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Adaptive Nonlinear Tracking Control of Kinematically Redundant Robot Manipulators with Sub-Task Extensions[1]

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Abstract: Past research efforts have focused on the end-effector tracking control of redundant robots because of their increased dexterity over their non-redundant counterparts. This work utilizes an adaptive full-state feedback quaternion based controller developed in [5] and focuses on the design of a general sub-task controller. This sub-task controller does not affect the position and orientation tracking control objectives, but instead projects a preference on the configuration of the manipulator based on sub-task objectives [18].

1 Introduction

In many robotic applications, the desired task is naturally defined in terms of end-effector motion. As a result, the desired robot trajectory is described by the desired position and orientation of a Cartesian coordinate frame attached to the robot manipulator's end-effector with respect to the base frame, also referred to as the *task-space*. Control of robot motion is then performed using feedback of either the joint variables (relative position of each robot joint pair) or the task-space variables. Unfortunately, joint-based control has the undesirable feature of requiring the solution of the inverse kinematics to convert the desired task-space trajectory into the desired joint space trajectory. In contrast, task-space control does not require the inverse kinematics; however, the precise tracking control of the end-effector orientation complicates the problem. For example, several parameterizations exist to describe the orientation angles, including minimum three-parameter representations (*e.g.*, Euler angles, Rodrigues parameters, *etc.*) and the non-minimum four-parameter representation given by the unit quaternion. Whereas the three-parameter representations always exhibit singular orientations (*i.e.*, the orientation Jacobian matrix in the kinematic equation is singular for some orientations), the unit quaternion-based approach can be used to represent the end-effector orientation without singularities. Thus, despite significantly complicating the control design, the unit quaternion seems to be the preferred method of formulating the end-effector orientation tracking control problem. Some past work that deals with task-space control formulation can be found in [2], [8], and [20]. Specifically, an experimental assessment of different end-effector orientation parameterization for task-space robot control was provided in [2]. One of the first results in task-space control of robot manipulators was presented in [8]. Resolved-rate and resolved-acceleration task-space controllers using the quaternion parameterization were proposed in [20].

In addition, the control problem is further complicated in the presence of kinematic redundancy. That is, to provide the end user with increased flexibility for executing sophisticated tasks, the next generation of robot manipulators will have more degrees of freedom than are required to perform an operation in the task-space. Since the number of joints in a redundant robot is greater than the dimension of the task-space, one can show that joint motion in the null-space of the Jacobian matrix exists that does not affect task-space motion (this phenomenon is commonly referred to as *self-motion*). As noted in [13], [14], and [17], there are generally an infinite number of solutions for the inverse kinematics of redundant robots. As a result, given a desired task-space trajectory, it is difficult to select a reasonable desired joint trajectory that satisfies the control requirements (*e.g.*, closed-loop stability and boundedness of all signals) and the sub-tasks. Thus, there is strong motivation for control of redundant robots to be done in the task-space. For work related to controllers for redundant robots, the reader is referred to [3], [6], [8], [15], [16], [19], [21] and the references therein.

This paper utilizes the adaptive full-state feedback quaternion based controller developed in [5] and focuses on the design of a general sub-task controller. The novelty of this work is the systematic integration of the sub-task controller while simultaneously achieving end-effector tracking. Other efforts have been proposed in [5], [6] and [21], but in these approaches, the sub-task objective is an *add-on* to the tracking objec-

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tive without integration into the stability analysis. In [5], a sub-task control signal was introduced and can be seen in equation (2.211) as $h(t)$. In the stability analysis of [5], this sub-task signal is inconsequential to the tracking control objective as long as $h(t)$ and $\dot{h}(t)$ remain bounded. This work will exploit the property of self-motion for redundant robot manipulators by designing a general sub-task controller that meets the above conditions while controlling the joint motion in the null-space of the Jacobian matrix to alleviate potential problems in the physical system or select configurations that are better suited for a particular application. In Section 2 the dynamic and kinematic models are described for a general redundant robot manipulator. In Section 3 the tracking objective is presented. The tracking closed-loop error system is presented in Section 4. In Sections 5 and 6 the general sub-task controller is presented. Concluding remarks are made in Section 7.

2 Robot Dynamic and Kinematic Model

The dynamic model for an n-joint ($n \geq 6$), revolute, direct drive robot manipulator is described by the following expression

$$M(\theta)\ddot{\theta} + V_m(\theta,\dot{\theta})\dot{\theta} + G(\theta) + F_d\dot{\theta} = \tau \qquad (1)$$

where $\theta(t), \dot{\theta}(t), \ddot{\theta}(t) \in \mathbb{R}^n$ denote the joint position, velocity, and acceleration in the joint-space, respectively. In (1), $M(\theta) \in \mathbb{R}^{n\times n}$ represents the inertia effects, $V_m(\theta,\dot{\theta}) \in \mathbb{R}^{n\times n}$ represents centripetal-Coriolis effects, $G(\theta) \in \mathbb{R}^n$ represents the gravity effects, $F_d \in \mathbb{R}^{n\times n}$ represents the constant positive definite diagonal dynamic frictional effects, $\tau(t) \in \mathbb{R}^n$ represents the control input torque vector. The subsequent development is based on the following properties [11].

Property 1: The inertia matrix $M(\theta)$ is symmetric and positive-definite, and satisfies the following inequalities

$$m_1 \|\xi\|^2 \leq \xi^T M(\theta)\xi \leq m_2 \|\xi\|^2 \qquad \forall \xi \in \mathbb{R}^n \quad (2)$$

where m_1, $m_2 \in \mathbb{R}$ are positive constants, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: The inertia and centripetal-Coriolis matrices satisfy the following skew symmetric relationship

$$\xi^T \left(\frac{1}{2}\dot{M}(\theta,\dot{\theta}) - V_m(\theta,\dot{\theta})\right)\xi = 0 \qquad \forall \xi \in \mathbb{R}^n \quad (3)$$

where $\dot{M}(\theta,\dot{\theta})$ denotes the time derivative of the inertia matrix.

Property 3: The left-hand side of (1) can be linearly parameterized as shown below

$$M(\theta)\ddot{\theta} + V_m(\theta,\dot{\theta})\dot{\theta} + G(\theta) + F_d\dot{\theta} = Y_g\left(\theta,\dot{\theta},\ddot{\theta}\right)\phi \quad (4)$$

where $\phi \in \mathbb{R}^p$ contains the constant system parameters, and the regression matrix $Y_g(\cdot) \in \mathbb{R}^{n\times p}$ contains known functions dependent on the signals $\theta(t)$, $\dot{\theta}(t)$, and $\ddot{\theta}(t)$.

Let $\mathcal{E}$ and $\mathcal{B}$ be orthogonal coordinate frames attached to the end-effector of a redundant robot manipulator and its inertial frame, respectively. The position and orientation of $\mathcal{E}$ relative to $\mathcal{B}$ are commonly represented by a homogeneous transformation matrix, $T(\theta) \in \mathbb{R}^{4\times 4}$ which is defined as [11]

$$T(\theta) \triangleq \begin{bmatrix} R(\theta) & p(\theta) \\ 0_{1\times 3} & 1 \end{bmatrix} \qquad (5)$$

where $0_{1\times 3} \triangleq \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, the vector $p(\theta) \in \mathbb{R}^3$ and the rotation matrix $R(\theta) \in \mathbb{R}^{3\times 3}$ represent the position and orientation of the end-effector coordinate frame, respectively. From this homogeneous transformation matrix, the constrained four-parameter unit quaternion representation can be used to develop the kinematic model. From (5), a relationship between the position and orientation of $\mathcal{E}$ relative to $\mathcal{B}$ can be developed as follows [12]

$$\begin{bmatrix} p \\ q \end{bmatrix} \triangleq \begin{bmatrix} f_p(\theta) \\ f_q(\theta) \end{bmatrix} \qquad (6)$$

where $f_p(\theta) \in \mathbb{R}^3$ and $f_q(\theta) \in \mathbb{R}^4$ are kinematic functions, $q(t) \triangleq \begin{bmatrix} q_o(t) & q_v^T(t) \end{bmatrix}^T \in \mathbb{R}^4$ with $q_o(t) \in \mathbb{R}$ and $q_v(t) \in \mathbb{R}^3$. The variable $q(t)$, as given in (6), denotes the unit quaternion [7]. The unit quaternion represents a *global nonsingular* parameterization of the end-effector orientation, and is subject to the constraint $q^T q = 1$. Note that, while $f_p(\theta)$ is directly obtained from (5), several algorithms exist to determine $f_q(\theta)$ from $R(\theta)$ ([7] and [9]). Conversely, $R(q)$ can be determined given the unit quaternion parameterization [7]

$$R(q) \triangleq \left(q_o^2 - q_v^T q_v\right) I_3 + 2q_v q_v^T + 2q_o q_v^\times \qquad (7)$$

where $I_3 \in \mathbb{R}^{3\times 3}$ is the standard identity matrix, and the notation $a^\times \in \mathbb{R}^{3\times 3}$ $\forall a = [a_1\ a_2\ a_3]^T$, denotes the following skew-symmetric matrix

$$a^\times \triangleq \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}. \qquad (8)$$

Velocity relationships can be formulated by differentiating (6) which can be written as

$$\begin{bmatrix} \dot{p} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} J_p(\theta) \\ J_q(\theta) \end{bmatrix}\dot{\theta} \qquad (9)$$

where $\dot{\theta}(t) \in \mathbb{R}^n$ denotes the velocity of $\mathcal{E}$ in a generalized coordinate system, and $J_p(\theta) \in \mathbb{R}^{3\times n}$, $J_q(\theta) \in \mathbb{R}^{4\times n}$ denotes the position and orientation Jacobian matrices, respectively. To facilitate the subsequent control development and stability analysis, the fact that $q(t)$ is related to the angular velocity of $\mathcal{E}$ relative to $\mathcal{B}$, denoted by $\omega(t) \in \mathbb{R}^3$ with coordinates expressed in $\mathcal{B}$, via the following differential equation ([1] and [12])

$$\dot{q} \triangleq B(q)\omega \tag{10}$$

where the Jacobian-type matrix $B(q) \in \mathbb{R}^{4\times 3}$ is defined as follows

$$B(q) \triangleq \frac{1}{2}\begin{bmatrix} -q_v^T \\ q_o I_3 - q_v^\times \end{bmatrix} \tag{11}$$

where $B(q)$ satisfies the following useful property

$$B^T(q)B(q) = I_3. \tag{12}$$

The final kinematic expression that relates the generalized Cartesian velocity to the generalized coordinate system is developed as follows

$$\begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(\theta)\,\dot{\theta} \tag{13}$$

where (9), (10) and (12) were utilized, and $J(\theta) \in \mathbb{R}^{6\times n}$ is defined as follows

$$J(\theta) \triangleq \begin{bmatrix} J_p(\theta) \\ B^T(q)J_q(\theta) \end{bmatrix}. \tag{14}$$

To facilitate the control development, the pseudo-inverse of $J(\theta)$ is denoted by $J^+(\theta) \in \mathbb{R}^{n\times 6}$, which is defined as follows

$$J^+ \triangleq J^T\left(JJ^T\right)^{-1} \tag{15}$$

where $J^+(\theta)$ satisfies the following equality

$$JJ^+ = I_6 \tag{16}$$

where $I_6 \in \mathbb{R}^{6\times 6}$ is the standard identity matrix. As shown in [13], the pseudo-inverse defined by (15) satisfies the Moore-Penrose Conditions given below

$$\begin{array}{ll} JJ^+J = J & J^+J\,J^+ = J^+ \\ \left(J^+J\right)^T = J^+J & \left(JJ^+\right)^T = JJ^+. \end{array} \tag{17}$$

In addition to the above properties, the matrix $(I_n - J^+J)$ satisfies the following useful properties

$$\begin{array}{l} (I_n - J^+J)(I_n - J^+J) = I_n - J^+J \\ (I_n - J^+J)^T = (I_n - J^+J) \\ J(I_n - J^+J) = 0 \\ (I_n - J^+J)J^+ = 0 \end{array} \tag{18}$$

where $I_n \in \mathbb{R}^{n\times n}$ is the standard identity matrix.

Remark 1 *During the control development, the assumption that the minimum singular value of the manipulator Jacobian, denoted by σ_m is greater than a known small positive constant $\delta > 0$, such that $\max\{\|J^+(\theta)\|\}$ is known a priori and all kinematic singularities are always avoided.*

Remark 2 *The dynamic and kinematic terms for a general revolute robot manipulator, denoted by $M(\theta)$, $V_m(\theta, \dot{\theta})$, $G(\theta)$, $J(\theta)$, and $J^+(\theta)$, are assumed to depend on $\theta(t)$ only as arguments of trigonometric functions, and hence, remain bounded for all possible $\theta(t)$. During the control development, the assumption will be made that if $p(t) \in \mathcal{L}_\infty$ then $\theta(t) \in \mathcal{L}_\infty$ (Note that $q(t)$ is always bounded since $q(t)^T q(t) = 1$).*

3 Task-Space Tracking

The objective for the redundant robotic system is to design a control input that ensures the position and orientation of $\mathcal{E}$ tracks the position and orientation of a desired orthogonal coordinate frame $\mathcal{E}_d$ where $p_d(t) \in \mathbb{R}^3$ denotes the position of the origin of $\mathcal{E}_d$, relative to the origin of $\mathcal{B}$ and the rotation matrix from $\mathcal{E}_d$ to $\mathcal{B}$ is denoted by $R_d(\cdot) \in \mathbb{R}^{3\times 3}$. The standard assumption that $p_d(t)$, $\dot{p}_d(t)$, $\ddot{p}_d(t)$, $R_d(\cdot)$, $\dot{R}_d(\cdot)$, and $\ddot{R}_d(\cdot) \in \mathcal{L}_\infty$ will be utilized in the subsequent stability analysis. The position tracking error $e_p(t) \in \mathbb{R}^3$ can be defined as follows

$$e_p \triangleq p_d - p \tag{19}$$

where $p(t)$ was defined in (5). If the orientation of $\mathcal{E}_d$ relative to $\mathcal{B}$ is described by the desired unit quaternion, $q_d(t) \triangleq \begin{bmatrix} q_{od}(t) & q_{vd}^T(t) \end{bmatrix}^T \in \mathbb{R}^4$, then similar to (7), the desired rotation matrix can be described as follows

$$R_d(q_d) = \left(q_{od}^2 - q_{vd}^T q_{vd}\right) I_3 + 2q_{vd}q_{vd}^T + 2q_{od}q_{vd}^\times. \tag{20}$$

As in (10), $q_d(t)$ is related to the desired angular velocity of $\mathcal{E}_d$ relative to $\mathcal{B}$, denoted by $\omega_d(t) \in \mathbb{R}^3$, through the kinematic equation

$$\dot{q}_d \triangleq B(q_d)\,\omega_d. \tag{21}$$

To quantify the difference between the actual and desired end-effector orientations, a rotation matrix $\tilde{R}(\cdot) \in \mathbb{R}^{3\times 3}$ of $\mathcal{E}$ with respect to $\mathcal{E}_d$ is defined as follows

$$\tilde{R} \triangleq R_d^T R = \left(e_o^2 - e_v^T e_v\right) I_3 + 2e_v e_v^T + 2e_o e_v^\times \tag{22}$$

where the unit quaternion tracking error, $e_q(t) \triangleq \begin{bmatrix} e_o(t) & e_v^T(t) \end{bmatrix}^T \in \mathbb{R}^4$ can be derived as follows (see [20] and Theorem 5.3 of [10])

$$e_q \triangleq \begin{bmatrix} e_o \\ e_v \end{bmatrix} = \begin{bmatrix} q_o q_{od} + q_v^T q_{vd} \\ q_{od}q_v - q_o q_{vd} + q_v^\times q_{vd} \end{bmatrix} \tag{23}$$

where $e_q(t)$ satisfies the constraint

$$e_q^T e_q = e_0^2 + e_v^T e_v = 1, \tag{24}$$

which indicates that

$$0 \leq \|e_v(t)\| \leq 1 \qquad 0 \leq |e_0(t)| \leq 1 \tag{25}$$

for all time.

Based on the above definitions, the end-effector position and orientation tracking objectives can be stated as follows

$$\|e_p(t)\| \to 0 \text{ and } \tilde{R}(e_q) \to I_3 \text{ as } t \to \infty, \tag{26}$$

respectively. The orientation tracking objective given in (26) can also be stated in terms of the unit quaternion error of (23). Specifically, it is easy to see from (24) that

$$\text{if } \|e_v(t)\| \to 0 \text{ as } t \to \infty, \text{ then } |e_0(t)| \to 1 \text{ as } t \to \infty; \tag{27}$$

hence, it can be stated from (22) and (27) that

$$\text{if } \|e_v(t)\| \to 0 \text{ as } t \to \infty, \text{ then } \tilde{R}(e_q) \to I_3 \text{ as } t \to \infty. \tag{28}$$

4 Task-Space Controller

Based on the open-loop kinematic tracking error system given in [5] and the subsequent stability analysis, the control input is designed as follows

$$\tau \triangleq Y\hat{\phi} + K_r r + (\Lambda J)^T \begin{bmatrix} e_p \\ e_v \end{bmatrix} \tag{29}$$

where $K_r \in \mathbb{R}^{n\times n}$ is a positive-definite, diagonal, control gain matrix, and $\hat{\phi}(t) \in \mathbb{R}^p$ denotes the parameter estimate vector which is updated according to

$$\dot{\hat{\phi}} \triangleq \Gamma Y^T r \tag{30}$$

with $\Gamma \in \mathbb{R}^{p\times p}$ being a positive-definite, diagonal, adaptation gain matrix. The auxiliary signal $r(t) \in \mathbb{R}^n$ can be defined as follows

$$r \triangleq u_d - \dot{\theta} \tag{31}$$

where $u_d(t) \in \mathbb{R}^n$ is an auxiliary control input defined as follows

$$u_d \triangleq J^+\Lambda^{-1} \begin{bmatrix} \dot{p}_d + K_1 e_p \\ -R_d^T \omega_d + K_2 e_v \end{bmatrix} + \left(I_n - J^+ J\right) h \tag{32}$$

where $K_1,\ K_2 \in \mathbb{R}^{3\times 3}$ are positive-definite, diagonal, control gain matrices, the matrix $\Lambda(t) \in \mathbb{R}^{6\times 6}$ is defined as follows

$$\Lambda \triangleq \begin{bmatrix} -I_3 & 0_{3\times 3} \\ 0_{3\times 3} & R_d^T \end{bmatrix} \tag{33}$$

where $0_{3\times 3} \in \mathbb{R}^{3\times 3}$ denotes a matrix of zeros, and $h(\theta) \in \mathbb{R}^n$ is the subsequently designed subtask controller signal. The linear parameterization $Y(p_d, \dot{p}_d, \ddot{p}_d, e_q, \theta, \dot{\theta}, h, \dot{h})\phi$ introduced in (29) is defined as follows

$$Y\phi \triangleq M\dot{u}_d + V_m u_d + G(\theta) + F_d\dot{\theta} \tag{34}$$

where $Y(\cdot) \in \mathbb{R}^{n\times p}$ denotes the measurable regression matrix, and $\phi \in \mathbb{R}^p$ represents the constant parameter vector (*e.g.*, mass, inertia, and friction coefficients). To obtain the closed-loop dynamics for $r(t)$, the time derivative of (31) is taken, pre-multiply the resulting equation by $M(\theta)$, and substitute (1) to obtain the following

$$M\dot{r} = -V_m r + Y\tilde{\phi} - K_r r - (\Lambda J)^T \begin{bmatrix} e_p \\ e_v \end{bmatrix} \tag{35}$$

where the parameter estimation error signal $\tilde{\phi}(t) \in \mathbb{R}^p$ is defined as follows

$$\tilde{\phi} \triangleq \phi - \hat{\phi}. \tag{36}$$

Remark 3 *A benchmark adaptive controller was utilized to compensate for the parametric uncertainties present in the dynamic model (e.g., mass, inertia, and friction coefficients). Alternatively, a robust or sliding mode controller could also be used to compensate for modeling uncertainties not restricted to parametric uncertainties (e.g. see [4]).*

The following theorem can be stated regarding the stability of the closed loop system.

Theorem 1 *The control law described by (29) guarantees global asymptotic end-effector position and orientation tracking in the sense that*

$$\|e_p(t)\| \to 0 \text{ as } t \to \infty \tag{37}$$

and

$$\tilde{R}(e_q(t)) \to I_3 \text{ as } t \to \infty, \tag{38}$$

as well as that all signals are bounded provided $h(\theta) \in \mathcal{L}_\infty$ and $\frac{\partial h(\theta)}{\partial \theta} \in \mathcal{L}_\infty$. (Note the assumption given in Remark 2 has been utilized.)

Proof: See [5] for proof.

5 Sub-Task Control Objective

In addition to the tracking control objective, there can be sub-task objectives that are required for a particular redundant robot application. To this end, the auxiliary control signal $h(\theta)$, as introduced in (32), allows for sub-task objectives to be integrated into the controller. This sub-task integration is completed by designing a framework that places preferences on desirable configurations where an infinite number of choices are available when dealing with the *self-motion* of the redundant robot. These sub-tasks are integrated through the joint motion in the null-space of the standard Jacobian matrix by designing $h(\theta)$. Theorem 1 requires that $h(\theta)$, $\frac{\partial h(\theta)}{\partial \theta} \in \mathcal{L}_\infty$, provided $\theta(t) \in \mathcal{L}_\infty$. Based on Remark 2, and the proof of Theorem 1 it is clear that $\theta(t) \in \mathcal{L}_\infty$. In the subsequent section, $h(\theta)$ will be designed to meet these conditions. In the event that a subsequently defined Jacobian-related matrix loses rank, the sub-task objective is not guaranteed. More specifically, if the Jacobian-related matrix maintains full rank, then the sub-task objective is met as proven in the subsequent stability analysis.

6 Sub-Task Closed-Loop Error System

In this section, a general sub-task closed-loop error system is developed. To this end, an auxiliary signal $y_a(t) \in \mathbb{R}^+$ is defined as follows

$$y_a \triangleq \exp\left(-\alpha\beta(\theta)\right) \tag{39}$$

where $\alpha \in \mathbb{R}^+$ is a constant, $\beta(\theta) \in \mathbb{R}^+$ is selected for each sub-task, and $\exp(\cdot)$ is the standard logarithmic exponential function. To determine the dynamics of $y_a(t)$, the time derivative of (39) is taken and can be written as follows

$$\dot{y}_a = J_s\dot{\theta} \tag{40}$$

where a Jacobian-type vector $J_s(t) \in \mathbb{R}^{1\times n}$ is defined as follows

$$J_s = \frac{\partial y_a}{\partial \theta}. \tag{41}$$

From (40), a substitution can be made for $\dot{\theta}(t)$ and the following expression for $\dot{y}_a(t)$ can be written as follows

$$\begin{aligned} \dot{y}_a &= J_s J^+ \Lambda^{-1} \begin{bmatrix} \dot{p}_d + K_1 e_p \\ -R_d^T \omega_d + K_2 e_v \end{bmatrix} \\ &\quad + J_s\left(I_n - J^+ J\right) h - J_s r \end{aligned} \tag{42}$$

where (31) and (32) were both utilized. Based on the dynamics of (42) and the subsequent stability analysis, the sub-task control input can be designed as follows

$$h \triangleq -k_{s1}\left[J_s\left(I_n - J^+J\right)\right]^T y_a \tag{43}$$

where $k_{s1} \in \mathbb{R}^+$ is a constant gain. After substituting (43) into (42), the following expression can be obtained

$$\begin{aligned} \dot{y}_a &= J_s J^+ \Lambda^{-1} \begin{bmatrix} \dot{p}_d + K_1 e_p \\ -R_d^T \omega_d + K_2 e_v \end{bmatrix} \\ &\quad - J_s r - k_{s1}\left\|J_s\left(I_n - J^+J\right)\right\|^2 y_a. \end{aligned} \tag{44}$$

Remark 4 *The auxiliary signal $y_a(t)$ in (39) was selected because of the useful properties of the logarithmic exponential function. From (39) it is clear that $0 < y_a(t) \leq 1$, and that as $\beta(\theta)$ increases, $y_a(t)$ decreases. This definition of $y_a(t)$ is arbitrary and many different positive functions could also be utilized.*

The following theorem can now be stated regarding the performance of the sub-task closed-loop error system.

Theorem 2 *The control law described by (43) guarantees that $y_a(t)$ is practically regulated (i.e., ultimately bounded) in the following sense*

$$|y_a(t)| \leq \sqrt{|y_a^2(t_0)|\exp\left(-2\gamma t\right) + \frac{\varepsilon}{\gamma}} \tag{45}$$

provided the following sufficient conditions hold

$$\left\|J_s\left(I_n - J^+J\right)\right\|^2 > \bar{\delta} \tag{46}$$

and

$$k_{s1} > \frac{1}{\bar{\delta}\delta_2} \tag{47}$$

where ε, γ, $\bar{\delta}$, $\delta_2 \in \mathbb{R}^+$ are constants.

Proof: See [18] for proof.

Remark 5 *For specific sub-task objectives (singularity avoidance, joint-limit avoidance, bounding the impact forces, and bounding the potential energy) and numerical simulation results the reader is referred to [18].*

Remark 6 *The sub-task objective is met only if the sufficient conditions as described by (46) and (47) are met. The sub-task objective is secondary to the tracking objective which is always guaranteed by Theorem 1. In the event that the sub-task controller attempts to force the robot manipulator's end-effector to take a path not allowed by the tracking controller, the condition in (46) will not be met; hence, the result of Theorem 2 will not hold. With this fact in mind, the formulation of the desired task-space trajectory and the sub-task objective require careful consideration to meet both the tracking and sub-task objective simultaneously.*

7 Conclusions

This work utilized an adaptive full-state feedback quaternion based controller developed in [5] and focused on the design of a general sub-task controller. This general sub-task controller was developed as to not affect the tracking control objective, and allows for the design of specific sub-task objectives.

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