

RoboTennis System

Part II: Dynamics and Control

L. Ángel, J. M. Sebastián, R. Saltaren and R. Aracil

Abstract— In this paper the equations of motion and the control of the RoboTennis system are presented. The dynamic model is based upon Lagrangian multipliers. The main innovation is the use of forearms of non-negligible inertias (1.315 Kg) in the dynamic model of the manipulator for the development of control strategies. A PD control law (nonlinear feedforward PD control) is applied. Several trajectories have been programmed and tested on the prototype. The experimental results demonstrated that the speed and acceleration of the robot can be satisfying the proposed task.

I. INTRODUCTION

THE dynamics of a manipulator plays an important role in achieving such high-speed performance. The development of a dynamical model is important in several ways. First, a dynamical model can be used for computer simulation of a robotic system. By examining the behavior of the model under various operating conditions, it is possible to predict how a robotic system will behave when it is built. Second, it can be used for the development of suitable control strategies. A sophisticated controller requires the use of a realistic dynamical model to achieve optimal performance under high-speed operations. Third, the dynamics analysis of a manipulator reveals all the joint reaction forces (and moments) needed for the design and sizing of links, bearings, and actuators.

There are two types of dynamical problems: direct dynamics and inverse dynamics. The direct dynamics problem is to find the response of a robot arm corresponding to some applied torques and/or forces. That is, given a vector of joint torques or forces, we wish to compute the resulting motion of the manipulator as a function of time. The inverse dynamics problem is to find the actuator torques and/or forces required to generate a desired trajectory of the manipulator. In general, the efficiency of computation for direct dynamics is not critical since it is used primarily for computer simulations of a manipulator. On the other hand, an efficient inverse dynamical model becomes extremely important for real-time feed forward control of a manipulator.

This paper presents the progresses on dynamics and motion control of the *RoboTennis* system, which proposes the design and construction of a high speed parallel robot that

will be used to perform complex tasks, i.e. playing table tennis with the help of a vision system. This integration is innovative regarding the kind of manipulator used (parallel robot) and the working speed of the system (4 m/s in the end-effector). The RoboTennis is inspired by the DELTA robot [1] and it is constructed with two purposes in mind. The first one is the development of a tool for use in visual servoing research. The second one is to evaluate the level of integration between a high-speed parallel manipulator and a vision system in applications with dynamic environments. The kinematics analysis (inverse kinematics and forward kinematics) and the optimal design of the parallel robot have been presented by Angel, *et al.* [2]. The structure of the robot is optimized from the view of both kinematics and dynamics respectively. The design method solves two difficulties: determining the dimensions of the parallel robot and selecting the actuators that will grant the implementation of the desired application (playing table tennis). The prototype built at the DISAM (Universidad Politecnica de Madrid) is presented in Fig. 1.

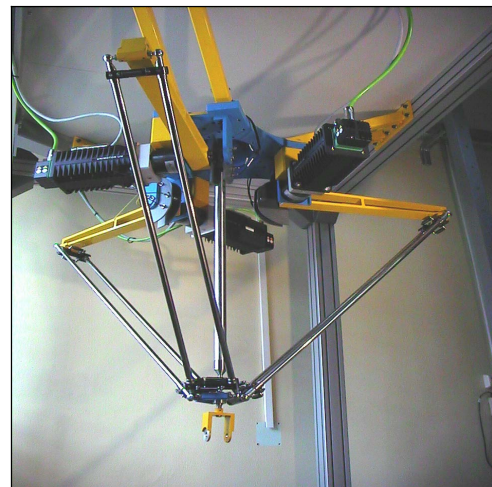


Fig. 1. Prototype of the RoboTennis System.

Generally, a parallel manipulator consists of two rings joined by more than one closed kinematics chain. This structure is more advantageous than that of a serial robot in terms of stiffness, precision, load capacity, speed and inertia in motion. These advantages are usually reached at the expense of reducing the workspace of the manipulator. Furthermore, due to the presence of multiple closed-loops chains in the structure of the manipulator, the kinematic and dynamic analysis is not as evident as in the case of serial robots.

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L. Angel is with the Universidad Pontificia Bolivariana, Facultad de Ing. Electronica, Bucaramanga, Colombia (e-mail: langel@etsii.upm.es).

J.M. Sebastian, R. Saltaren and R. Aracil are with the Universidad Politecnica de Madrid, DISAM, Madrid, España. (e-mail: {jsebas, rsaltaren, aracil}@etsii.upm.es).

Several approaches for the dynamical analysis of parallel manipulators have been proposed, including the Newton-Euler formulation [3], [4], [5], the Lagrangian formulation [6], [7], [8], and the principle of virtual work [9], [10], [11], [12]. The traditional Newton-Euler formulation requires the equations of motion to be written once for each body of a manipulator, which inevitably leads to a large number of equations and results in poor computational efficiency. The Lagrangian formulation eliminates all of the unwanted reaction forces and moments at the outset. It is more efficient than the Newton-Euler formulation. However, because of the numerous constraints imposed by closed loops of a parallel manipulator, deriving explicit equations of motion in terms of a set of independent generalized coordinates becomes a prohibitive task. To simplify the problem, additional coordinates along with a set of Lagrangian multipliers are often introduced. The principle of virtual work appears to be the most efficient method of analysis. A comparison study of the inverse dynamics of manipulators with closed-loop geometry can be found in [13].

Fast and precise robot manipulation requires control algorithms that make the best use of the information extracted from the dynamic analysis of the robot. Several approaches have been used to characterize the dynamics of the DELTA robot. The most common approaches are based upon application of the virtual work principle [14], [15], Lagrange formulations [6], the application of Hamilton's Principle [16], and the direct application of the Newton's equations of motion [4]. In these works, the DELTA robot prototype used for experimental results has forearms with negligible inertia. This assumption allowed using a simplified dynamic model of the robot.

In this paper the equations of motion and the control of the RoboTennis system are presented. The dynamic model is based upon Lagrangian multipliers. This approach is selected to give a more complete characterization of the manipulator dynamics, with the most significant assumptions all treating the manipulator as a system of rigid bodies connected by frictionless kinematic pairs. The main innovation is the use of forearms of non-negligible inertias (1.315 Kg) in the dynamic model for the development of control strategies. To analyze the performance characteristics such as trajectory and control, a PD control law (nonlinear feedforward PD control) is applied.

II. DYNAMIC MODELING

In this section we analyze the dynamics of the roboTennis system. The coordinate frames, link lengths, and the joint angles of the manipulator are shown in Fig. 2. α_1 , α_2 , and α_3 are the actuated joints.

The Lagrangian equations are written in terms of a set of redundant coordinates. Therefore, the formulation requires a set of constrain equations derived from the kinematics of a mechanism. These constraint equations and their derivates are adjoined to the equations of motion to produce a number of equations that is equal to the number of unknowns.

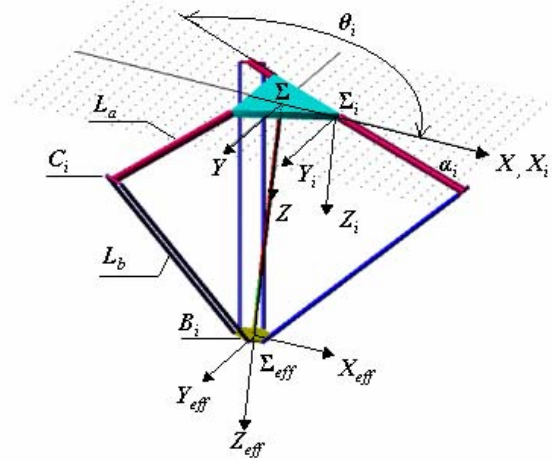


Fig. 2 Geometric model of the RoboTennis.

The Lagrangian equations can be written as

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j + \sum_{i=1}^k \lambda_i \frac{\partial f_i}{\partial q_j} \quad \text{for } j = 1 \text{ to } n, \quad (1)$$

where f_i denotes the i th constraint function, k is the number of constraint functions, n is the redundant coordinates, and λ_i is the Lagrangian multiplier.

Solving the equations of the motion is made easier by arranging the Lagrangian equations into two sets. One contains the Lagrangian multipliers as the only unknowns, and the other contains the generalized forces contributed by the actuators as the additional unknowns. Let the first k equations be associated with the redundant coordinates and the remaining $n-k$ equations are associated with the actuated joint variables.

For the roboTennis, the Lagrange's equations will be employed by introducing three redundant coordinates, x , y , and z . Thus we have

$$q_j = \{x, y, z, \alpha_1, \alpha_2, \alpha_3\} \quad (2)$$

as the generalized coordinates vector.

Equation (1) represents a system of six equations with six variables, where the six variables are λ_i for $i = 1, 2$ and 3 , and the three actuator torques, Q_j for $j = 4, 5$ and 6 . The external generalized forces, Q_j for $j = 1, 2$ and 3 , are zero since there is no externally applied force at the mobile ring and the joints at C_i are passive.

This formulation requires three constraint equations, f_i for $i = 1, 2, 3$, that are written in terms of the generalized coordinates. The constraint equations have been presented in [2]. These are obtained from the fact that the distance between joints B and C is always equal to the length of the forearm, L_b ; that is,

$$f_i = \|\mathbf{B}_i \mathbf{C}_i\|^2 - L_b^2 = 0 \quad (3)$$

$$= (x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 - L_b^2 = 0$$

with

$$a_{i1} = (x - x_i), \quad a_{i2} = (y - y_i), \quad a_{i3} = (z - z_i), \quad \text{and}$$

$$b_{ii} = L_a [(\Delta r - xc\theta_i - ys\theta_i)s\alpha_i + zc\alpha_i].$$

for $i=1, 2, \text{ and } 3$.

For a given position (x, y, z) on the desired trajectory, from the equations of motion given by (1) one can with ease obtain the required multipliers $\lambda_i, i = 1, 2, 3$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = \sum_{i=1}^3 \lambda_i A_{i1}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} = \sum_{i=1}^3 \lambda_i A_{i2} \quad (4)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{z}} \right) - \frac{\partial L}{\partial z} = \sum_{i=1}^3 \lambda_i A_{i3}.$$

Once the Lagrange multipliers are found, the actuator torques can be determined directly from the three remaining equations. Specifically, the second set of the equations can be written as

$$\tau_1 = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\alpha}_1} \right) - \frac{\partial L}{\partial \alpha_1} - \lambda_1 A_{14}$$

$$\tau_2 = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\alpha}_2} \right) - \frac{\partial L}{\partial \alpha_2} - \lambda_2 A_{25} \quad (5)$$

$$\tau_3 = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\alpha}_3} \right) - \frac{\partial L}{\partial \alpha_3} - \lambda_3 A_{36}$$

where τ_i is the actuator torque for the i^{th} arm.

To apply (4) and (5), an expression must be developed for the Lagrange function, L . The Lagrange function is defined as the difference between the kinetic energy of the system, T , and its potential energy, V .

The total kinetic energy of the robot is

$$T = T_c + \sum_{i=1}^3 (T_{ai} + T_{bi}), \quad (6)$$

where, T_c is the kinetic energy of the mobile platform, T_{ai} is the kinetic energy of the arm and the rotor i , and T_{bi} is the kinetic energy of the forearm i . Specifically,

$$T_{ai} = \frac{1}{2} m_a I_i \dot{\alpha}_i^2,$$

$$T_{bi} = \frac{1}{2} m_b \left[\frac{1}{3} (\dot{x}c\theta_i + \dot{y}s\theta_i + L_a \dot{\alpha}_i s\alpha_i)^2 + \right.$$

$$\left. \frac{1}{3} (-\dot{x}s\theta_i + \dot{y}c\theta_i)^2 + \right.$$

$$\left. \frac{1}{3} (\dot{z} - L_a \dot{\alpha}_i c\alpha_i)^2 - \right.$$

$$L_a (\dot{x}c\theta_i + \dot{y}s\theta_i) \dot{\alpha}_i s\alpha_i +$$

$$L_a \dot{z} \dot{\alpha}_i c\alpha_i],$$

$$T_c = \frac{1}{2} m_c (\dot{x}^2 + \dot{y}^2 + \dot{z}^2),$$

where m_a, m_b and m_c are the mass of the arm, the forearm and the mobile platform, respectively. I_i is the momentum of inertia of arm+motor, and θ_i is the rotation angle of the system Σ_i , with respect to the absolute reference frame, Σ .

Assuming that the acceleration of gravity (g) points in the z direction, the total potential energy of the robot relative to the plane x - y on the fixed platform is

$$V = V_c + \sum_{i=1}^3 (V_{ai} + V_{bi}), \quad (8)$$

where, V_c is the potential energy of the mobile platform, V_{ai} is the potential energy of the arm i , and V_{bi} is the potential energy of the forearm i . Specifically,

$$V_{ai} = -\frac{1}{2} m_a g L_a s\alpha_i,$$

$$V_{bi} = -\frac{1}{2} m_b g (z + L_a s\alpha_i), \quad (9)$$

$$V_c = -m_c g z.$$

A. Verification of the dynamic model

The verification of the dynamic model uses a simulation model for the manipulator developed in ADAMS® (Fig. 3).

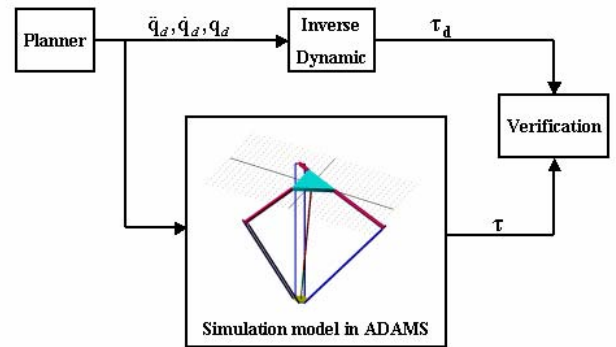


Fig. 3 Verification of the dynamic model using ADAMS.

The torques for the desired trajectory have been computed and compared to the results obtained for the simulation

model ADAMS. The optimal parameters of the robot used for the simulation and the desired trajectory are given in [2].

The parameters of the test motion are: start point: (0,0,750) mm, end point: (-200,200,750) mm, maximum velocity: 2 m/s, and maximum acceleration: 4g. The results are presented in Figs. 4 and 5. The largest difference in torques values between the dynamic model proposed in this paper and the ADAMS model is about 12%. Assuming that the dynamic model has forearms with negligible inertia, the difference in torques values is about 48%. In this case, the errors caused by the simplification are very significant.

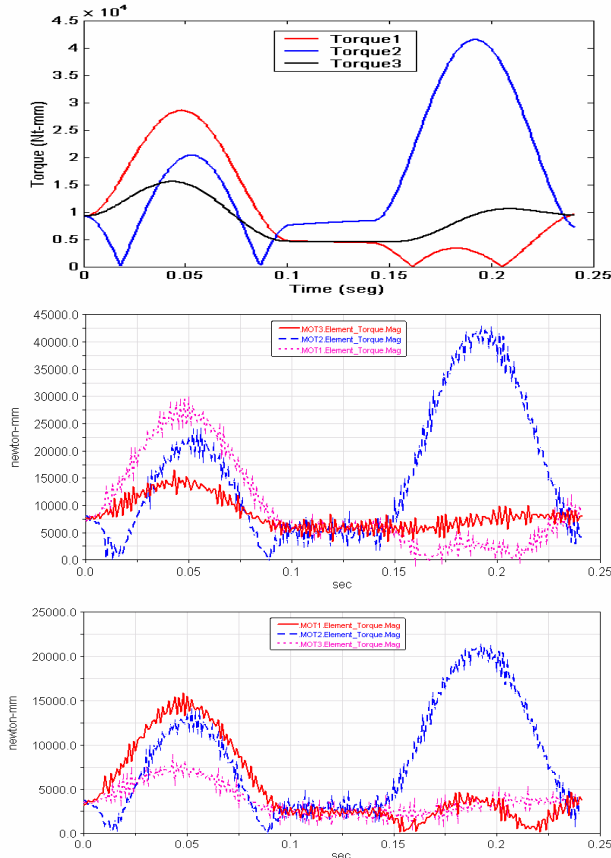


Fig. 4 Torques obtained using: (a) inverse dynamic, (b) ADAMS model, (c) ADAMS model assuming that the dynamic model has forearms with negligible inertia.

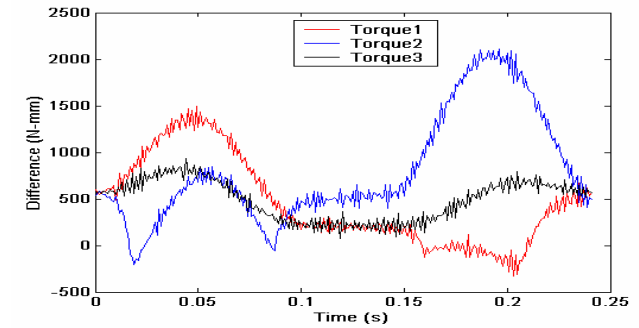
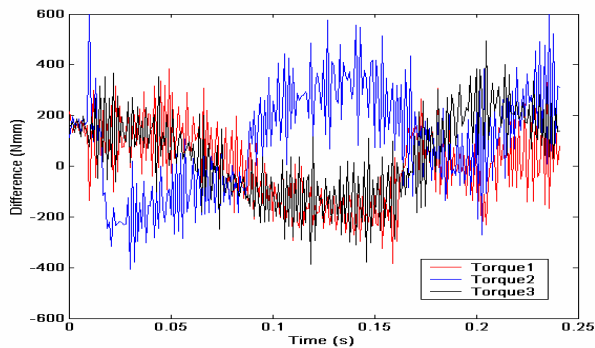


Fig. 5 Difference between the dynamic model proposed to RoboTennis and: (a) ADAMS model, (b) ADAMS model assuming that the dynamic model has forearms with negligible inertia.

III. CONTROL AND EXPERIMENTAL RESULTS

The motor drivers of the robot provide the hardware velocity servo and the torques cannot be sent directly to the motors drives. The torque command has be converted into the velocity command as

$$\boldsymbol{\tau} = K_V (\dot{\boldsymbol{\alpha}}_{ref} - \dot{\boldsymbol{\alpha}}) \quad (10)$$

where $\boldsymbol{\tau}$ denotes the motor torque vector, $\dot{\boldsymbol{\alpha}}_{ref}$ is the velocity command vector and $\dot{\boldsymbol{\alpha}}$ is the current motor angular velocity vector.

To analyze the performance characteristics such as trajectory and control, the following PD control law (nonlinear feedforward PD control) is applied:

$$\dot{\boldsymbol{\alpha}}_{ref} = K_p (\boldsymbol{\alpha}_d - \boldsymbol{\alpha}) + K_d (\dot{\boldsymbol{\alpha}}_d - \dot{\boldsymbol{\alpha}}) + K_V^{-1} \boldsymbol{\tau}_d \quad (11)$$

where $\boldsymbol{\tau}_d$ denotes the torque vector calculated from the motion equation (1) using the target trajectory, and K_p and K_v are constant gain matrices satisfying $K_v, K_p \geq 0$.

Several trajectories have been programmed and tested on the prototype. The trajectories are generated in the Cartesian space and use 616 polynomials that grant continuous accelerations and finite jerks. As an example we show the results obtained in the performance of a desired motion. The motion consists of 20 chained trajectories to approximate the behaviour of the end-effector to the behaviour it would have while performing the proposed specialized task (playing table tennis).

The complete control system is implemented on the DSPACE 1103 card. The main tasks of the real-time process are: to generate the trajectory, to evaluate the dynamic model and to control the system. The control task is defined as a periodical task at a 1 KHz. The desired motion lasts approximately 3 s.

Figure 6 shows the evolution in the position of the end effector for the proposed motion. In this experimental result we see the high speed and precision of the prototype (position error < 10 mm (Fig. 7)). The operational position is calculated using the current joint positions given by the motor encoders and the application of the forward kinematic model.

Due to the large distance ($>350\text{mm}$) covered by each of the trajectories of the motion, the results show the non-linear dynamics of the system. The end-effector has a maximum speed of 4 m/s (Fig. 8) and a maximum acceleration of $10g$. These results prove that the system is able of performing the desired task.

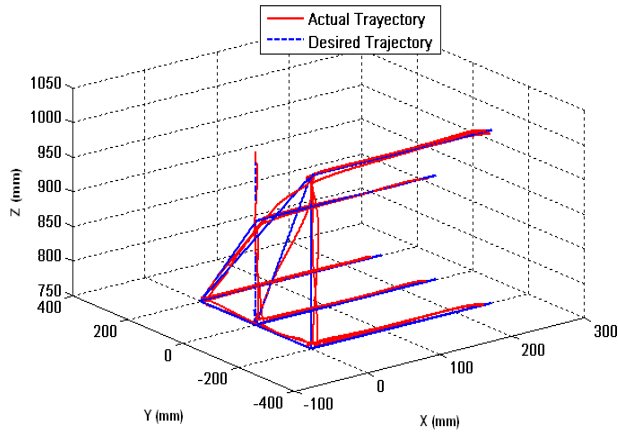


Fig. 6. Position profile of the end-effector in the performance of the desired motion.

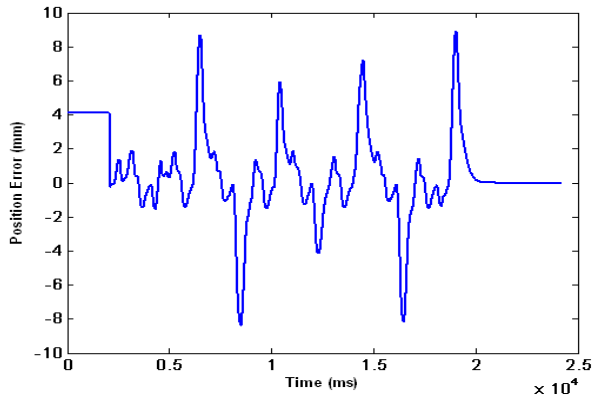


Fig. 7. Position Error of the end-effector in the performance of the desired motion.

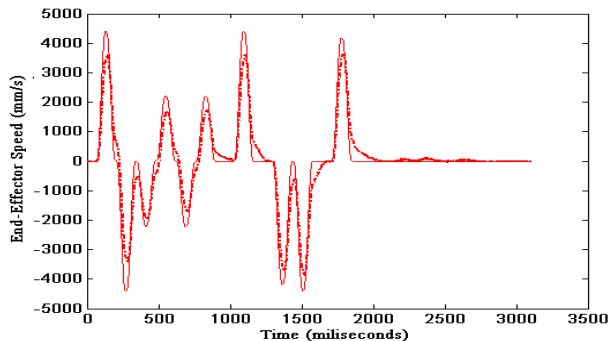


Fig. 8. Speed Profile of the end-effector in the performance of desire motion.

IV. CONCLUSIONS

RoboTennis is an innovative platform, based on a parallel manipulator, which will allow performing complex tasks, as is playing table tennis, by using visual feedback. This paper describes the analysis, design, dynamic modeling and preliminary experiences of control of the system. The experiments demonstrate that the prototype is capable of performing complex tasks in unstructured environments. The ongoing works are: calibration of the camera-end-effector, calculation of the image Jacobian, and estimation of the ball speed from several consecutive images by using the Kalman filter. The future work is the integration between the visual and robotic systems.

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