Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Observer-based optimal control of dry clutch engagement

Pietro Dolcini* Hubert Béchart
Centre Technique Renault de Lardy
1 Allée Cornuel Lardy 91510 France
{pietro.dolcini,
hubert.bechart}@renault.com

Carlos Canudas de Wit
Laboratoire d'Automatique de Grenoble
UMR CNSR 5528
B.P. 46, St. Martin d'Hèrez 38402 France
carlos.canudas-de-wit@lag.ensieg.inpg.fr

Abstract— **Optimal control of the engagement of a dry clutch is examined paying particular attention to the driver's comfort. Due to the strong constraints and large errors of the torque control for internal combustion engines, only the normal force on the clutch disks is considered as controlled input. The resulting analytically derived controller is used in an observer-based trajectory tracking system whose performances have been tested on a highly realistic nonlinear model yielding very good results.**

I. INTRODUCTION

The engagement control of automotive dry clutch is getting attention from automotive industry as a mean of enhancing the comfort of manual transmission (MT) passenger cars. The increasing torque of modern engines coupled with the stiffness reduction of the driveline make very hard for the driver to meet standard comfort requirements with a manually operated clutch, particularly in the standing start scenario where the energy dissipated in the clutch is maximal. The introduction of an automated manual transmission (AMT) or, eventually, a clutch-by-wire system is seen as a mean of easing the driver task and, thus, enhancing his satisfaction about the car.

The control of powertrain systems is an ample and well-established research field covering thermic, chemical, mechanical and electric systems. The driveline alone has been the subject of different studies concerning, for example, optimal shift strategies, hybrid vehicle management, engine torque estimation, dry clutch controlled sliding and engagement control.

Literature on dry clutch engagement control for AMT transmissions is quite ample and many different approaches have been proposed: quantitative feedback theory [1], fuzzy control [2] [3], model predictive control [4] and decoupling control [5]. Also optimal control of a dry clutch has been studied in some detail [6] and [7].

Our previous article [8] introduced a finite time, optimal control based engagement strategy employing the clutch torque as the only controlled input. This limitation is motivated by the poor performances of the torque control for internal combustion engines under dynamic load particularly for the low rotation speeds found during a standing start scenario. The present paper extends the dynamic model to include a Dual Mass Flywheel (DMFW), analyses the robustness of the control against engine torque disturbances and introduces a nonlinear adaptive observer.

II. ENGAGEMENT PROBLEM

Internal combustion engines cannot descend below a minimum critical speed called *idle speed*[1]. In order to assure a smooth transition of the vehicle speed from zero to a minimal stabilised speed several coupling devices are used; on manual transmission or automated manual transmission vehicles a dry clutch, first introduced in the automotive field by Karl Benz in 1885 for his Benz-Velo, is used.

A dry clutch behaviour can be thought as composed by two running modes; a sliding mode where the torque is generated by the friction of the clutch disk against the flywheel and the pressure plate, and a non-sliding mode where the clutch behaves as a simple connecting link[2]. During the sliding phase the clutch torque Γ_c is proportional to the normal force F_n exerted by the washer spring on the friction surfaces. The amount of normal force is controlled by acting on the washer spring's end either through an hydraulic system connected to the clutch pedal or an hydraulic actuator, in the case of manual transmissions or automated manual transmission vehicles respectively.

During a standing start the normal force is gradually increased from zero to a maximum value whose amount gives the sportiness of the manoeuver. Under the clutch torque action the driveline, and thus the vehicle, accelerate; since Γ_c is usually higher than the engine torque Γ_e the engine slows down and the clutch sliding speed is gradually reduced to zero. Once the sliding phase over, the clutch acts as a linking element; Γ_c equals Γ_e minus the engine inertial reaction torque. This sudden torque change excites the driveline generating highly uncomfortable longitudinal oscillations. Fig. 1 shows a standing start simulation with subsequent driveline oscillations.

Usually an experienced driver can avoid, or at least minimise, the occurrence of this situation but the concurrent increase in engine torque output and reduction of the transmission stiffness, due to tougher NVH standards, make the

* Corresponding author, supported by the Association Nationale de la Recherche Technique

[1] for robustness reasons the idle speed of an engine is slightly higher than the actual minimum speed.

[2] Actually inside the clutch disk there is spring-damper system designed to filter out the engine acyclicity. Since its working frequency is sensibly higher than the one involved in driveline oscillations, its presence will be ignored in our analysis.

0-7803-9568-9/05/$20.00 ©2005 IEEE

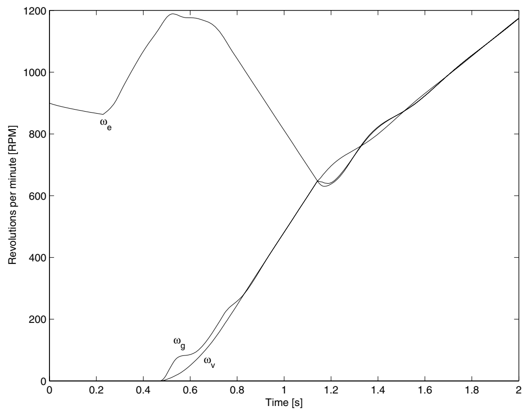

Fig. 1. Simulation of a standing start of a mid-sized car.

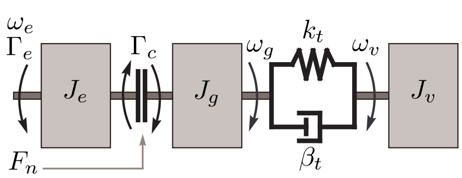

Fig. 2. Simplified powertrain scheme

driver's task harder and reduce the perceived comfort. Since the automated manual transmission is an inexpensive upgrade from a classical manual transmission the importance of developing oscillation-preventing control strategies becomes evident.

III. REDUCED MODEL FOR CONTROL

The driveline reduced model, valid during the clutch sliding phase, is a simple four state linear system shown in Fig. 1 whose dynamic equations are:

$$\begin{cases} J_e\dot{\omega}_e = \Gamma_e - \gamma F_n \\ J_g\dot{\omega}_g = \gamma F_n - k_t\theta - \beta_t(\omega_g - \omega_v) \\ J_v\dot{\omega}_v = k_t\theta + \beta_t(\omega_g - \omega_v) \\ \dot{\theta} = \omega_g - \omega_v \end{cases} \tag{1}$$

where w_e, w_g and w_v are respectively the engine, gearbox and vehicle revolution speeds, J_e is the engine inertia, J_g and J_v are respectively the gearbox inertia and the equivalent vehicle inertia both measured on the engine side of the gearbox, θ the transmission torsion, k_t and β_t the transmission stiffness and friction coefficients, Γ_e the engine torque, F_n the normal force on the clutch friction surfaces and $\gamma = 2\mu_d R_c$ where μ_d is the dynamic clutch fiction coefficient and R_c is the mean radius of the clutch disks. This model is sufficiently simple to be used for designing the controller while still capturing the main dynamic of the the driveline since the driveline poles are dominant when the first gear is selected.

IV. CONTROLLER DESIGN

A. Control objectives

The control objective is reaching the clutch's engagement, i.e. $\omega_e = \omega_g$, while assuring the comfort and minimising the dissipated energy and actuator activity. A finite-time optimal control approach with prescribed final states has been chosen since it assures the engagement to take place in a limited time window. The optimal controller is designed for the simplified system (1) with Γ_c as controlled input and Γ_e as known non-controlled input.

Oscillations induced in the powertrain by the sudden change of torque due to the end of the sliding phase are an important element of the engagement comfort. In order to suppress, or at least reduce, these oscillations it is necessary that the torque $\Gamma_c(t_s^-)$ transmitted by the clutch just before the engagement instant t_s equals the engine torque $\Gamma_e(t_s^+)$ minus the engine inertia reaction torque. Expressing this equivalence in terms of the speeds at the sides of the clutch gives the so called *no-lurch condition* [5] which requires that $\dot{\omega}_e(t_s^-) - \dot{\omega}_g(t_s^-) = 0$. This condition does not guarantee the driveline equilibrium after the engagement; numerical simulations, in fact, show that even when this condition is met the surplus elastic energy in the transmission can cause residual oscillations.

Ideally after the engagement all the powertrain elements should rotate with same speed and acceleration:

$$\omega_e = \omega_g = \omega_v$$
$$\dot{\omega}_e = \dot{\omega}_g = \dot{\omega}_v = \frac{\Gamma_e}{(J_e+J_g+J_v)}$$

Defining $z_1 = \omega_e - \omega_g$ and $z_2 = \omega_g - \omega_v$, through simple algebraic manipulation the simplified system (1) can be written as:

$$\begin{cases} \dot{z}_1 = \frac{\beta_t}{J_g}z_2 + \frac{k_t}{J_g}\theta + \frac{1}{J_e}\Gamma_e - \frac{1}{J_{t1}}\Gamma_c \\ \dot{z}_2 = -\frac{\beta_t}{J_{t2}}z_2 - \frac{k_t}{J_{t2}}\theta + \frac{1}{J_g}\Gamma_c \\ \dot{\theta} = z_2 \end{cases} \tag{2}$$

where

$$J_{t1} = \frac{J_e J_g}{J_e + J_g} \quad J_{t2} = \frac{J_g J_v}{J_g + J_v}$$

The final boundary conditions that assure a perfect equilibrium at the end of the engagement for this model are:

$$\begin{gathered} z_1(t_f) = 0 \quad z_2(t_f) = 0 \\ \theta(t_f) = \frac{1}{k_t}\frac{J_v\Gamma_e(t_f)}{J_e+J_g+J_v} \quad \Gamma_c(t_f) = \frac{(J_g+J_v)\Gamma_e(t_f)}{J_e+J_g+J_v} \end{gathered} \tag{3}$$

Due to the physical structure of the clutch the main limitation of the actuator is the slew rate of the normal force F_n. Adding the extra state equation $\dot{\Gamma}_c = u$ to the simplified system and weighting the new input u allows to limit this aspect of the actuator activity since $\Gamma_c = \gamma F_n$.

Finally, in order to increase the driver's comfort, the peak clutch torque during the engagement should be as low as possible while still archiving the specified final conditions.

B. Optimisation problem

All the control objectives defined in the previous subsection define the following optimisation problem: finding $u(t)$ on $T = [t_0, t_f]$ such that minimises the following quadratic value function:

$$J = \frac{1}{2}\int_{t_0}^{t_f} \left[z^T\mathbf{Q}z + u^T\mathbf{R}u\right] \mathrm{d}t$$

under the constraint:

$$\dot{z} = \mathbf{A}_z z + \mathbf{B}_{z1}\Gamma_e + \mathbf{B}_{z2}u \tag{4}$$

where:

$$\mathbf{A}_z = \begin{bmatrix} 0 & -\frac{\beta_t}{J_g} & \frac{k_t}{J_g} & -\frac{1}{J_{t1}} \\ 0 & \frac{\beta_t}{J_{t2}} & -\frac{k_t}{J_{t2}} & -\frac{1}{J_g} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{B}_{z1} = \begin{bmatrix} \frac{1}{J_e} \\ 0 \\ 0 \\ 0 \end{bmatrix} \mathbf{B}_{z2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c \end{bmatrix} \mathbf{R} = \begin{bmatrix} 1 \end{bmatrix}$$

with prescribed initial and final states:

$$z_1(t_0) = z_{10} \quad z_2(t_0) = z_{20}$$
$$\theta = \theta_0 \quad \Gamma_c(t_0) = 0$$

$$z_1(t_f) = 0 \quad z_2(t_f) = 0$$
$$\theta(t_f) = \frac{1}{k_t}\frac{J_v\Gamma_e}{J_e+J_g+J_v} \quad \Gamma_c(t_f) = \frac{(J_g+J_v)\Gamma_e}{J_e+J_g+J_v}$$

under the assumption that Γ_e is a measured non-controlled input. Even dropping the hard final state constraints in favour of a heavy weighting of the final states in the cost function does not allow the use of a LQ controller since (4) is not stabilisable under the same assumptions made in subsection IV-C for Γ_e.

The more general differential analysis theory [9] defines the optimal input $u(t)$ as:

$$u = -\lambda_4$$

where λ_4 is defined by the following Two Point Boundary Value Problem (TPBVP):

$$\dot{x} = \mathbf{A}_L x + \mathbf{B}_L\Gamma_e \tag{5}$$

where Γ_e is a known non-controllable input and:

$$x = \begin{bmatrix} z_1 & z_2 & \theta & \Gamma_c & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{bmatrix}^T$$

$$\mathbf{A}_L = \begin{bmatrix} \mathbf{A}_z & -\mathbf{B}_{z2}\mathbf{R}^{-1}\mathbf{B}_{z2}^T \\ -\mathbf{Q} & -\mathbf{A}_z^T \end{bmatrix} \mathbf{B}_L = \begin{bmatrix} \mathbf{B}_{y1} \\ 0 \end{bmatrix}$$

with the following boundary conditions:

$$\begin{array}{ll} z_1(t_0) = z_{10} & z_1(t_f) = z_{1f} \\ z_2(t_0) = z_{20} & z_2(t_f) = z_{2f} \\ \theta(t_0) = \theta_0 & \theta(t_f) = \theta_f \\ \Gamma_c(t_0) = \Gamma_{c0} & \Gamma_c(t_f) = \Gamma_{cf} \end{array}$$

C. Analytic solution

Solving the TPBVP implies finding $\lambda_1(t_0)\ldots\lambda_4(t_0)$ that satisfy the boundary conditions in t_f effectively transforming (5) in an initial value problem (IVP). Standard TPBVP numerical resolution methods, such as the shooting method, are not an interesting option for online implementation of the controller due to their calculation cost.

Assuming Γ_e constant[3] over the interval T, (5) can be written as an homogeneous linear system with a non-controllable constant state Γ_e:

$$\dot{x} = \mathbf{A}_L x + \mathbf{B}_L\Gamma_e \quad \Rightarrow \quad \dot{\chi} = \bar{\mathbf{A}}_L\chi$$

where

$$\chi = \begin{bmatrix} z_1 & z_2 & \theta & \Gamma_c & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \Gamma_e \end{bmatrix}^T$$

Since the system is linear:

$$\chi(t_f) = e^{\bar{\mathbf{A}}_L t_f}\chi(t_0) = \Phi_{t_f}\chi(t_0)$$

Defining:

$$\chi(t_0) = \Big[\underbrace{z_{10}\ z_{20}\ \theta_0\ \Gamma_{c0}}_{z_0}\ \ \underbrace{\lambda_{10}\ \lambda_{20}\ \lambda_{30}\ \lambda_{40}}_{\lambda_0}\ \ \Gamma_e \Big]^T$$

$$\chi(t_f) = \Big[\underbrace{z_{1f}\ z_{2f}\ \theta_f\ \Gamma_{cf}}_{z_f}\ \ \underbrace{\lambda_{1f}\ \lambda_{2f}\ \lambda_{3f}\ \lambda_{4f}}_{\lambda_f}\ \ \Gamma_e \Big]^T$$

we get:

$$\begin{bmatrix} z_f \\ \lambda_f \\ \Gamma_e \end{bmatrix} = \begin{bmatrix} \varphi_{11} & \varphi_{12} & \varphi_{13} \\ \varphi_{21} & \varphi_{22} & \varphi_{23} \\ \varphi_{31} & \varphi_{32} & \varphi_{33} \end{bmatrix} \begin{bmatrix} z_0 \\ \lambda_0 \\ \Gamma_e \end{bmatrix} \tag{6}$$

whose first line defines the linear system:

$$\varphi_{12}\lambda_0 = z_f - \varphi_{11}z_0 - \varphi_{13}\Gamma_e$$

which, since φ_{12} is invertible, defines λ_0 as a function of z_0, z_f and Γ_e. Once this initial value is known the feedback controller is defined by an opportune partition of $\bar{\mathbf{A}}_L$:

$$\dot{\lambda} = \mathbf{A}_\lambda\lambda + \mathbf{B}_\lambda \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$
$$u = \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix}\lambda$$

D. Resulting control law

The clutch torque:

$$\Gamma_c(t) = \int_{t_0}^{t} u(\tau)\mathrm{d}t \quad \forall t \in [t_0, t_f]$$

where $u(\tau)$ is defined by the dynamic feedback:

$$\dot{\lambda} = \mathbf{A}_\lambda\lambda + \mathbf{B}_\lambda \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$
$$u = \begin{bmatrix} 0 & 0 & 0 & -1 \end{bmatrix}\lambda$$

is the optimal, finite time, clutch engagement control relative to the weight function:

$$J = \int_{t_0}^{t_f} \left[a\,z_1^2(t) + b\,z_2^2(t) + c\,\Gamma_c^2 + u^2(t)\right]\mathrm{d}t$$

for the system (1) under the assumption that Γ_e is a known constant or linearly time dependant input. The initial state of the controller are defined by the a linear combination of the initial and final states:

$$\lambda_0 = \varphi_{12}^{-1}\left(z_f - \varphi_{11}z_0 - \varphi_{13}\Gamma_e\right)$$

[3]The same line of reasoning holds for Γ_e linearly time-dependant.

E. Controller robustness analysis

The analytic solution of the TPBVB has been obtained under the assumption that the engine torque is constant or linearly time dependent. As pointed out in the introduction the torque control under dynamic charge for low revolution speeds is quite inaccurate thus the robustness of the proposed controller has to be investigated.

Decomposing the perturbed engine torque Γ_e in its nominal component $\bar{\Gamma}_e$ and the perturbation $\tilde{\Gamma}_e$ we have, by linearity:

$$\begin{aligned} x(t) &= \bar{x}(t) + \tilde{x}(t) \\ \dot{\bar{x}} &= \mathbf{A}_L \bar{x} + \mathbf{B}_L \bar{\Gamma}_e \\ \dot{\tilde{x}} &= \mathbf{A}_L \tilde{x} + \mathbf{B}_L \tilde{\Gamma}_e \end{aligned}$$

Since the minimum target that has to be assured is $z_1(t_f) \simeq 0$ our analysis will be limited to this quantity:

$$\begin{aligned} z_1(t_f) &= \bar{z}_1(t_f) + \tilde{z}_1(t_f) = \tilde{z}_1(t_f) \\ &= \mathbf{C}_L\, e^{\mathbf{A}_L t_f} \tilde{x}(t_0) + \int_{t_0}^{t_f} \mathbf{C}_L\, e^{\mathbf{A}_L (t_f - \tau)} \mathbf{B}_L \tilde{\Gamma}_e(\tau) \mathrm{d}\tau \\ &= \int_{t_0}^{t_f} \mathbf{C}_L\, e^{\mathbf{A}_L (t_f - \tau)} \mathbf{B}_L \tilde{\Gamma}_e(\tau) \mathrm{d}\tau = \Phi(\tilde{\Gamma}_e) \end{aligned}$$

where:

$$\mathbf{C}_L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

and the initial error $\tilde{x}(t_0)$ has been assumed equal to zero.

Defining $\mathbb{G}$ as the set of all possible perturbation signals $\tilde{\Gamma}_e$, $\Phi(\tilde{\Gamma}_e) : \mathbb{G} \to \mathbb{R}$ can be seen as a linear operator from the $\mathbb{G}$ space to the real space. The induced $\mathcal{L}_2$ norm on Φ is:

$$\begin{aligned} \|\Phi(\tilde{\Gamma}_e)\|_2^2 &= sup_{\tilde{\Gamma}_e} \frac{\|\Phi(\tilde{\Gamma}_e)\|_2^2}{\|\tilde{\Gamma}_e\|_2^2} \\ &= \left(\int_{t_0}^{t_f} \mathbf{C}_L\, e^{\mathbf{A}_L (t_f - \tau)} \mathbf{B}_L \mathrm{d}\tau \right)^2 sup_{\tilde{\Gamma}_e} \frac{\left(\int_{t_0}^{t_f} \tilde{\Gamma}_e(\tau) \mathrm{d}\tau \right)^2}{\int_{t_0}^{t_f} \tilde{\Gamma}_e^2(\tau) \mathrm{d}\tau} \end{aligned}$$

Since a zero crossing $\tilde{\Gamma}_e$ cannot yield an extremum, we can assume without loss of generality that either $\tilde{\Gamma}_e \geq 0\ \forall t$ or $\tilde{\Gamma}_e \leq 0\ \forall t$; applying the Chebyshev integral inequality for $\tilde{\Gamma}_e \geq 0$ (a symmetrical line of reasoning holds in the opposite case) we get:

$$\left(\int_{t_0}^{t_f} \tilde{\Gamma}_e(\tau) \mathrm{d}\tau \right)^2 \leq (t_f - t_0) \int_{t_0}^{t_f} \tilde{\Gamma}_e^2(\tau) \mathrm{d}\tau \qquad (7)$$

Since (7) becomes an equality for a constant $\tilde{\Gamma}_e$, the norm of the Φ operator is:

$$\|\Phi(\tilde{\Gamma}_e)\|_2^2 = (t_f - t_0) \left(\int_{t_0}^{t_f} \mathbf{C}_L\, e^{\mathbf{A}_L (t_f - \tau)} \mathbf{B}_L \mathrm{d}\tau \right)^2.$$

Since $\mathbf{A}_L$ presents unstable poles this norm increases exponentially with time and has high values even for short $t_f - t_0$ intervals ($\|\Phi(\tilde{\Gamma}_e)\|_2 \approx 1.54$ for a mid-sized car with $a = b = c = 1$ and $t_f = 1s$) show a strong sensibility to perturbations on Γ_e.

F. Trajectory tracking

In order to assure the engagement in spite of the engine torque perturbations the overall control structure has been modified.

The optimal control, which, as seen in the previous section, is highly sensitive to the perturbations, has been connected to a linear unperturbed model in order to generate a target optimal trajectory y^* in the z state space and an optimal clutch torque Γ_c^*. A feed-forward/output-feedback trajectory tracking is then employed to drive the perturbed system. The final structure, including the MIMO adaptive observer introduced in the next section, is shown in Fig. 3.

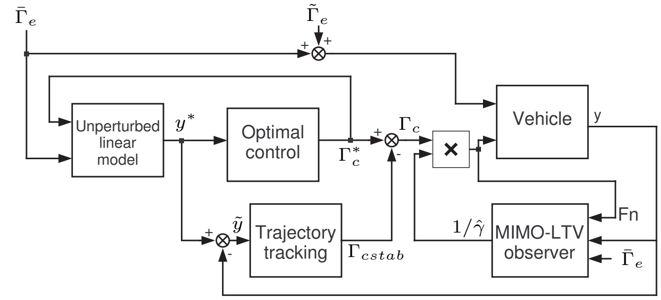

Fig. 3. Complete controller structure.

The Φ operator norm using this control scheme can be shown, by simple algebraic manipulation, being equal to:

$$\|\Phi(\tilde{\Gamma}_e)\|_2^2 = (t_f - t_0) \left(\int_{t_0}^{t_f} \mathbf{C}_{z1}\, e^{(\mathbf{A}_z + \mathbf{B}_{z2} \mathbf{K}_{\Gamma_e} \mathbf{C}_z)(t_f - \tau)} \mathbf{B}_{z1} \mathrm{d}\tau \right)^2$$

where:

$$\mathbf{C}_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad \mathbf{C}_{z1} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

With a correct choice of the feedback matrix $\mathbf{K}_{\Gamma_e}$, the Φ norm can be made arbitrary small.

V. OBSERVER DESIGN

In a production AMT vehicle only ω_e, ω_g, ω_v and the position x_c of the clutch hydraulic actuator are directly measured. The engine torque Γ_e is estimated by the engine control unit and the actual normal force F_n exerted by the pressure plate on the clutch disk can be obtained by x_c under the assumption that the washer spring characteristic is known. Since $\Gamma_c = \gamma F_n$, an estimation of the friction coefficient $\gamma = 2\mu_d R_c$ is needed in order to apply a the optimal control output.

The system (1) can be written as:

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}_1 u(t) + \gamma \mathbf{B}_2 F_n(t) \\ \dot{\gamma} = 0 \\ y = \mathbf{C}x \end{cases} \qquad (8)$$

where $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_2$, $\mathbf{C}$ are constant matrices, F_n the normal force on the clutch disks, γ the friction coefficient and ω_e, ω_g and ω_v the measured outputs y.

Literature on adaptive observers for LTV SISO systems is quite ample; [10] extends these observers to the class of MIMO LTV systems of whom (8) is a particular case.

Using the results of [10], under the following sufficient hypotheses:

- $\gamma = 2\mu_d R_c$ constant[4]

[4]Due to the heating and wear effects on the contact surfaces the friction coefficient μ_d varies but can be assumed constant for a single engagement event.

- exists a matrix $\mathbf{K}$ | the system $\dot{\eta} = (\mathbf{A} - \mathbf{KC})\eta$ is stable; i.e. the couple $(\mathbf{A}, \mathbf{C})$ is detectable.
- $F_n(t)$ is persistently exciting, i.e. $\exists\ \delta, T > 0$ such that the following inequality is satisfied:

$$\int_t^{t+T} \Upsilon^T(\tau)\mathbf{C}^T\mathbf{C}\Upsilon(\tau)\mathrm{d}\tau \geq \delta I \qquad \forall t$$

where

$$\dot{\Upsilon} = [\mathbf{A} - \mathbf{KC}]\,\Upsilon + \mathbf{B}_2 F_n$$

the system:

$$\begin{cases} \dot{\hat{x}} = \mathbf{A}\hat{x} + \mathbf{B}_1 u + \hat{\gamma}\mathbf{B}_2 F_n + \left[\mathbf{K} + \Upsilon\mathbf{B}_2\Upsilon^T\mathbf{C}^T F_n\right][y - \mathbf{C}\hat{x}] \\ \dot{\hat{\gamma}} = \mathbf{B}_2\Upsilon^T\mathbf{C}^T\,[y - \mathbf{C}\hat{x}]\,F_n \end{cases}$$

is a global exponential adaptive observer for (8).

Since the couple $(\mathbf{A} - \mathbf{KC}, \mathbf{B}_2)$ is controllable and $(\mathbf{A} - \mathbf{KC}, \mathbf{C})$ observable the persistent excitation condition is easily verified with for $F_n(t) \geq 0$; a detailed justification is given in the appendix. Having that the simplified model is valid only during the sliding phase, we can also assume that $\omega_e \neq \omega_g$ without loss of generality. Physically this implies that the driveline state and the friction coefficient can only be observed when $\Gamma_c \neq 0$, i.e. when the optimal control is active.

VI. NUMERICAL RESULTS

A. Simulation model

In order to obtain more realistic simulations a highly detailed nonlinear powertrain model has been used. This model, first introduced in our previous paper [8], has been improved including a nonlinear stiffness Dual Mass Flywheel (DMFW).

The dynamic equations describing the powertrain model, whose simplified scheme is shown in Fig. 4, are:

$$\begin{cases} J_e\dot{\omega}_e = \Gamma_e - k_{dm}(\theta_{dm})\theta_{dm} - \beta_{dm}(\omega_e - \omega_{dm}) \\ J_{dm}\dot{\omega}_{dm} = k_{dm}(\theta_{dm})\theta_{dm} + \beta_{dm}(\omega_e - \omega_{dm}) - \Gamma_c \\ J_g\dot{\omega}_g = \Gamma_c - \Gamma_d/\alpha \\ J_{tl}\dot{\omega}_{tl} = \Gamma_d/2 - k_{tl}\theta_{tl} - \beta_{tl}(\omega_{tl} - \omega_{wl}) \\ J_{tr}\dot{\omega}_{tr} = \Gamma_d/2 - k_{tr}\theta_{tr} - \beta_{tr}(\omega_{tr} - \omega_{wr}) \\ J_{wl}\dot{\omega}_{wl} = k_{tl}\theta_{tl} + \beta_{tl}(\omega_{tl} - \omega_{wl}) - R_w F_{xl} \\ J_{wr}\dot{\omega}_{wr} = k_{tr}\theta_{tr} + \beta_{tr}(\omega_{tr} - \omega_{wr}) - R_w F_{xr} \\ \dot{\theta}_{dm} = \omega_e - \omega_{dm} \\ \dot{\theta}_{tl} = \omega_{tl} - \omega_{wl} \\ \dot{\theta}_{tr} = \omega_{tr} - \omega_{wr} \\ M\dot{v} = F_{xl} + F_{xr} \end{cases}$$

where ω_e is the engine rotational speed; ω_d the DMFW secondary mass speed; ω_g the gearbox speed; ω_{tr} and ω_{tl} the right and left transmission speeds; ω_{wr} and ω_{wl} right and left wheel speeds; v the vehicle speed; θ_{dm} the DMFW internal torsion; θ_{tr} and θ_{tl} the right and left transmission torsion; Γ_e the engine torque; Γ_c the clutch torque and Γ_d the differential torque; F_{xr} and F_{xl} the right and left longitudinal wheel friction forces; $k_{dm}(\theta_{dm})$ the nonlinear DMFW stiffness coefficient; k_{tr} and k_{tl} the right and left transmission stiffness coefficients; β_{tr} and β_{tl} the right and left transmission damping coefficients; R_w the wheel radius and α the gearbox reduction.

The clutch torque Γ_c generation, defined by a normal force controlled LuGre model, is modeled through a nonlinear spring-damper dynamic system whose internal state is analogous to the bristle flection in the bristle friction model. This solution assures a simple continuous, albeit nonlinear, alternative to the linear piecewise switching model usually proposed in literature. Detailed explication of the model and its parameters is can be found in [11].

Right and left tire longitudinal friction forces are defined by a similar averaged lumped LuGre model [12] and the differential torque Γ_d is defined by imposing $\omega_g = 1/2(\omega_{tr} + \omega_{tl})$.

B. Simulation results

The complete control system has been tested on the nonlinear simulation model using actual car parameters (Renault Megane II 2.0 petrol engine) simulating a standing start scenario. In order to verify the robustness of the proposed control law worst case $\pm 20\%$ constant offsets on the engine torque have been introduced together with an initial estimation error of the friction coefficient of $\pm 100\%$; numerical results confirm very good perturbation rejection. In Fig. 5 a standing start simulation using the complete control system is shown, together with a non-stabilised optimal control and nominal run for comparing purposes. Since in this case the estimated friction coefficient is just half of the actual one (-100% estimation error) the open loop optimal control generates a clutch torque twice the needed value causing a strong excitation of the driveline and a serious drop in ω_e which would probably lead to an engine shutdown on a real car. The proposed control law, thanks to the concurrent action of the MIMO observer and trajectory tracking, on the other hand, assures a comfortable engagement despite the serious perturbations

It's worth pointing out that since the trajectory stabilisation is done in the z state space the final revolution speeds of the driveline and vehicle acceleration are different from the ones obtained on the nominal run while still satisfying the constraints imposed on the final state.

VII. CONCLUSIONS

The engagement of a dry clutch for an automated manual transmission vehicle under realistic control and resources constraints has been considered. The finite-time optimal control problem obtained from this specifications has been addressed using the dynamic lagrangian method with analytical solution of the resulting Two Point Boundary Value Problem. As a result of the robustness analysis of the optimal control against engine torque perturbations a trajectory stabilisation feedback has been added. The control law thus obtained has the structure of a dynamic feedback with initial states defined by a linear combination of initial and target driveline states. An exponentially globally convergent nonlinear adaptive observer is used to estimate the clutch friction coefficient and the driveline state. The proposed complete controller structure has been tested using a novel, highly realistic,

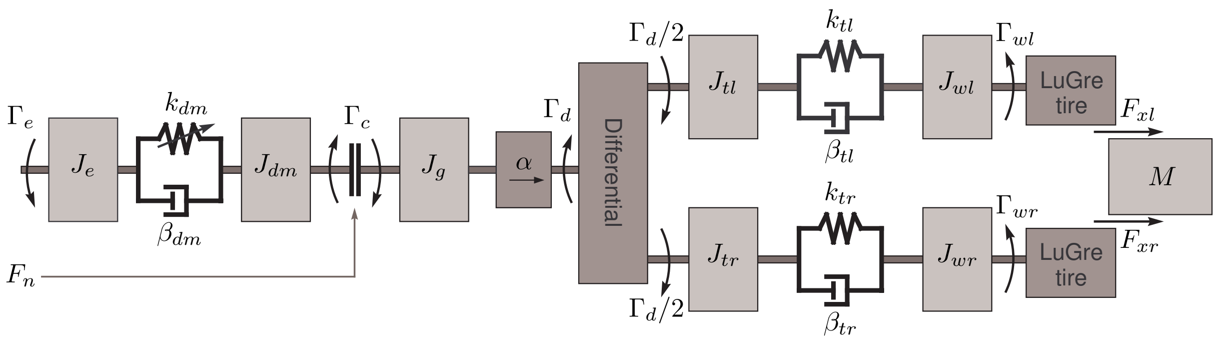

Fig. 4. Powertrain scheme

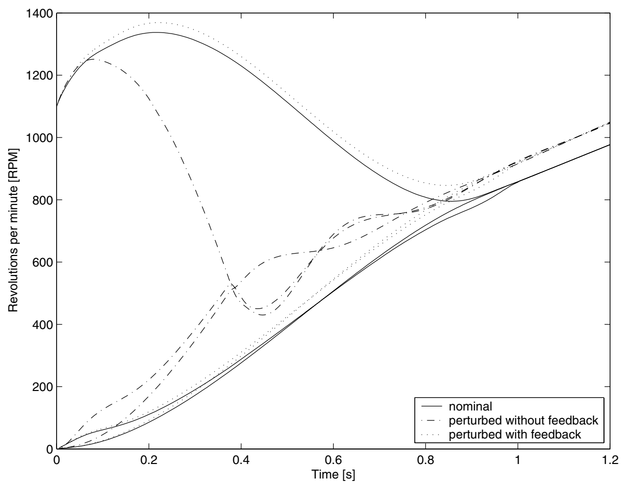

Fig. 5. Results of engine torque (constant offset 20% of nominal value) and friction coefficient (100% constant initial estimation error) perturbations and trajectory stabilisation.

nonlinear model on the standing start scenario with a worst case perturbation on the engine torque with very good results.

The resulting engagement control system will soon be tested on a specially equipped AMT production vehicle.

APPENDIX
PERSISTENT EXCITATION CONDITION

Lemma 1: Given the controllable and observable linear system:

$$\begin{cases} \dot{x} = \mathbf{A}x + \mathbf{B}u \\ y = \mathbf{C}x \end{cases}$$

$y(\tau) = 0\ \forall \tau \in (0, t)$ implies $u(\tau) = 0$ on the same interval.

Proof: $y(t) = 0$ over the interval $(0, t)$ implies $y^{(k)}(t) = 0\ \forall k \in \mathbb{N}$ over the same interval. By the definition of the observability matrix $\mathcal{O}$:

$$\mathcal{O}\, x(t) = \left[\begin{array}{cccc} y(t) & y'(t) & \cdots & y^{(n-p)} \end{array} \right] = 0$$

where $n = rank(A)$ and $p = rank(C)$. Since, by hypothesis, $rank(\mathcal{O}) = n$ we have $x(t) \equiv 0$ on the given interval. Using the same line of reasoning on the controllability matrix $\mathcal{C}$ we have $\mathcal{C}x = 0 \Rightarrow u(t) \equiv 0$ on $(0, t)$. ■

Defining $\upsilon = C\Upsilon$ the persistent excitation conditions becomes:

$$\exists \delta, T > 0 \text{ s.t.} \int_t^{t+T} \upsilon^T \upsilon \mathrm{d}\tau = \|\upsilon\|^2 \geq \delta$$

Since δ is arbitrary this condition is not satisfied only if $\upsilon(t) \equiv 0$. The linear system defined by the triplet $(\mathbf{A} - \mathbf{KC}, \mathbf{B}_2, \mathbf{C})$ is controllable and observable; applying the previous lemma the persistent excitation condition is not satisfied if $F_n(t) = 0$ over $(t, t + T)$ thus $F_n > 0$ easily satisfies the condition.

REFERENCES

[1] J. Sliker and R. Loh, "Design of robust vehicle launch control systems," *IEEE Trans. on Control System Technology*, vol. 4, no. 4, pp. 326–335, 1996.
[2] H. Tanaka and H. Wada, "Fuzzy control of engagement for automated manual transmission," *Vehicle system dynamics*, vol. 24, pp. 365–366, 1995.
[3] S. Shuiwen, G. Anlin, L. Bangjie, Z. Tianyi, and F. Juexin, "The fuzzy control of a clutch of an electronically controlled automatic mechanical transmission," *JSAE Technical Paper Series*, no. 9530805, 1995.
[4] A. Bemporad, F. Borrelli, L. Glielmo, and F. Vasca, "Hybrid control of dry clutch engagement," *Proc. of European Control Conf.*, 2001.
[5] F. Garofalo, L. Glielmo, L. Iannelli, and F. Vasca, "Smooth engagement of automotive dry clutch," *Proc. of the* 40^{th} *IEEE CDC*, pp. 529–534, 2001.
[6] L. Glielmo and F. Vasca, "Optimal control of dry clutch engagement," *SAE*, no. 2000-01-0837, 2000.
[7] F. Garofalo, L. Glielmo, L. Iannellli, and F. Vasca, "Optimal tracking for automotive dry clutch engagement," *Proc. of the* 15^{th} *IFAC congress*, 2002.
[8] P. Dolcini, C. Canudas de Wit, and H. Béchart, "Improved optimal control of dry clutch engagement," *Proc. of the* $16^t h$ *IFAC World Congress*, 2005.
[9] S. Agrawal and B. Fabien, *Optimisation of Dynamic Systems*. Kluwer Academic Publishers, Netherlands, 1994.
[10] Q. Zhang, "Adaptative observer for mimo linear time varying systems," *INRIA research report - theme 4*, no. INRIA/RR-4111-FR+ENG, 2001.
[11] H. Olsson, K. Åström, C. Canudas de Wit, M. G. fvert, and P. Lischinsky, "Friction model and friction compensation," 1997.
[12] C. Canudas de Wit, M. Petersen, and A. Shiriaev, "A new nonlinear observer for tire/road distributed contect friction," *Proc. of the Conf. on Decision and Control*, 2003.