_m + \gamma^2 J_e \quad (13)$$

$$B = B_m + \gamma^2 B_e \quad (14)$$

where P_m denotes the wind power from (1) and P_e represents the electrical power generated by dc generator. It is clear from the previous section that P_e is related to the field and armature currents of the generator:

$$P_e = T_e \omega_e = K_1 i_f i_a \omega_e. \quad (15)$$

Finally, the WECS model is obtained by combining the dynamics of dc generator with that of the turbine and is given by:

$$\left\{ \begin{array}{l} \dfrac{di_f}{dt} = \dfrac{V_f}{L_f} - \dfrac{R_f i_f}{L_f} \\ \dfrac{di_a}{dt} = \dfrac{\gamma K_1 i_f \omega}{L_a} - \dfrac{V_a}{L_a} - \dfrac{R_a i_a}{L_a} \\ \dfrac{d\omega}{dt} = \dfrac{\frac{1}{2} C_p(\lambda, \beta) \rho \pi R^2 u^3}{J\omega} - \dfrac{\gamma K_1 i_f i_a}{J} - \dfrac{B\omega}{J} \end{array} \right. \quad (16)$$

III. ESTIMATION OF C_p

Our main objective is to design a power coefficient estimator based upon the WECS model given in (16). For this, we regard C_p as a state variable instead of a parameter and assume that the power coefficient is unknown and piecewise constant so that $\frac{dC_p}{dt} \simeq 0$ on the time intervals where C_p is constant. As a result, system (16) can be augmented by including the dynamics of C_p; yielding a system of order 4 given by:

$$\left\{ \begin{array}{l} \dfrac{di_f}{dt} = \dfrac{V_f}{L_f} - \dfrac{R_f i_f}{L_f} \\ \dfrac{di_a}{dt} = \dfrac{\gamma K_1 i_f \omega}{L_a} - \dfrac{V_a}{L_a} - \dfrac{R_a i_a}{L_a} \\ \dfrac{d\omega}{dt} = \dfrac{1}{2} \dfrac{C_p(\lambda, \beta) \rho \pi R^2 u^3}{J\omega} - \dfrac{\gamma K_1 i_f i_a}{J} - \dfrac{B\omega}{J} \\ \dfrac{dC_p}{dt} = 0 \end{array} \right. \quad (17)$$

The estimation of C_p can be based on these four equations[13]. However, in order to reduce computation complexity we look for a reduced order model from which the estimation of C_p can be made. For this, we shall assume that the values of i_f, i_a, ω and V_a are measured: that is they are accessible outputs of the WECS. This is a reasonable hypothesis since in practice all these measurements can be obtained. The voltage V_f and the wind speed u are inputs to the WECS. Since i_f, i_a, u and ω are measured, they can be injected directly into mechanical equation of the WECS. As a result, only the mechanical dynamics of the WECS can be considered for the estimation of C_p. More precisely, the following second order reduced model can be employed for C_p estimation:

$$\left\{ \begin{array}{l} \dfrac{d\omega}{dt} = \dfrac{1}{2} \dfrac{C_p(\lambda, \beta) \rho \pi R^2 u^3}{J\omega} - \dfrac{\gamma K_1 i_f i_a}{J} - \dfrac{B\omega}{J} \\ \dfrac{dC_p}{dt} = 0 \\ y = \omega \end{array} \right. \quad (18)$$

where i_f, i_a and u are viewed as inputs to the system and ω as the output.

In what follows, we are going to use this model for the design of the estimator for the estimation of the power coefficient. The reduced order system (18) can be written in a matrix form as follows:

$$\left\{ \begin{array}{l} \dot{x} = Ax + \varphi(i_f, i_a) \\ y = Cx \end{array} \right. \quad (19)$$

where

$$\begin{array}{rcl} x & = & \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \omega \\ C_p \end{pmatrix}, \; A = \begin{pmatrix} -\mu & \sigma \\ 0 & 0 \end{pmatrix} \\ \varphi(i_f, i_a) & = & \begin{pmatrix} -\dfrac{\gamma K_1 i_f i_a}{J} \\ 0 \end{pmatrix}, \text{ and } C = \begin{pmatrix} 1 & 0 \end{pmatrix} \end{array}$$

with

$$\mu = \frac{B}{J} \text{ and } \sigma = \frac{1}{2} \frac{\rho \pi R^2 u^3}{J\omega}.$$

The estimator takes the following form:

$$\dot{\hat{x}} = A\hat{x} + \varphi(i_f, i_a) + K_P(y - \hat{y}) \quad (20)$$

where the K_P coefficients are chosen in order to impose a stable dynamic to the observer error dynamic. For this, we impose double real poles that we can place using the parameter θ:

$$p(\lambda) = \det\left[\lambda I_2 - (A - KC)\right] = \lambda^2 + (K_{P1} + \mu)\lambda + K_{P2}\sigma$$

where I_2 stands for the identity matrix of order 2. In order to choose a double negative real poles, we impose $\theta\sigma$ as a double eigenvalue of $(A - KC)$ where $\theta > 0$ is a tuning parameter, that is

$$p(\lambda) = \lambda^2 + 2\theta\sigma\lambda + (\theta\sigma)^2.$$

By identification of coefficients, we obtain

$$K_P = \begin{pmatrix} K_{P1} \\ K_{P2} \end{pmatrix} = \begin{pmatrix} 2\theta\sigma - \mu \\ \theta^2\sigma \end{pmatrix}.$$

In effect, by setting, $\varepsilon = x - \hat{x}$, it can be verified that the error dynamics of the observer is given by

$$\begin{array}{rcl} \dot{\varepsilon} & = & (A - K_P C)\varepsilon \\ & = & \sigma \begin{pmatrix} -2\theta & 1 \\ -\theta^2 & 0 \end{pmatrix} \varepsilon = \sigma A_0 \varepsilon. \end{array}$$

Since the eigenvalues of A_0 are both equal to $-\sigma\theta$ and $\sigma > 0$, we therefore conclude that the error dynamics is stable. Consequently, the estimator/observer (20) is an exponential observer for system (19).

In summary, the estimator for C_p is given by:

$$\left\{ \begin{array}{l} \dfrac{d\hat{\omega}}{dt} = \sigma\hat{C}_p - \mu\hat{\omega} - \dfrac{\gamma K_1 i_f i_a}{J} + (2\theta\sigma - \mu)(\omega - \hat{\omega}) \\ \dfrac{d\hat{C}_p}{dt} = \theta^2\sigma(\omega - \hat{\omega}) \end{array} \right. \quad (21)$$

A block diagram of the proportional estimator is given in Fig. 3.

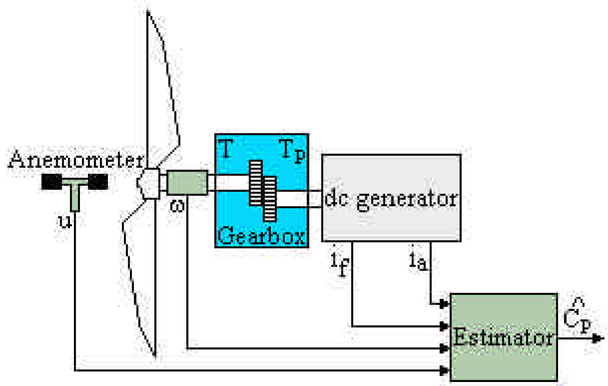

Fig. 3. WECS estimator schematic

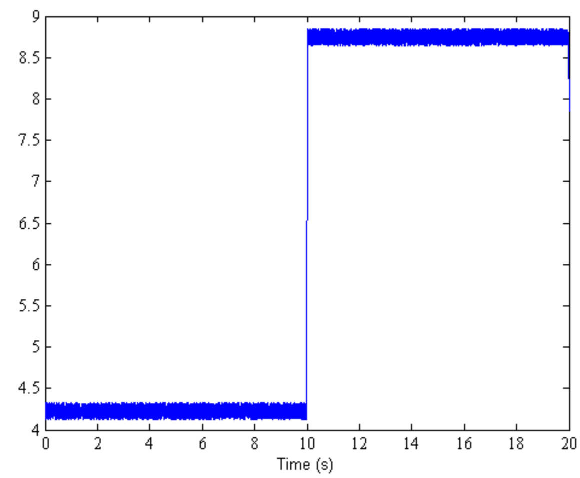

Fig. 4. Wind speed $u(t)$ signal with measurment noise

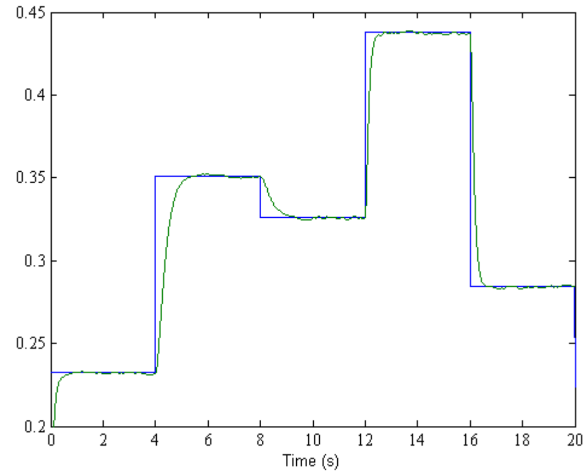

Fig. 5. Estimation of C_p with no disturbance($\theta = 2$)

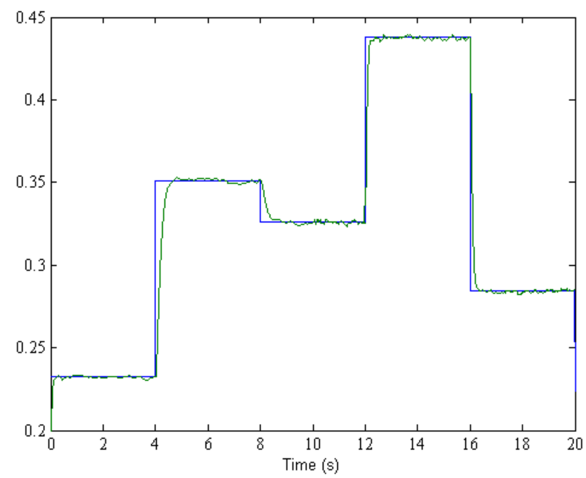

Fig. 6. Estimation of C_p with no measurment noise ($\theta = 4$)

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, we evaluate the performance of the estimator designed in the previous section. The following numerical values of the model parameters are used for simulation studies:

$V_f = 240\text{V}$	$R_f = 60$
$L_f = 60\text{H}$	Rated $V_a = 240\text{V}$
$R_a = 1.2$	$L_a = 0.01\text{H}$
$K_1 = 0.353\text{NmA}^{-2}$	$B_e = 0.011\text{Nm}$
$J_e = 0.208\text{kgm}^2$	$\rho = 1.25\text{kgm}^{-3}$
$R = 1.5\text{m}$	$\gamma = 1 : 23.4$
$J_m = 0.3\text{kgm}^2$	$B_m = 0.0151\text{Nm}$.

For simulation purposes, we have assumed that V_a is modelled as a purely resistive load so that $V_a = R_l i_a$. The numerical value used for the simulations is $R_l = 15.57$.

Two sets of simulations were carried out, one without and one with measurement noise $d(t)$. For these simulations the values of $\theta = 2$ and $\theta = 4$ were adopted. The wind speed profile, shown in Fig. 4, is modelled as a step change with an additional uniform random noise.

Fig. 5 shows the estimation of C_p when $\theta = 2$ with no measurement noise. The power coefficient C_p and the estimated power coefficient $\hat{C}_p$ are shown. The piecewise square wave function represents the reference power coefficient C_p. It can be easily seen that the estimator provides estimation of C_p with good convergence properties.

Fig. 6 illustrates the estimator behaviour for $\theta = 4$ with no disturbance. Again the power coefficient C_p and the estimated power coefficient $\hat{C}_p$ are shown. Here we can see that the convergence is faster as expected for higher gain values.

Fig. 7 shows the profile of the measurement noise $d(t)$.

Fig. 8 shows the estimation of C_p when $\theta = 2$ under noisy measurements. The estimator again provides an estimation of C_p with satisfactory convergence properties.

Fig. 9 shows the estimation of C_p when $\theta = 4$ under noisy measurements. The estimator again provides

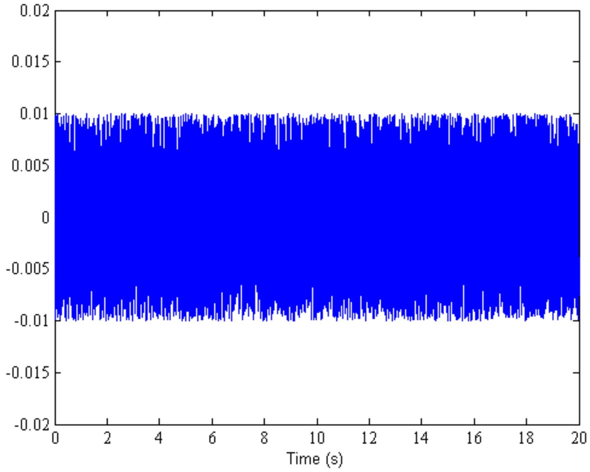

Fig. 7. The measurment noise $d(t)$

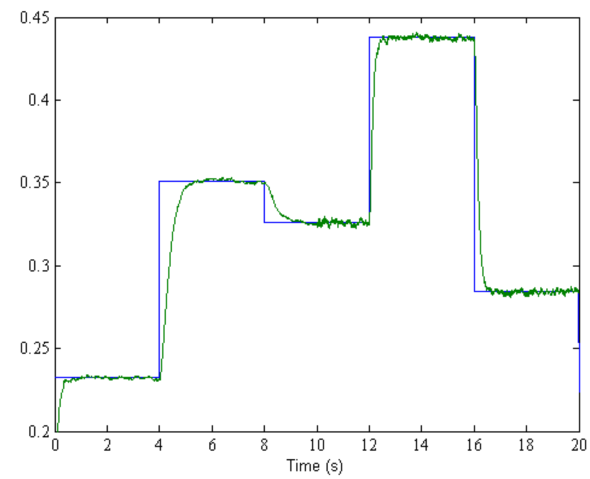

Fig. 8. Estimation of C_p with measurment noise ($\theta = 2$)

estimation of C_p with good convergence properties and faster response than in Fig. 8. However, the noise is amplified to the high gain employed. It can be noted that the noise amplification is higher in the time interval 10s to 20s. This is due to the fact that $K_{P1} = 2\theta\sigma - \mu = \frac{\theta\rho\pi R^2 u^3}{J\omega} - \frac{B}{J}$ is a function of the cube of the wind speed which is also affected by an additive random noise as mentioned previously. As a result, as the value of u increases the noise affecting the wind speed is also amplified by a cube factor. Consequently, a trade-off is necessary with respect to noise amplification and fast convergence. One way to overcome this problem would be to employ a PI observer as described in [14] instead of the previous proportional observer; since it is shown that the PI observer has the property of attenuating measurement noise.

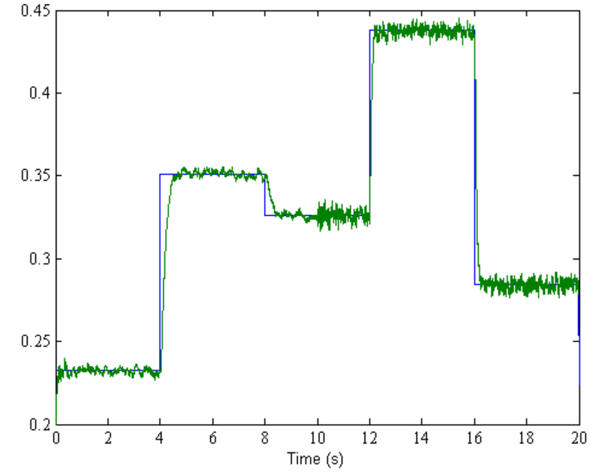

Fig. 9. Estimation of C_p with measurement noise ($\theta = 4$)

Some Remarks

- It is important to realise that the wind speed u measured by the anemometer is the wind speed as measured at a particular point in the area swept by the rotor. This is not an ideal representation as it assumes that the wind speed is uniform over that area and does not take into consideration spatial fluctuations which can be eliminated using spatial filtering.
- Future work would be to consider the case when the power coefficient is not assumed to be a piecewise constant function in order to provide a more realistic representation of the practical system. However, current suitable anemometer technology has a measurement frequency of around 1Hz, subsequently the assumption that C_p is piecewise constant is not too restrictive.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, the design of a proportional observer for the estimation of the power coefficient in a wind energy conversion system has been presented and its performance evaluated by simulation studies. The results obtained have verified the estimator's capability of supplying good estimates of the power coefficient. It is also shown that the estimator can effectively handle measurement noise while retaining satisfactory convergence qualities provided that the observer gain is not excessively high. The proposed observer design has many other important advantages: it is versatile, allows fewer turbine parameter dependence of the associated control system and as such can easily be extended to any other WECS where different generators and/or turbine types are used. Further work would be to develop a maximum power controller for an adjustable speed WECS based on the implementation of the estimated power coefficient.

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