Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

Theory for a Control Architecture of Fuzzy Discrete Event Systems for Decision Making

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Abstract—**Since we introduced a method for control of fuzzy discrete event systems (FDES) in 2001, we have been focusing our attention on medical applications. In this paper, we propose a new control architecture of FDES that includes a fuzzy objective generator for generating optimal control objectives online and an online optimal control scheme using both disablement and enforcement. The optimal control problem is nontrivial because its performance index is state dependent and hence not monotonic. Furthermore, the state space of a FDES is infinite in general. We show that our online approach can solve this problem efficiently. The architecture is general and can be used for decision making in many complex systems. We demonstrate the usefulness of the architecture by applying it to HIV/AIDS treatment planning, because it poses some of the most difficult treatment challenges in medicine. We build a FDES decision model from expert's knowledge, treatment guidelines, clinic trials, patient database statistics, and other information available in the medical literature. The system generates optimal control objectives for real patients from our database and applies our online approach to decide a regimen for each patient.**

Index Terms—**Discrete event systems, fuzzy logic, decision making, AIDS, HIV.**

I. Introduction

The supervisory control theory of discrete event systems (DES) (see, for example, [RW87, LW88, CL99]) has been applied to many engineering fields. In most of engineering applications, the states of a discrete event system are crisp. However, this is not the case in many other applications in complex systems such as biomedical systems and economic systems. For instance, it is vague when a company's financial situation is said to be "good". Furthermore, the transition from one state to another is also vague. It is hard to say at what point exactly a company's condition has changed from "good" to "bad". For such applications, we must introduce fuzzy logic to discrete event systems.

In a conference paper [LY01] and then a journal paper [LY02], we first proposed a new method for control of fuzzy discrete event systems (FDES). [LY01] also introduces observability of FDES, which extends the observability theory of crisp discrete event systems [LW88]. [LY02] further extends the results to control of FDES by considering an optimal control problem.

Since the publication of [LY01, LY02], other researchers have continued the work in FDES. For example, [Q2005] studies the controllability theorem and nonblocking controllability theorem for FDES and derives the conditions for the existence of supervisors for FDES.

Meanwhile, our focus has been on applying the FDES theory to clinical treatment planning, in particular, to HIV/AIDS treatment, because it is one of the most challenging treatment decision processes in medicine. We study real patient's data and drug regimens, using the most up-to-date treatment practices. Our results have been presented in two conference papers [LYL04, YLL04]. In [LYL04], an online approach is proposed to solve an optimal control problem of FDES, which is at the heart of our application. The optimal control problem is nontrivial because its performance index is state dependent and hence not monotonic. Furthermore, the state space of a fuzzy discrete event system is infinite in general. We show that an online approach can solve this problem efficiently. In [YLL04], we apply FDES theory to design a treatment planning system for HIV/AIDS patients who have never previously received antiretroviral therapy (ART). We show how to design such a system. We also statistically evaluate the preliminary results produced by the system in comparison with two HIV/AIDS specialists on our team. The results indicate strong agreement between the physicians and the fuzzy discrete event system.

Despite the progress made in [LYL04, YLL04], we realize that the old control architectures are not suitable for our application and other applications in complex systems. This is because, in particular, optimization objectives depend on each particular case and the corresponding input data, and are not as obvious and easy to obtain as in engineering applications. Therefore, we propose a new control architecture in this paper.

In the architecture, two major modeling parts are FDES decision model and fuzzy objective generator. Both parts are built from expert's knowledge, statistics of relevant database, and other information available in the literature. We will outline how these two parts were built using the

Manuscript received March 1, 2005. This work was supported in part by NIH under grant 1 R21 EB001529-01A1 and by NSF under grant INT-0213651.

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0-7803-9568-9/05/$20.00 ©2005 IEEE

example of HIV/AIDS treatment, but we will omit some details for the sake of brevity. The fuzzy objective generator which produces optimization objectives on a case-by-case basis for any given patient's conditions is a new feature in this architecture. For example, for HIV/AIDS treatment, this generator takes a patient's conditions (e.g., plasma HIV RNA levels and CD4 counts - important measures of the extent of the disease) as inputs and determines the objectives for optimization in terms of a performance index.

The optimization is done using online optimal control. We extend the results in [LYL04] to accommodate the performance index obtained by the fuzzy objective generator. For the applications that we have in mind, the performance index is often state dependent rather than event dependent, as it is in most optimal control problems in crisp discrete event systems. Therefore, the performance index is not monotonic, which makes the problem much more difficult. The online approach investigated in this paper can solve such an optimal control problems efficiently. Online approaches for controlling crisp discrete event systems were first proposed in [CLL92, CLL94, HL94]. Our control mechanism includes both disablement and enforcement, which is similar to a generalized framework proposed in [LLL98].

This paper is divided into two main parts. We first present the control architecture and theoretical issues related to the architecture. We then illustrate the theoretical results with the example of our application for HIV/AIDS treatment.

II. Control Architecture

Our proposed control architecture is shown in Figure 1. It consists of three blocks: (1) Fuzzy objective generator, (2) FDES decision model, and (3) FDES optimal control. We discuss these blocks in this section.

A. FDES Decision Model

The FDES decision model describes various decisions available and their expected results. We have proposed that the best way to build such a model is to use a fuzzy automaton [LY01, LY02], denoted as

$$G = (Q, \Sigma, \delta, q_o).$$

In G, $Q = [0,1]^n$ is the fuzzy state space, with n being the number of states. A state vector $q \in Q$ is a vector $q = [v_1, v_2, ..., v_n]$, where $v_i \in [0,1]$ is the possibility (membership function) that the system is in state i. $\Sigma = \{\sigma^1, \sigma^2, ..., \sigma^m\}$ is the set of events. Each event σ^k is represented by a matrix $\sigma^k = [\sigma^k_{ij}]_{n \times n}$, where σ^k_{ij} is the likelihood that if σ^k occurs, the system will move from state i to state j. δ describes the state (vector) transition: If the current state vector is q and event σ^k occurs, then the next state vector is $q' = q \circ \sigma^k$, where $\circ$ is some fuzzy operation specified by δ. $q_o \in Q$ is the initial state vector. Note that unlike a crisp DES, whose state space is usually finite, the state space of a fuzzy DES is usually infinite.

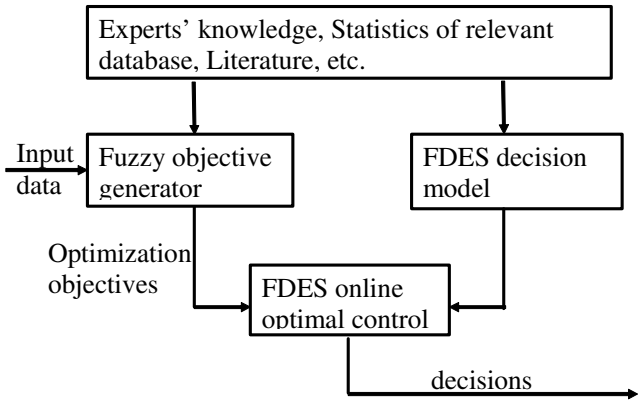

Figure 1. Control architecture.

In general, the FDES decision model may consist of N components for different aspects of the decision making, modeled by N fuzzy automata

$$G_1 \quad G_2 \quad ... \quad G_N .$$

Their state vectors are denoted by $q_1 \quad q_2 \quad ... \quad q_N$ respectively. Their event sets are denoted by $\Sigma_1 \quad \Sigma_2 \quad ... \quad \Sigma_N$ respectively, which may or may not be disjoint. $\Sigma = \Sigma_1 \cup \Sigma_2 \cup ... \cup \Sigma_N$ is the set of all events in the system.

In the HIV/AIDS treatment planning example to be discussed below, the FDES decision model consists of four fuzzy automata, one for each of the following four aspects (factors) considered by doctors in deciding which drug regimen to use: potency of the regimen, adherence to the regimen, adverse events caused by the regimen, and future drug options if the current regimen fails. Each fuzzy automaton has 3 or 4 states. For example, the fuzzy automaton for potency has the following three states: Initial (pre-treatment), High, and Medium (referring to the expected potency of the regimens). The events describe the use of a particular regimen. For potency, the events are represented by 3×3 matrices. These fuzzy automata (including event matrices) are obtained from expert's knowledge, treatment guidelines, clinic trials, patient database statistics, and other information available in the medical literature [YLL04, LYL04].

B. Optimization Objectives

The FDES decision model describes the anticipated results of various decisions. Which result is expected to be optimal for a particular case depends on its circumstances as specified by the input data and may change from case to

case. In our architecture, this is formalized as how to determine optimization objectives for a particular case.

In our control architecture, the optimization objectives are determined by (1) a fuzzy function (or mapping) from the input data to fuzzy discrete states describing the desired outcomes, and (2) a mapping from the fuzzy discrete states to weight vectors. The weight vector of a particular case describes the optimization objectives for that case. Formally, this can be modeled as follows:

Let P be a set of input data.

Let $S = S_1 \times S_2 \times ... \times S_L$ be a new set of *L* fuzzy discrete states. Here $s_i \in S_i = [0,1]^{l_i}$ describes the *i*th aspect of the desired outcomes and l_i is the number of states in S_i.

Let $W = W_1 \times W_2 \times ... \times W_N$ be a set of weight vectors. Here $w_j \in W_j = R^{n_j}$ is the weight vector corresponding to the jth automaton G_j in the FDES decision model and n_j is the number of states in G_j.

The optimization objectives can be formally determined by two mappings *f* and *e*:

$$P \xrightarrow{f} S \xrightarrow{e} W .$$

To specify *f*, we often consider $f = f_1 \times f_2 \times ... \times f_L$, where $f_i : P \to S_i$ can be obtained by a set of fuzzy rules combined with various fuzzy operations. For example, doctors may use the following rule to determine the desired treatment outcome for potency: "If patient's CD4 is less that 100, use a regimen with high expected potency".

In our example of HIV/AIDS treatment, many rules have been obtained by extensive interviews with HIV/AIDS specialists in our team. This process is very time-consuming and often requires several iterations. To speed this process, the physician's implicit decision rule structures may also be solicited and discovered by statistical analyses of simulation experiments on their therapeutic decisions and we have recently begun work developing this approach. Fuzzy operations can also be used to combine some of these rules and different weights can be used for different rules. The final output of *f* is the desired treatment choices represented by fuzzy discrete states. For example, for a particular patient, the desired treatment choice might be an anticipated membership of 0.65 for "medium potency" with a corresponding membership of 0.35 for "high potency".

A weight vector for optimization is then determined from the fuzzy discrete states obtained from *f*. This relationship is described by the mapping *e*. In our example of HIV/AIDS treatment, the mapping *e* is determined as follows. We first determine the weight vectors for the "extreme" states, for example, the state of medium potency (medium potency with membership 1 and high potency with membership 0). This is done by asking doctors' choices of treatment under various states and trying to match their choices as closely as possible. Since doctors often disagree with each other on the best treatment, this process can be complicated, but we have developed some effective procedures for doing this [YLL04]. After determining the weight vectors for the extreme states, we use linear interpolation to determine the weight vectors for other states. Statistical approaches may also allow for simultaneous determination of appropriate weight vectors for all states.

We use the following notation for the weight vectors: If the fuzzy state of the FDES decision model is $q_1 \quad q_2 \quad ... \quad q_N$, then the corresponding weight vector is denoted by $w_1 \quad w_2 \quad ... \quad w_N$. The performance index for state $q = [q_1, q_2, ..., q_N]$ is then given by

$$J(q) = w_1^T q_1 + w_2^T q_2 + ... + w_N^T q_N .$$

C. Online Optimization

Online optimization is achieved by applying optimal control of fuzzy discrete event systems. To this end, we assume that some of the events can be disabled and some of the events can be enforced. The events that can be disabled are called controllable events. The events that can be enforced are called enforceable events. The set of controllable events is denoted by Σ_c ($\Sigma_c \subseteq \Sigma$). The set of enforceable events is denoted by Σ_f ($\Sigma_f \subseteq \Sigma$). This control mechanism is similar to a generalized framework for crisp DES proposed in [LLL98].

Control is achieved by disabling some controllable events and/or enforcing some enforceable events. Note that the control mechanisms are crisp in that if an event is disabled, then its occurrence can be prevented with certainty. Similarly, if an event is enforced, then it will definitely occur.

Our goal is to maximize the performance index J. We do this using an online approach. Online (optimal) control for crisp DES has been studied in the literature. For crisp DES, to design an optimal control online, we do the following [CLL92]: After each occurrence of events, the controller will evaluate the possible future execution of the system and determine which events to disable and which events to enforce. Therefore, we will construct a forward-looking tree as shown in Figure 2.

For a fuzzy DES, the construction of this tree is different than for the tree construction for a crisp DES. Each node in the tree no longer represents a crisp state, but rather a fuzzy state with vector $q = [q_1, q_2, ..., q_N]$, where q_i is the state vector of component G_i. If an event σ occurs in the system, the next state vector is $q' = [q'_1, q'_2, ..., q'_N]$, where q'_i is calculated as follows. If σ is an event in G_i, then $q'_i = q_i \circ \sigma$. If σ is not an event in G_i, then $q'_i = q_i$. In this way, each node is represented by the corresponding state vector q. The root of the tree represents the current state vector. How big the tree is depends on the number of forward-looking steps, denoted

by L. Looking ahead further will produce better results, but also requires more computation. Also, a variable look-ahead policy may be used by specifying some intelligent terminating conditions [CLL94]. The main difference between a crisp DES tree and the fuzzy DES tree is that there are more branches in the fuzzy DES tree, because the system can now be partially in more than one state and, therefore, more events are possible at each node in the fuzzy DES tree. We denote the forward-looking tree by $T(q_o)$. For a node q in $T(q_o)$, the next node after executing an event σ is denoted by $child(q,\sigma)$.

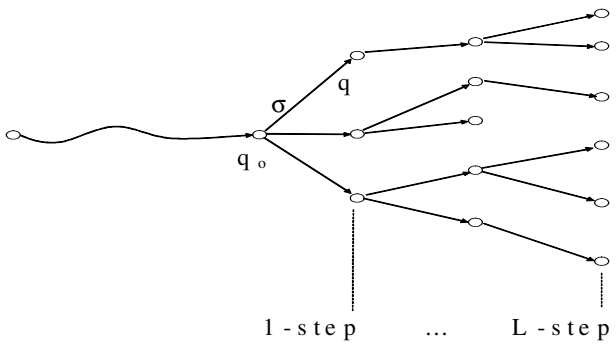

Figure 2. Forward-looking tree for online control synthesis.

In our HIV/AIDS treatment planning example, a one-step tree corresponds to first round treatments and two-step tree corresponds to both first and second round treatments. Each branch of the tree represents the use of a particular regimen.

After constructing the forward-looking tree, we use the output of fuzzy objective generator to compute the performance index $J(q)$ for each node. $J(q)$ is a function of $q=[q_1,q_2,...,q_N]$:

$$J(q)=w_1^T q_1+w_2^T q_2+...+w_N^T q_N .$$

Our goal is to design a control (using disablement and enforcement) so that the above performance index can be maximized at the *L*th step of the forward looking tree, that is,

$$\max_{\gamma} J(q_L),$$

where γ is the control (disablement and enforcement) and q_L is the nodes at the *L*th step of the forward looking tree.

The control is specified by events to be disabled or enforced at each node in $T(q_o)$, which is given by two control maps:

$$C_d^*: T(q_o)\to 2^{\Sigma_c}$$
$$C_f^*: T(q_o)\to \Sigma_f\cup\{\varepsilon\}$$

where $C_d^*(q)$ is the set of events to be disabled at node q and $C_f^*(q)$ is the event to be enforced at node q. If $C_f^*(q)=\varepsilon$, then no event is enforced. We assume that enforcement has priority over disablement. That is, if $C_f^*(q)=\sigma$, then the only event will occur in q is σ, even if it is disabled ($\sigma\in C_d^*(q)$).

To compute $C_d^*(q)$ and $C_f^*(q)$, we first calculate the performance achievable at each node q in $T(q_o)$ under control, called controlled performance and denoted by $J_c(q)$, by recursively backtracking from *L*-step nodes at the boundary of $T(q_o)$ as follows.

For a *L*-step node q, the controlled performance is equal to its performance, because no control is given at q

$$J_c(q)=J(q).$$

If the controlled performance for all (*i+1*)-step nodes is calculated, then the controlled performance of an *i*-step node can be calculated by considering the following two cases.

(1) No event is enforced. In this case, the system will either stay in q or one of the enabled events σ will occur, taking the system to $child(q,\sigma)$. Since we do not know which will actually occur, the controlled performance is the minimum of all possibilities:

$$\min_{\sigma\notin C_d^*(q)}\{J(q),J_c(child(q,\sigma))\}.$$

Obviously, to maximize the above minimum, the controller needs to disable as many events as possible (Note that enforcement will overwrite disablement). Therefore $C_d^*(q)=\Sigma_c$ for all $q\in T(q_o)$.

(2) Event $\sigma'\in\Sigma_f$ is enforced. In this case, the controlled performance is

$$J_c(child(q,\sigma')).$$

Since both (1) and (2) is a valid control, the optimal control will be the one maximizing

$$J_c(q)=\max_{\sigma'\in\Sigma_f\cup\{\varepsilon\}}\{J_c(child(q,\sigma')),\ \min_{\sigma\notin\Sigma_c}\{J(q),J_c(child(q,\sigma))\}\}$$

III. Example of HIV/AIDS Treatment

In this section, we illustrate our new architecture by applying it to HIV/AIDS treatment. We selected HIV/AIDS treatment as our application area because it arguably involves the some of most complex therapeutic decision processes in medical science. This complexity is partly due to the fact that there is no cure for HIV/AIDS. Therefore, the current treatments can only suppress the HIV virus (by reducing the plasma HIV RNA levels) and boost the immune system (by increasing the CD4 counts). This is done by using a combination of two or more classes of drugs to create a drug regimen. Since the number of drugs available is rather limited and it is easy for the virus to develop resistance if a patient does not adhere to the medication, it is not a trivial task to decide which regimen

to use for a particular patient. If a wrong decision is made, the patient may run out of options on available drugs much sooner than otherwise.

In deciding which regimen to use, doctors consider mainly the following four factors:

Potency: This may be counterintuitive, but unlike treatments of most other diseases, it is not always desirable to use the most potent regimen for HIV/AIDS treatment. In fact, at the initial stage after HIV infection, it is probably best not to use any regimen at all. As one can see, this factor alone makes HIV/AIDS treatment more complex than treatments of most other diseases.

Adherence: Again, unlike treatments of most other diseases, adherence is critical to the success of any HIV/AIDS regimen. Missing more than 5% of doses reduces the probability that a patient will enjoy the maximum benefits of antiretroviral therapy and increases the risk of the virus eventually developing resistance to the drugs or the classes of drugs in the regimen. Once ART resistance has developed within a patient, the use of these drugs will be lost forever for that patient. Since there are only very limited number of drugs, such loss of drugs will significantly shorten the life of a patient. Studies of HIV/AIDS and other chronic diseases demonstrate that patients typically take 50-60% of prescribed doses of long-term medications; so achieving ideal levels of adherence with antiretroviral medications is very challenging.

Adverse events (AE): Adverse events include side effects and toxicity. Side effects are common in HIV/AIDS treatment, especially during the early stages of therapy. Typical symptoms include abdominal discomfort, loss of appetite, diarrhea, nausea and vomiting. Toxicity may cause liver problems, osteopenia/osteoporosis, leukopenia, pancreatitis, hyperglycemia, etc. and in some instances can be fatal.

Future drug options (FDO): Perfect adherence with complex regimens is difficult for patients to maintain over long periods of time. Development of drug resistance frequently occurs. Therefore, before deciding on a regimen, it is important to consider the future drug options after the resistance occurs. In other words, difficult regimens must be sequenced appropriately to preserve maximal availability of drugs.

The above four factors are interrelated. For example, adverse events may cause non-adherence, which in turn will reduce the potency of a regimen.

Based on patient's conditions, as observed by the doctor and reflected in the test data, the doctor must decide which regimen to use. Typically, the decision process of a doctor can be conceptualized into two steps. In the first step, the doctor decides patient's desired treatment outcomes. For example, is it best to treat the patient with a regimen that has a medium potency, easy adherence, low adverse events, and high future drug options or a regimen that has a high potency, moderate adherence, medium adverse events, and high future drug options? After this is decided, the doctor then selects a regimen that can best achieve the desired treatment outcomes in the second step.

Our FDES methodology and architecture follows the same line of reasoning as used by the doctors. We first decide patient's desired treatment outcomes using fuzzy rules and associated methods. We then use an (online) optimization to determine which regimen to use. Both processes depend on the FDES decision model that we must first establish.

Our FDES decision model consists of four components for the four factors considered in the treatment, modeled by four fuzzy automata

$$G_1 \quad G_2 \quad G_3 \quad G_4 .$$

G_1 models potency. It has three states: Initial, High, and Medium. Before treatment, G_1 is at the Initial state. After treatment using a particular regimen, G_1 moves to the High (potency) state or Medium (potency) state. Using a particular regimen is modeled as an event. For the first round of treatment, we consider four commonly used regimens. Therefore, we define the following four events (CBV etc. are acronyms for drug names).

α_1 : Using regimen CBV+NFV;

α_2 : Using regimen CBV+EFV;

α_3 : Using regimen CBV+NVP;

α_4 : Using regimen CBV+ABC.

For the second round of treatment, we consider six regimens, which are grouped into three classes. Therefore, we define the following three events.

β_1: Using NNRTI regimens EFV+ABC+d4T or EFV +TDF+ddI;

β_2: Using PI regimens ATV+ABC+d4T or Fos-Amp +TDF+ddI;

β_3: Using boosted PI regimen ATV/r+ABC+ddI or LPV/r+ABC+d4T;

The matrices describing these events in G_1 are obtained from the characteristics of these regimens. After obtaining $G_i, \quad i = 1,2,3,4$, we can construct the forward-looking tree for one or two rounds of treatment as shown in Figure 3. In the figure, the dotted lines stand for the second round regimens that are most likely to be unavailable because of the drug resistance. Therefore, we will not consider these regimens.

After we obtain the FDES decision model and construct the forward-looking tree, the next step is to calculate the performance index $J(q)$ for each node in the tree:

$$J(q) = w_1^T q_1 + w_2^T q_2 + w_3^T q_3 + w_4^T q_4 .$$

To this end, we need to determine the weight vectors $w_i, \quad i = 1,2,3,4$, which varies from patient to patient.

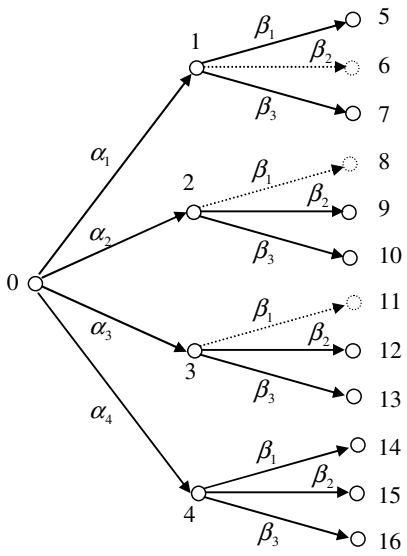

Figure 3. Forward-looking tree for two rounds of HIV/AIDS treatments.

The weight vectors for a particular patient are obtained from two mappings $P \xrightarrow{f} S \xrightarrow{e} W$. Let us discuss the second mapping from fuzzy discrete states describing patient's desired treatment outcomes *S* to the weight vectors *W* first. The mapping is obtained by asking experts about their ranking of regimens for different "extreme cases" of patients' desired treatment outcomes. For desired potency, there are two extreme cases: High and Medium. The experts are asked to rank the four regimens (for the first round) for these combinations.

The first mapping in $P \xrightarrow{f} S \xrightarrow{e} W$ is the mapping from patient's conditions and clinical test data *P* to fuzzy discrete states describing patient's desired treatment outcomes *S*. This mapping is implemented as a set of weighted fuzzy rules. These rules are based on experts' opinion on what are the patient's desired treatment outcomes given patient's conditions and clinical test data.

After obtaining performance index $J(q)$, we perform optimization to decide which regimen to be used for the patient. We compare our computer results with the actual prescriptions given by doctors. We find that our computer results match with the actual prescriptions reasonable well.

IV. CONCLUSIONS

We proposed a new control architecture and approach to fuzzy discrete event systems. This architecture is particularly suitable for decision making in complex systems such as economical systems and biomedical systems. We demonstrated the approach by applying it to HIV/AIDS treatment and testing our approach on real patient data. Our results show that the approach is promising.

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