

Parameters identification in a physical system for thermal efficiency prediction

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Abstract—Thermal efficiency prediction for heat exchangers is a great challenge for environmental purposes. In spite of technological developments in order to optimize materials or design, fouling phenomena are quite difficult to take into account. This important constraint for heat exchangers efficiency has to be carefully estimated and new sensors are developed in order to inform experimenters about fouling thickness evolution. In this communication, a mathematical model for fouling state prediction is proposed. Thermal system analysis is performed and from experimental observations, identification methodologies are carried out and lead to the identification of unknown model parameters.

I. INTRODUCTION

THIS communication is dedicated to parameters identification for a thermal system encountered in an industrial framework. The application purpose is the design of a new probe in order to accurately estimate in situ not only the exchange coefficient but also the fouling thickness of cross flow tubular heat exchangers from reliable transient state identification methods. Frequently, in industrial gaseous systems, the effectiveness of heat exchangers can decrease because of fouling deposition onto the heat transfer surface. Some gas-side fouling measuring devices have been envisaged. All of them allow to obtain physical information on fouling. Nevertheless, they are still far from giving thermal information and taking into account the deposit phenomena involved.

Investigating heat transfer in a heat exchanger is crucial to determine optimal condition and to ensure safety processes. In fouling conditions, the formulation of a predictive mathematical model requires the identification of unknown model parameters. The non-stationary state and non-linearity of the studied process reduce the possibility of using many traditional design-and-theoretical and experimental methods. The approach based on observations of the system state to specify the unknown parameters is known as inverse method (in other words, to find not causal-sequential, as in direct problems, but rather sequential-causal quantitative relations). Inverse heat transfer problems are

widely investigated (see [1]) since the formulation of ill-posed inverse problem by Hadamard: an inverse problem is well-posed if it satisfies the three Hadamard's conditions of existence, uniqueness and stability. If any of the previous conditions are not satisfied, then the problem is ill-posed. For continuous diffusive systems such the thermal process studied in this paper, the inverse operator is ill-conditioned and the presence of measurement noises in the observed data makes the problem unstable; the inverse problem is then ill posed. Overcoming the ill-posedness of inverse problem is known as regularization. The key issue in solving inverse problems, is how to introduce just enough prior information to obtain a satisfactory result [2]. Several techniques for regularizing and solving ill-posed problems have been proposed and used. One first simple idea consisted on choosing a restricted class of inputs : steps, band limited signals, polynomial bases ... A more attractive technique, however, does not use constraint on the input signal structure but build a regularization operator depending on a numerical parameter called the regularization parameter. Tikhonov first suggests the use a smoothing function to account for prior information. The determination of the optimal value for the smoothing parameter remains an open problem. A commonly used procedure is based on the Morozov's discrepancy principle ([3] and the references therein) assuming that a bound on measurement errors statistics is known. In other situations, a Bayesian inference is preferred and the maximum entropy principle makes it possible to take into account any prior knowledge on the unknown parameters and measurement noises.

In the specific framework of inverse heat transfer problem, three classes can be considered:

- inverse problems that arise in the diagnostic and identification of physical processes,
- inverse problems that arise in the design of engineering products,
- inverse problems that arise in the control of processes and systems.

The problem of the determination of a model for predicting fouling state evolution in a heat exchanger depends on the previous first class. In the following, the experimental situation and the partial differential equations system describing the process state evolution are exposed. Then a sensitivity analysis is performed in order to ensure efficient identification conditions. State observations are then exposed and several identification methods are investigated. Finally, model prediction is compared to experimental measurements.

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II. THERMAL SYSTEM

A. Experimental situation

Heat exchangers design and optimization are essential to ensure environmental protection since they are commonly used in industrial processes for energy management. In the specific framework of energy recovering from polluting effluents, the deposition of unwanted residues of particulate matter on exchange surfaces constitutes the fouling [4]. The effectiveness of cross flow tubular heat exchangers for instance decreases because of this foulant deposition. Besides predictive maintenance approaches developed in [5], technological developments have been investigated in the last decades in order to develop sensors dedicated to fouling state prediction (see for example [6]-[13]). However, even if physical observations on fouling are obtained, heat transfers are not well described (from the thermal point of view) and a loss of efficiency of the heat exchanger is often detected late in industrial conditions.

Mathematical modelization of fouling phenomenon has been widely investigated from the pionner studies [14] and give relations based on deposition and removal mechanisms (see for example [15]-[20]). Models are often based on strong empirical considerations and less attention is focused and thermal effect of fouling. In the following, heat transfer is modelized in a simplified geometry for both clean and fouling situation.

Let us consider the heating wire and the thermocouple described in figure 1 where T is the temperature, convective exchange coefficient is denoted by h and ambient temperature is T_∞ .

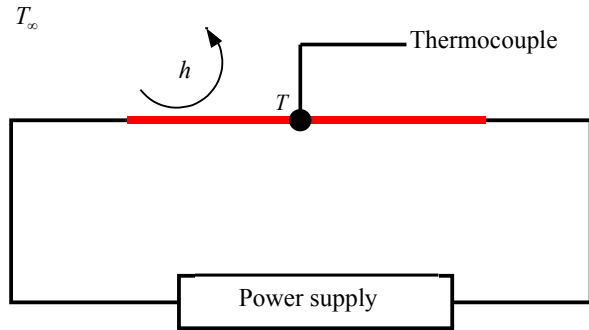


Fig. 1. Simplified experimental situation

Heating wire and thermo physical parameters are given in the following table. In the next paragraph, equation describing heat transfer in the heating wire is proposed.

TABLE 1

Symbol	Quantity	
L	length [m]	$L = (27.8 \pm 0.1) 10^{-2}$
r	radius [m]	$r = (1.49 \pm 0.01) 10^{-4}$
S	surface [m ²]	$S = (2.6 \pm 0.01) 10^{-4}$
V	volume [m ³]	$V = (1.95 \pm 0.01) 10^{-8}$
λ	thermal conductivity [W.m ⁻¹ .K ⁻¹]	$\lambda = 19.5 \pm 0.1$
ρC_p	volumic heat capacity [J.m ⁻³ .K ⁻¹]	$\rho C_p = (3.35 \pm 0.01) 10^6$

B. Mathematical model

Let us consider the following notations :

- h is the convective exchange coefficient [W.m⁻².K⁻¹]
- t is the time variable [s]
- $\phi(t) = Q.H(t)$ is the power delivered by joule effect [W],
- $H(t)$ is the Heaviside function.

Then, temperature evolution of the heating wire is described by :

$$\rho C_p V \frac{dT}{dt} = \phi(t) - hS(T - T_\infty) \quad (1)$$

Solution of (1) is given by :

$$T(t) = \frac{Q}{hS} \left(1 - e^{-\frac{hS}{\rho C_p V} t} \right) + T_\infty \quad (2)$$

In gaseous systems, while the deposition of particulate matter leads to fouling conditions, both the heat exchange coefficient on the heating wire surface h and the material property $\rho C_p V$ are modified. In order to propose an efficient identification method based on the resolution of an inverse problem, a sensitivity analysis is performed in the following paragraph.

III. SENSITIVITY ANALYSIS

Let us denote by $\beta = [h, \rho C_p V]$ the unknown parameters. The sensitivity coefficient versus parameter β_i ,

$$(i=1,2) \text{ at instant } t_k \text{ is : } X_i(t_k, \beta) = \frac{\partial T(t_k, \beta)}{\partial \beta_i}.$$

Comparisons are performed by introducing :

$$X_i^* = \beta_i \frac{\partial T}{\partial \beta_i} = \beta_i X_i. \text{ On figure 2, sensitivity functions are}$$

presented according to the following values : $Q = 0.02 W$, $T_\infty = 293 K$. Convective exchange coefficient is fixed : $h = 35 W.m^{-2}.K^{-1}$.

It is shown on curve (+) that system state depends on h at each instant. However, sensitivity to $\rho C_p V$ is neglected for $t > 50 s$, see curve (*). Moreover for $t < 2 s$, relation between h and $\rho C_p V$ avoid identification. Then in order to identify the unknown parameters from state observations :

- $\rho C_p V$ has to be identified from observations obtained in $t \in [2, 50]$,
- h can be identified from long time measurements.

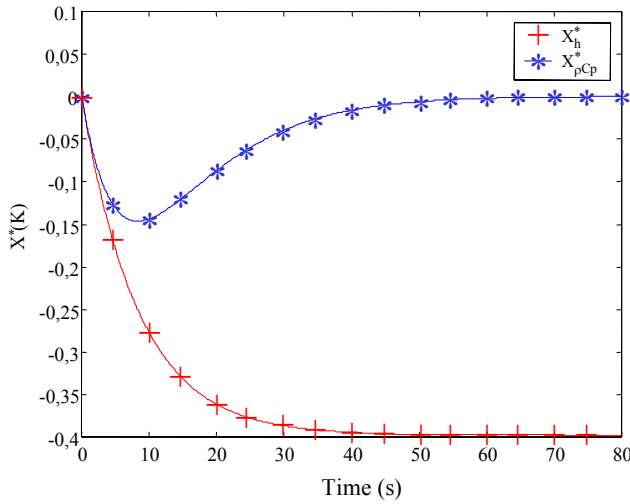


Fig. 2. Sensitivity functions

In the next paragraphs, the following notations are considered, $H = hS$ and $C = \rho C_p V$.

IV. IDENTIFICATION METHODS

A. Method 1

This method is based on a trivial formulation deduced from (2), considering linearization around initial known values (H_0, C_0) and introducing : $f(H, C, t) = 1 - e^{-\frac{H}{C}t}$:

$$T(t_k) - T_\infty = \frac{Q}{H} \left(f(H_0, C_0, t_k) + \Delta H \cdot \frac{\partial f}{\partial H} + \Delta C \cdot \frac{\partial f}{\partial C} \right) \quad (3)$$

where : $\frac{\partial f}{\partial H} = \frac{t}{C_0} e^{-\frac{H_0}{C_0}t}$ and $\frac{\partial f}{\partial C} = -\frac{H_0}{C_0^2} t e^{-\frac{H_0}{C_0}t}$. Then

identification problem is formulated as :

$$[T]_{n \times 1} = [X]_{n \times 4} \cdot [B]_{4 \times 1} \quad (4)$$

where : $[T]_{n \times 1} = \begin{bmatrix} T(t_1) \\ \vdots \\ T(t_n) \end{bmatrix}$ corresponds to the n observations,

$$[X]_{n \times 4} = \begin{bmatrix} 1 & f(H_0, C_0, t_1) & \left(\frac{\partial f}{\partial H} \right)_{t=t_1} & \left(\frac{\partial f}{\partial C} \right)_{t=t_1} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & f(H_0, C_0, t_n) & \left(\frac{\partial f}{\partial H} \right)_{t=t_n} & \left(\frac{\partial f}{\partial C} \right)_{t=t_n} \end{bmatrix} \quad \text{and}$$

$$([B]_{4 \times 1})^T = \left[T_\infty \quad \frac{Q}{H} \quad \frac{Q}{H} \Delta H \quad \frac{Q}{H} \Delta C \right].$$

Then, from given value of (H_0, C_0) and knowing Q , state system observations $T_{k=1, \dots, n}$ lead to the determination of T_∞ , H and ΔC :

$$[B]_{4 \times 1} = \left([X]_{4 \times n}^T \cdot [X]_{n \times 4} \right)^{-1} \cdot [X]_{4 \times n}^T \cdot [T]_{n \times 1} \quad (5)$$

This relation is illustrated in thermal sciences context in [21]. Experimental results are shown on the following figure where fouling conditions corresponds to a thin coating.

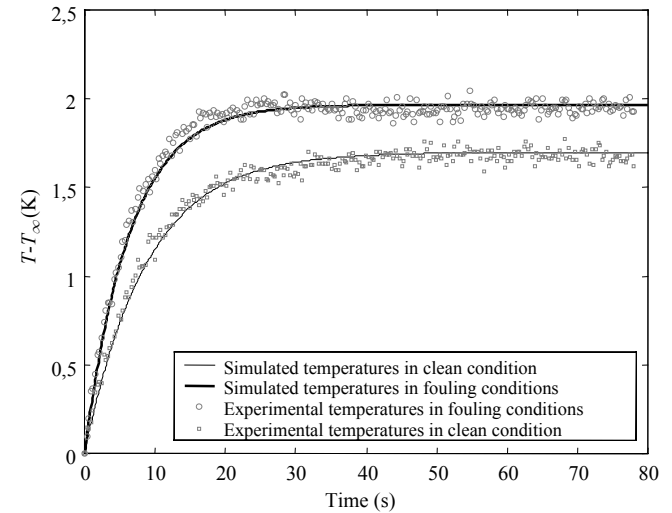


Fig. 3. Comparisons between experimental results and simulations

Results obtained, from method 1, are :

TABLE 2		
	$h \ [W.m^{-2}.K^{-1}]$	$\Delta \rho C_p V \ [J.K^{-1}]$
clean condition	$45.2 \pm 5.0 \cdot 10^{-3}$	
fouling conditions	$39.0 \pm 3.8 \cdot 10^{-3}$	$0.035 \pm 3.5 \cdot 10^{-3}$

Residual values between simulated $T(\beta)$ and measured temperatures T_m are shown on the following figures.

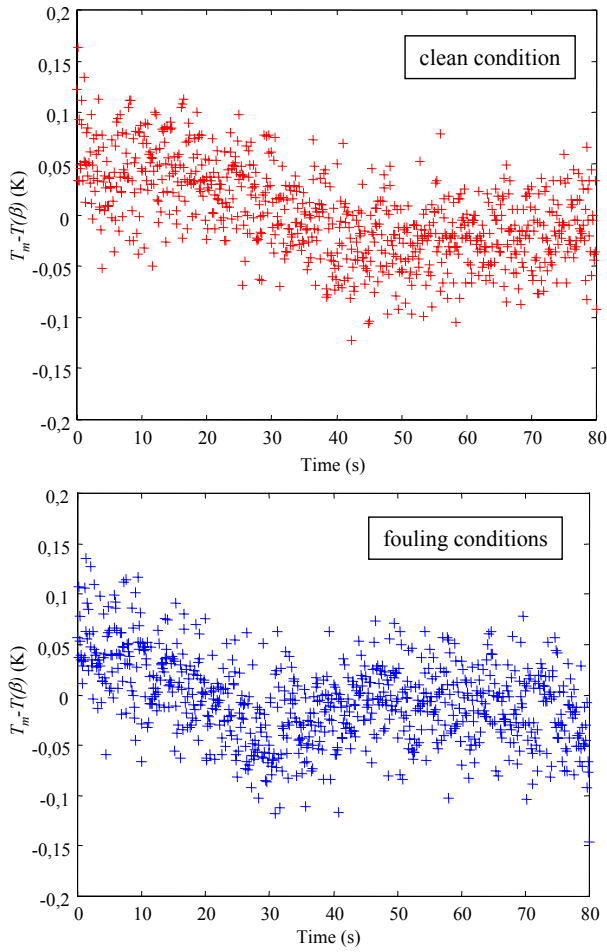


Fig. 4. Residual values between experimental and simulated results

Figures 3 and 4 ensure that unknown parameters $\beta = [H, \Delta C]$ are identified and values given in table 2 can be taken into account. However, it is important to notice that prior informations on the heating source Q and the wire surface S are required. For situations where inaccuracies are not neglected on S or where heating source is noisy disturbed, method 1 can lead to erroneous estimations.

B. Method 2

The aim of this method is to identify simultaneously $\left(\frac{\Delta H}{H}, \frac{\Delta C}{C}\right)$ from experimental sensitivities. Firstly, considering only the convective heat transfer coefficient variation, it comes :

$$T(H + \Delta H, C, t) = T(H, C, t) + \Delta H \frac{\partial T}{\partial H} \quad (6)$$

Moreover, since $T(H, C, t) = \frac{Q}{H} \left(1 - e^{-\frac{Ht}{C}}\right)$, then :

$$T(H + \Delta H, C, t) = \frac{Q}{H + \Delta H} \left(f(H, C, t) + \frac{\Delta H}{C} te^{-\frac{Ht}{C}} \right)$$

and :

$$\begin{aligned} T(H + \Delta H, C, t) &= \frac{Q}{H \left(1 + \frac{\Delta H}{H}\right)} \left(f(H, C, t) + \frac{\Delta H}{C} te^{-\frac{Ht}{C}} \right) \\ &= \frac{Q}{H} \left(1 - \frac{\Delta H}{H}\right) \left(f(H, C, t) + \frac{\Delta H}{C} te^{-\frac{Ht}{C}} \right) \\ &= \frac{Q}{H} f(H, C, t) + \frac{Q \Delta H}{H C} te^{-\frac{Ht}{C}} \\ &\quad - \frac{Q \Delta H}{H^2} f(H, C, t) - \frac{Q}{H^2} \frac{(\Delta H)^2}{C} te^{-\frac{Ht}{C}} \end{aligned}$$

then,

$$\begin{aligned} T(H + \Delta H, C, t) &= T(H, C, t) \\ &\quad + \frac{\Delta H}{H} \left(t \frac{\partial T(H, C, t)}{\partial t} - T(H, C, t) \right) \end{aligned}$$

Thus, according to (6),

$$\frac{\partial T(H, C, t)}{\partial H} = \frac{1}{H} \left(t \frac{\partial T(H, C, t)}{\partial t} - T(H, C, t) \right) \quad (7)$$

Moreover, by the same way, the following relation is formulated :

$$\frac{\partial T(H, C, t)}{\partial C} = -\frac{t}{C} \frac{\partial T(H, C, t)}{\partial t} \quad (8)$$

Then (7) and (8) are used in the following equation :

$$T(H + \Delta H, C + \Delta C, t) = T(H, C, t) + \Delta H \frac{\partial T}{\partial H} + \Delta C \frac{\partial T}{\partial C}$$

and lead to :

$$\begin{aligned} T(H + \Delta H, C + \Delta C, t) &= T(H, C, t) \\ &\quad + \frac{\Delta H}{H} \left(t \frac{\partial T(H, C, t)}{\partial t} - T(H, C, t) \right) \\ &\quad - \frac{\Delta C}{C} t \frac{\partial T(H, C, t)}{\partial t} \end{aligned} \quad (9)$$

where $T(H + \Delta H, C + \Delta C, t)$ is the experimental temperature measurement in fouling conditions.

Thus :

$$\begin{aligned} \int_0^t \frac{1}{t} (T(H + \Delta H, C + \Delta C, t) - T(H, C, t)) dt &= \quad (10) \\ &\quad + \frac{\Delta H}{H} \left(T(H, C, t) - \int_0^t T(H, C, t) dt \right) - \frac{\Delta C}{C} T(H, C, t) \end{aligned}$$

Let us denote by :

$$\mathbf{M}'_1(t) = \int_0^1 T(\mathbf{H} + \Delta\mathbf{H}, \mathbf{C} + \Delta\mathbf{C}, t) dt$$

$$\mathbf{M}''_1(t) = \int_0^1 T(\mathbf{H}, \mathbf{C}, t) dt$$

$$\mathbf{M}_1(t) = \mathbf{M}'_1(t) - \mathbf{M}''_1(t)$$

Then the following equation is considered :

$$[\mathbf{M}_1]_{n \times 1} = [\mathbf{X}]_{n \times 2} \cdot [\mathbf{B}]_{2 \times 1} \quad (11)$$

where : $[\mathbf{M}_1]_{n \times 1} = \begin{bmatrix} \mathbf{M}_1(t_1) \\ \vdots \\ \mathbf{M}_1(t_n) \end{bmatrix}$, $[\mathbf{B}]_{2 \times 1} = \begin{bmatrix} \frac{\Delta\mathbf{H}}{\mathbf{H}} \\ \frac{\Delta\mathbf{C}}{\mathbf{C}} \end{bmatrix}$ and

$$[\mathbf{X}]_{n \times 2} = \begin{bmatrix} T(\mathbf{H}, \mathbf{C}, t_1) - \mathbf{M}''_1(t_1) & -T(\mathbf{H}, \mathbf{C}, t_1) \\ \vdots & \vdots \\ T(\mathbf{H}, \mathbf{C}, t_n) - \mathbf{M}''_1(t_n) & -T(\mathbf{H}, \mathbf{C}, t_n) \end{bmatrix}$$

Then, relation : $[\mathbf{B}]_{2 \times 1} = ([\mathbf{X}]_{2 \times n}^T \cdot [\mathbf{X}]_{n \times 2})^{-1} \cdot [\mathbf{X}]_{2 \times n}^T \cdot [\mathbf{M}_1]_{n \times 1}$

leads to identification of $\left(\frac{\Delta\mathbf{H}}{\mathbf{H}}, \frac{\Delta\mathbf{C}}{\mathbf{C}}\right)$ by considering the measurement data shown on figure 3 :

	$\frac{\Delta\mathbf{H}}{\mathbf{H}}$	$\frac{\Delta\mathbf{C}}{\mathbf{C}}$
fouling conditions	0.1549 ± 9.610^{-5}	0.5280 ± 5.410^{-5}

Method 2 does not inform about (\mathbf{H}, \mathbf{C}) but only about the variation $\left(\frac{\Delta\mathbf{H}}{\mathbf{H}}, \frac{\Delta\mathbf{C}}{\mathbf{C}}\right)$. However, it is important to notice that prior informations on nuisance parameters (such as the heating source the heating source Q and the wire surface S) are not required in order to obtain an accurate estimation.

C. Method 3

For longer steady-state observations, the following situation is encountered :

$$\int_0^1 T(\mathbf{H}, \mathbf{C}, t) dt \ll T(\mathbf{H}, \mathbf{C}, t) \quad (12)$$

If relation (12) is satisfied, method 2 is modified since $[\mathbf{X}]_{n \times 2}$ is badly conditioned and $\frac{\Delta\mathbf{H}}{\mathbf{H}}$ and $\frac{\Delta\mathbf{C}}{\mathbf{C}}$ have to be identified on different time samples [22]. Let us consider

that for $t > t^*$, $t \frac{\partial T(\mathbf{H}, \mathbf{C}, t)}{\partial t} \ll T(\mathbf{H}, \mathbf{C}, t)$. Then (9) is modified :

$$\begin{aligned} T(\mathbf{H} + \Delta\mathbf{H}, \mathbf{C} + \Delta\mathbf{C}, t) &= T(\mathbf{H}, \mathbf{C}, t) + \frac{\Delta\mathbf{H}}{\mathbf{H}} (-T(\mathbf{H}, \mathbf{C}, t)) \\ &= T(\mathbf{H}, \mathbf{C}, t) \left[1 - \frac{\Delta\mathbf{H}}{\mathbf{H}} \right] \end{aligned}$$

Let us denote by :

$$\mathbf{M}'_0(t) = \int_0^1 T(\mathbf{H} + \Delta\mathbf{H}, \mathbf{C} + \Delta\mathbf{C}, t) dt$$

$$\mathbf{M}''_0(t) = \int_0^1 T(\mathbf{H}, \mathbf{C}, t) dt$$

$$\text{then : } \frac{\Delta\mathbf{H}}{\mathbf{H}} = \frac{\mathbf{M}'_0(t)}{\mathbf{M}''_0(t)} - 1.$$

Once $\frac{\Delta\mathbf{H}}{\mathbf{H}}$ is identified, the following relation issued

from (10) is considered for the identification of $\frac{\Delta\mathbf{C}}{\mathbf{C}}$ at any time t' :

$$\frac{\Delta\mathbf{C}}{\mathbf{C}} = \frac{1}{T(\mathbf{H}, \mathbf{C}, t')} \left[-\mathbf{M}_1(t') + \frac{\Delta\mathbf{H}}{\mathbf{H}} (T(\mathbf{H}, \mathbf{C}, t') - \mathbf{M}''_1(t')) \right]$$

According to method 3, parameters identification results are shown in table 3.

	$\frac{\Delta\mathbf{H}}{\mathbf{H}}$	$\frac{\Delta\mathbf{C}}{\mathbf{C}}$
fouling conditions	0.1561	0.5242

Identification results obtained according to both methods (2 & 3) considering exactly the same measurements are quite similar for the encountered situation. In fact, matrix $[\mathbf{X}]_{n \times 2}$ is not very badly conditioned ($cond(\mathbf{X}) \leq 7$) and (12) is not satisfied. In such a situation method 2 does not lead to erroneous estimation. For longer steady-state observations, (12) is verified : method 2 would not ensure a simultaneous correct estimation of the variation $\left(\frac{\Delta\mathbf{H}}{\mathbf{H}}, \frac{\Delta\mathbf{C}}{\mathbf{C}}\right)$. Then method 3 has to be implemented.

V. CONCLUDING REMARKS

In this communication, several identification methods have been exposed and compared in an experimental situation. The thermal system describing thermophysical modifications due to fouling conditions has been presented

and a sensitivity analysis has been performed. It has been shown that the convective heat exchange coefficient and the variation of a volumic thermal characteristic of the system depend on the perturbation generated by fouling. Considering state system observations (obtained by thermocouple), inverse methods are implemented and lead to efficient parameters identification.

Identification methods 2 and 3 are used in order to estimate the modification not only of the heat transfer coefficient but a global thermophysical characteristic of the system. Experimental sensitivity functions are obtained from the temporal variation of the experimental temperatures and not from the nominal values of the forward model as they are usually.

VI. FURTHER INVESTIGATIONS

The presented methods have been successfully implemented for the conception of a new probe. Its main originality lie in the fact that it replaces one part (or the totality) of exchanger tube and it is constructed from the same materials to be subject to the same thermohydraulic and fouling conditions as exchanger. Its principle is based on the analysis of the transient temperature response of the system to a thermal excitation. An experimental device has been built up, in order to test the thermal transient probe. An analytical 3D forward model for transient heat transfer is developed. The thermal quadrupole formalism using integral transforms methods [23] is used to model the three-dimensional transient heat transfer in the studied multilayered system. The interest of this approach lies in fact that a linear relationship is given between the input and the output temperature and heat flux after a double Laplace Fourier transform. The main advantage of this temperature-flux vector representation is to make possible the analytical modeling of multimaterials by multiplying the corresponding quadrupole matrices. Then, it allows to optimise the probe sizing.

REFERENCES

- [1] O. M. Alifanov, *Inverse heat transfer problems*, Springer-Verlag, 1994, pp. 348.
- [2] A. Mohammad-Djafari, J. Giovannelli, G. Demoment and J. Idier, "Regularization, maximum entropy and probabilistic methods in mass spectrometry data processing problems", *International Journal of Mass Spectrometry*, vol. 215, 2002, pp. 175–193.
- [3] W. B. Muniz, F. M. Ramos and H. F. Velho De Campos, "Entropy and Tikhonov-base regularization techniques applied to the backwards heat equation", *Computers and mathematics with applications*, vol. 40, 2000, pp. 1071-1084.
- [4] T. R. Bott, "Heat exchanger fouling. The Challenge. Fouling Mechanisms : Theoretical and Practical Aspects", presented at the 1992 Eurotherm Seminar 23, 3-10, Grenoble, France.
- [5] G. P. Sullivan, W. D. Hunt, A. P. Melendez and R. Pugh, "Operations and maintenance, Best practices, A guide to achieving operational efficiency", *US Department of Energy*, 2002.
- [6] H. H. Krause, P. W. Cover, W. E. Berry and R.A. Olexsey, "Corrosion and deposit from combustion of solid waste. Part VII : Coincineration of refuse and sewage sludge", *Journal Engineering for Power*, 102:698-705, 1980.
- [7] A. L. Plumley and W. R. Rocznik, "Natural occurring high-chloride coal and superheater corrosion", *Journal Engineering for Power*, 104:874-884, 1982.

- [8] D. E. Rosner and R. M. Atkins, *Experimental studies of salt / ash deposition rates from combustion products using optical techniques*, in Fouling and slagging resulting from impurities in combustion gases, R. W. Bryers Edition, New York, 1983.
- [9] J. D. Isdale, A. C. Scott and G. Cartwright, *The use of probes to assess fouling in dirty gas streams*, in Installation effects in ducted fan systems, Institution of Mechanical Engineers, Londres, 1984.
- [10] J. Marner, K. S. MacDavid and L. J. Muzio, *Development of a gas side measuring device*, in Heat Transfer Fundamentals Design, Applications and Operating Problems, R. K. Shaw Editions, 1987.
- [11] W. J. Marner, "Progress in gas-side fouling of heat transfer surfaces", *Applied Mechanical Review*, 43: 35-66, 1990.
- [12] M. Sohal and S. Field. "Testing of a probe to measure fouling in an industrial flue gas stream", *Heat Transfer Engineering*, 14(2), 1993.
- [13] A. K. Temu, E. Naess and O. K. Sonju, "Development and testing of a probe to monitor gas side fouling in cross flow", *Heat Transfer Engineering*, 23, 2002.
- [14] D. Q. Kern and R. E. Seaton, "A theoretical analysis of thermal surface fouling", *Britanic Chemical Sciences* 4, 5:258-262, 1959.
- [15] S. K. Beal. "Deposition of particles in turbulent flow on channel or pipe walls", *Nuclear Science and Engineering*, 1970.
- [16] A. P. Watkinson and N. Epstein, "Particulate fouling of sensible heat exchangers", presented at the 1970 4th International Heat Transfer Conference, 1:1-12.
- [17] D. Thomas and U. Grigull, "Experimental investigation of the deposition of suspended magnetite from the fluid flow in steam generating boiler tubes", *Brennst-Warme Kraft*, 109-115, 1974.
- [18] N. Epstein, "Fouling in heat exchangers", presented in Heat Transfer 1978: 6th International Heat Transfer Conference, 235-253.
- [19] N. F. Glen and J. H. Howarth, "Modelling refuse incineration fouling", presented in 2nd United Kingdom Conference on Heat Transfer, 1988, Institute of Mechanical Engineering, 1:401- 420.
- [20] D. Bouris and G. Bergeles, "Particle-Surface Interactions in Heat Exchanger Fouling", *Journal of Fluids Engineering*, 118:574-581, 1996.
- [21] J. V. Beck and K. J. Arnold, *Parameter estimation in engineering and science*, New-York, Editions Wiley, 1977.
- [22] A. Degiovanni, M. Laurent, *Une nouvelle technique d'identification de la diffusivité thermique pour la méthode flash*, *Revue de Physique appliquée*, vol 21, 229-237, 1986.
- [23] D. Maillet, S. André, J.C. Batsale, A. Degiovanni and C. Moyne, *Thermal quadrupoles, solving the heat equation through integral transforms*, Editions Wiley and Sons, 2000.