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Helicopter Flight Control Law Design Using H_∞ Techniques

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Abstract—**This paper describes the development of H_∞ controllers for two helicopter models. The study was first carried out using rigid body state feedback only and was then extended to include the rotor flap and lag states. It was observed that the H_∞ controllers gave better robustness and performance than the baseline controllers. Moreover, using rotor state feedback, it was possible to design high bandwidth controllers.**

TABLE I
NOTATION

Symbol	Quantity	Units
θ, ϕ, ψ	Pitch, roll and yaw attitude	deg
u, v, w	Longitudinal, lateral and normal velocity	ft/s
p, q, r	Roll, pitch and yaw rate	deg/s
h	Climb rate	ft/s
a1s	Lateral cyclic blade angle	deg
b1s	Longitudinal angle blade angle	deg
theta0	Main rotor collective blade angle	deg
thetatr	Tail rotor collective blade angle	deg
XA	Lateral stick input	inches
XB	Longitudinal stick input	inches
XC	Collective lever input	inches
XP	Pedal input	inches
β_{1s}, β_{1c}	Sine and cosine non-rotating flap degrees of freedom	deg
ξ_{1s}, ξ_{1c}	Sine and cosine non-rotating lead-lag degrees of freedom	deg

I. INTRODUCTION

HELICOPTER control law design is a difficult task owing to the inherent complexity and nonlinearity of the dynamics and the high levels of inter-axis coupling present in this type of aircraft. In addition, many mathematical models are unable to predict cross-coupling satisfactorily, thus making accurate modelling difficult and controller design challenging; see [1] and the references therein. In degraded visual conditions and during the execution of certain manoeuvres, the pilot's task becomes even more difficult, and special forms of control augmentation are needed to reduce the pilot's workload and to improve the handling qualities. The traditional approach to controller design, based on single-input, single-output classical control theory, appears to be less appropriate as the stringent requirements set out in the Aeronautical Design Standard ADS-33 [2] become an increasingly important part of the certification process of what are, in fact, complex *multivariable* systems. Part of the solution to this problem may be found in more advanced control and modelling techniques.

Over the years, different design methodologies have been applied to the design of control laws for helicopters, with varying levels of success [3], [4], [5], [6], [7], [8], [9]. Among these techniques, H_∞ control has paved its way since the late 1980's to reach a fairly mature stage today. H_∞ techniques feature some very desirable characteristics in the sense that they are multivariable and can also provide robust stability for systems subject to uncertainties. The first mixed sensitivity H_∞ controller was synthesized by Tombs [10] using a model of the Westland Lynx helicopter. This work was taken up by Walker and Postlethwaite [11] where H_∞ loop-shaping methods were used to develop controllers, also for the Lynx.

In [1] and [12], Walker investigated the use of H-infinity optimal control in a framework of traditional decoupling of longitudinal and lateral/directional controls. During flight testing of the resulting control law, the pilots detected little or no coupling [1, 12]. This paper builds on the foregoing work by using a fully decoupled scheme for the controller synthesis based on H_∞ loop-shaping techniques. The model used was the FLIGHTLAB Generic Rotorcraft (FGR), configured so as to be representative of the UH-60 Black Hawk [13]. The resulting H_∞ controllers have been compared to the baseline FGR aircraft with its mechanical flight control system.

In the design of high performance rotorcraft, it is useful to be able to use high levels of feedback in the flight control system to improve disturbance rejection and tracking. However, the strong coupling between the rotor and fuselage dynamics in certain frequency ranges limits the use of high gain feedback.

To overcome this problem, rotor states can (at least in

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theory) be used in the design, alongside the rigid body state measurements, thus directly controlling the rotor dynamics. The study has also been extended to investigate the benefits of feeding back the flap and lag rotor states in the design of a pitch-roll controller for the Bell 412 helicopter model.

The paper is organized as follows. Section II gives a brief overview on the models used. The baseline FGR flight control system is discussed in Section III. In Section IV, the H_∞ loop-shaping design procedure is explained. Some comparison between the baseline SCAS and the H_∞ controllers is presented in Section V. The design of control laws using rotor state feedback is discussed in Section VI. Finally, some conclusions are presented in Section VII.

II. HELICOPTER ANALYSIS MODELS

A. *The FLIGHTLAB Generic Rotorcraft (FGR) model*

The FLIGHTLAB Generic Rotorcraft Model [13] is a nonlinear simulation model of the UH60 Black Hawk. The model is a total force, large angle representation in six rigid body degrees of motion. Rotor blade flapping, lagging and hub rotational degrees of freedom are also represented. The model, complete with control system consisting of digital and analogue Stability and Control Augmentation Systems (SCAS's), interlinks and actuators, is programmed in the "FLIGHTLAB" package.

Mechanical interlinks are used in the FGR Mechanical Flight Control System (MFCS), also known as the mixing unit, to compensate for the coupling between the different control axes. The primary actuation system is driven by the MFCS. The actuators are modelled by second order systems with a bandwidth of approximately 11Hz and a damping ratio of 0.7.

The baseline model used in this study for the controller synthesis is a 9-state linearization of the nonlinear FGR model at a speed of 40 knots. The states of the rigid body model are ψ, θ, ϕ, u, v, w, p, q and r. The control inputs are lateral cyclic angle, longitudinal cyclic angle, main rotor collective angle and tail rotor collective angle. The controlled outputs were chosen as h, θ, ϕ, and r. The pitch and roll rates were also fed back to improve damping.

The nominal model is unstable in the longitudinal axis with an unstable low frequency phugoid-like mode while the other rigid body modes are all stable.

B. *The Bell 412 Helicopter Model*

The Bell 412 model used in this paper was also developed in FLIGHTLAB. The model is still undergoing development, but preliminary validation against flight test data is encouraging. The Bell 412 is a four-bladed, twin-engine helicopter having a hingeless rotor system.

III. FGR FLIGHT CONTROL SYSTEM

The flight control system for the FGR model comprises the mechanical flight control system (MFCS) and the Stability and Control Augmentation System (SCAS). The SCAS is made up of the Stability Augmentation System (SAS), the Pitch Bias Actuator (PBA), the Flight Path Stabilization (FPS) and the Stabilator. It should be noted that the FPS (which helps in attitude hold and turn coordination) and the stabilator (which regulates the tail incidence) have not been used in this analysis. These automatic control systems collectively enhance the stability and control characteristics of the FGR. The control system incorporates the sensors, shaping networks, logic switching, authority limits and actuators.

The MFCS is made up of the longitudinal cyclic control mixing unit, the lateral cyclic control mixing unit, the collective mixing unit and the tail rotor mixing unit. Mechanical interlinks are also used to compensate for the coupling between the different channels. The overall flight control system is complex and appears to have evolved.

The FGR SCAS is a 3-axis coupled controller which gives a rate command response type in pitch, roll and yaw. The inputs to the controller are the feedback signals ϕ, θ, p, q, u, v *and* r and the pilot demands *XA, XB, XC and XP.* The outputs are lateral cyclic angle *a1s*, longitudinal cyclic angle *b1s,* main rotor collective angle *theta0* and tail rotor collective *thetatr.*

The SCAS has a good roll-off around the crossover frequency and has low gain at high frequency, as we would expect from a classical point of view. The singular values of the closed loop transfer matrix from [*XA, XB, XC, XP*] to [ϕ, θ, h, r] are shown in Fig. 1.

The baseline SCAS was simulated at a range of flight conditions. As an example, the simulation results on the linear rigid body model at 40 knots are shown in Fig. 2 for a pulse input on lateral cyclic. It can be observed that the closed loop system is stable but there exists considerable

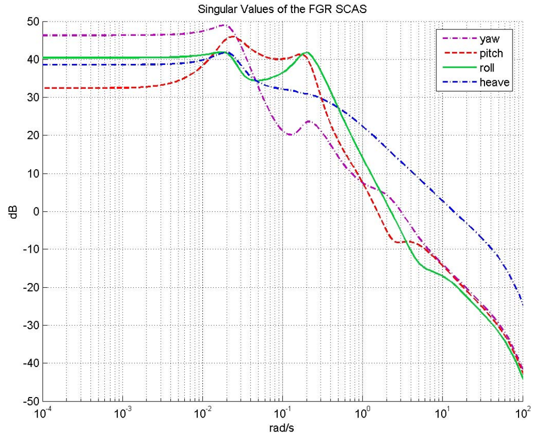

Fig. 1. Singular values of the baseline FGR SCAS

cross coupling between the channels.

To address the shortcomings of the FGR SCAS, newer design methods such as the H_∞ optimization merit investigation; e.g. to try to improve the robustness and performance of the system, as well as potentially reducing the overall complexity.

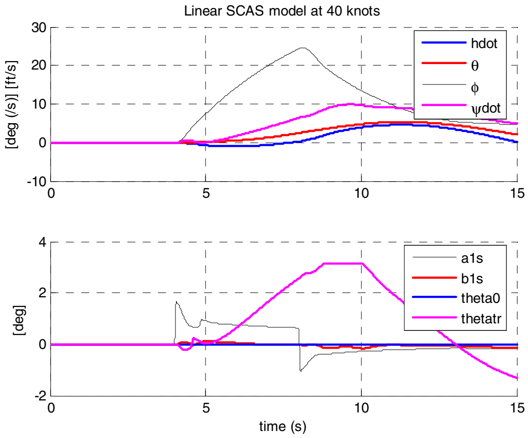

Fig. 2. Responses to a pulse input on XA for the FGR SCAS

IV. CONTROLLER DESIGN

Motivated by the ideas described in [1], it was decided to use a decoupled control scheme in this study in a view to minimize coupling between the channels and obtain good tracking. Thus, the design was performed with one controller for each of the four axes – pitch control (longitudinal cyclic to θ, q), roll control (lateral cyclic to ϕ, p), collective control (main rotor collective to h) and yaw control (tail rotor collective to r). The structure adopted for the control system is shown in Fig. 3. The controllers have full authority over the four primary controls (*a1s*, *b1s*, *theta0*, *thetatr*) and have been designed to provide an attitude command, attitude hold (ACAH) response type in pitch and roll and a rate command (RC) response type in yaw. A two-degree of freedom structure was chosen where the demands are ϕ_{dem}, θ_{dem}, h_{dem} and r_{dem}.

The design methodology used to synthesize the controller is based on H_∞ robust stabilization combined with classical loop-shaping, as proposed by McFarlane and Glover [14]. The method works by shaping the open-loop singular values of the nominal plant with pre/post compensators, followed by H_∞ optimization. This technique addresses robust stability and performance objective simultaneously. After investigating the open loop singular values, each input channel was augmented with a proportional-plus-integral type pre-compensator, W_1. The integral part was used to boost low-frequency gain and improve performance (i.e. to ensure zero steady-state error when tracking an attitude, good output decoupling, good disturbance rejection at the plant input and output), whereas the proportional part was used to rectify the phase lag added by the integrators at cross-over, to increase robustness and to adjust control actuation. The post-compensators W_2 were chosen as identity matrices.

Fig. 4 shows the singular values of the closed loop system. It can be observed that the loop gain is high at low frequencies and low at high frequencies with a roll-off of about 30 dB/decade around the crossover frequencies, thus ensuring good robust stability and performance. The controllers effectively stabilize the unstable phugoid mode and generally improve the damping of the rigid body modes.

The control law, which was originally designed at the 40 knot operating point, was simulated through a range of flight conditions on both the linear 9-state rigid body model as well as on the nonlinear FLIGHTLAB model. A high bandwidth (about 11 Hz) second order actuator as well as rate limiters were used in each channel. For illustration, only the primary linear responses for the 40-knot flight condition

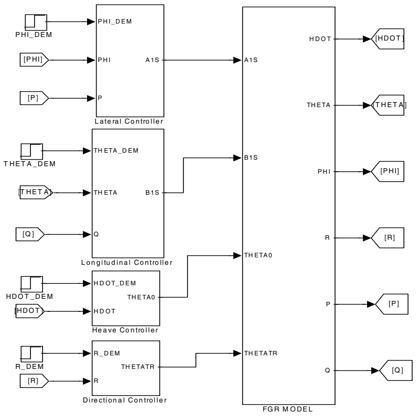

Fig. 3. H_∞ controller structure

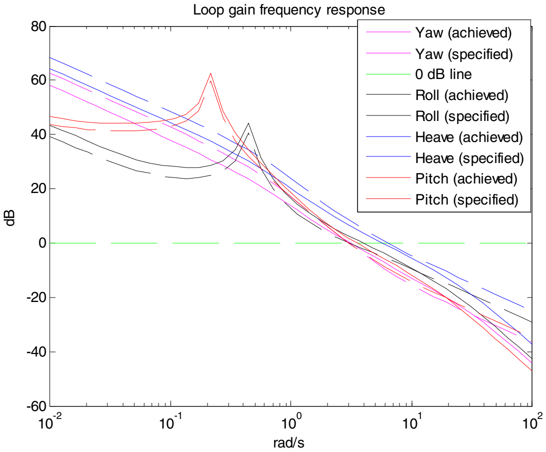

Fig. 4. Loop gain frequency response

will be shown. To enable a back-to-back comparison with the baseline FGR SCAS, similar types of pulse inputs (applied at time=4s and held for 4s) as used in Section 3 are also used to simulate the H_∞ control laws. The responses to a pulse demand on lateral cyclic is shown in Fig. 5. It can be observed that the cross couplings are small compared to those of the baseline SCAS (see Fig. 2).

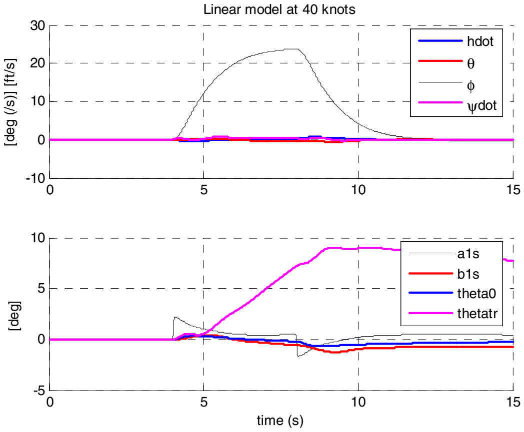

Fig. 5. Responses to lateral cyclic pulse demand

V. FURTHER COMPARISON BETWEEN THE FGR SCAS AND H_∞ CONTROLLER

A. *Handling Qualities Assessment*

Predicted Levels of Handling Qualities (HQs) have been evaluated to test compliance with the Aeronautical Design Standard ADS-33 [2]. This assessment was further used to provide a comparative study of the handling qualities of the FGR baseline SCAS and the H_∞ controller. The results are summarized in Table 2 and Table 3 for the set of criteria tested (for the hover and low speed requirement). For brevity, only the results for the most stringent MTE (Target Acquisition and Tracking) are presented.

TABLE 2

HANDLING QUALTIES RATING

ADS33 criteria	FGR SCAS		H_∞ controller	
	Pitch	Roll	Pitch	Roll
Short term response to control input	Level 1	Level 1	Level 1	Level 1
Short term response to disturbance input	Level 2	Level 2	Level 1	Level 1
Moderate amplitude pitch (roll) attitude changes	Level 2	Level 2	Level 1	Level 1
Large amplitude pitch (roll) attitude changes	Level 1	Level 1	Level 2	Level 1
Pitch due to roll (roll due to pitch) coupling	Level 2	Level 1	Level 1	Level 1

Compared to the baseline FGR SCAS, it can be observed that the H_∞ controller gave generally better performance, having more Level 1 handling qualities rating and having its worst behaviour (Level 2) for only two of the ADS-33 criteria tested.

TABLE 3

HANDLING QUALTIES RATING

ADS33 criteria	FGR SCAS	H_∞ controller
Moderate amplitude heading changes	Level 2	Level 1
Yaw due to collective coupling	Level 1	Level 2

B. *Robustness Analysis Using the Structured Singular Value*

The method of robustness analysis used is based on the structured singular value, μ. In order to apply μ-analysis tools, it is necessary to represent the original uncertain system as a Linear Fractional Transformation (LFT)-based uncertainty model [15], [16], [17]. Three uncertain parameters were considered and they were mass (*m*), centre of gravity position (*Xcg*) and rolling moment of inertia (*Ixx*). We assume that these parameters vary in the range *Xcg* = [344,366] inches, *m* = [13821,19821] slug and *Ixx* = [5966.4, 15910] slug/ft^2. These values, taken from [14], correspond to a variation of approximately [–4.4%, +1.7%] in the nominal *Xcg*, [-17.8%, +17.8%] in the nominal *m*, and [-40%, +40%] in the nominal *Ixx*.

The approach to LFT-modelling adopted in this paper is based on the method described in [15]. This technique uses the differences in the laniaries trims that arise from the nonlinear simulation of the FGR model over all combinations of the extreme points of the uncertain parameters to form a multi-model state description, from which an affine parameter-dependent representation can be extracted. Hence, fictitious inputs and outputs, corresponding to the differences in the state-space matrices, are added to the original plant model. Finally, after some further manipulation, an LFT-based uncertainty description can be formed. In this case, eight linear models, corresponding to the vertices of the three uncertain parameters, were used to generate the LFT-based uncertainty model.

The closed-loop systems were then formed by connecting the controllers to the LFT-based uncertainty model. It was found that the LFT-based uncertainty model was made up of 25 fictitious inputs/outputs (uncertainties). Since the uncertain parameters considered were all real, it was necessary to resort to real-μ algorithms in order to compute bounds on μ. The new algorithm developed in [17] was thus used for this purpose and the results are shown in Fig. 6.

It can be observed that both systems are robustly stable ($\mu < 1$). The values of μ obtained with the H_∞ controller and the baseline SCAS were found to be about 0.33 and 0.47 respectively. Clearly, it can be deduced that, for the set of uncertain parameters considered, the H_∞ controller is more robustly stable than the baseline SCAS.

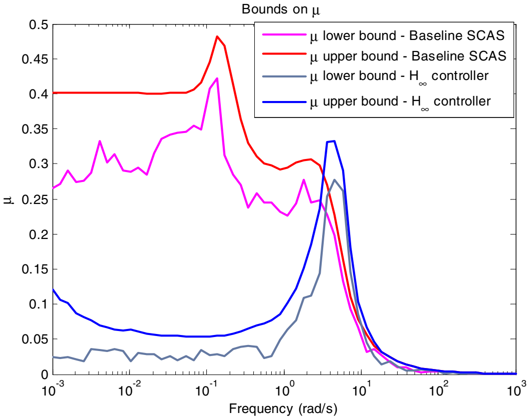

Fig. 6. Robustness analysis: lower and upper bounds on μ

VI. CONTROLLER DESIGN USING ROTOR STATE FEEDBACK

Our use of rotor state feedback follows the study described in [18] on an articulated rotor helicopter configuration in low speed flight. As shown in [18], the use of rotor flap feedback allows high gain feedback in the design and also significantly reduces noise sensitivity, thereby enhancing the robustness and performance of the resulting controllers.

A similar study has been carried out using the Bell 412 helicopter model, wherein the effects of feeding back the rotor flap modes were investigated. The results can be found in [19] and shows similarity with the results obtained in [18].

The next step consists of designing a pitch-roll control law using the fuselage and rotor state measurements for the Bell 412 at the hover flight condition. The feedbacks used were the four body angular measurements (p, q, θ, ϕ) and the rotor lag and flap state measurements and their derivatives ($\beta_{1c}, \beta_{1s}, \beta_{1c}, \beta_{1s}, \zeta_{1c}, \zeta_{1s}, \zeta_{1c}, \zeta_{1s}$). The control inputs were longitudinal cyclic blade angle and lateral cyclic blade angle.

The general control structure is shown in Fig. 7.

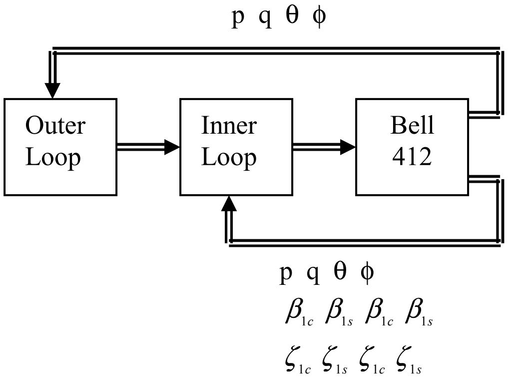

Fig. 7. Controller structure for the Bell 412

The control loops were broken into an inner and an outer pitch-roll loop, each of which will be detailed shortly. The overall objective in the design was to achieve a high-gain attitude command, attitude hold system in the pitch-roll axes.

A. *Inner Loop Design*

The inner pitch-roll loops are high gain and are used to set the basic crossover characteristics of the pitch and roll axes. These loops use body angular measurements as well as the rotor-state measurements. Both sets of measurements are essential in this inner loop since the coupling between the fuselage and the rotor is a primary factor in high feedback gain situations [18]. The pitch and roll axes are considered together since they are coupled in the high frequency range.

The inner loop was designed using LQR techniques and the performance index to be minimized was

$$J = \int_0^\infty \left(\phi^2 + \theta^2 + \frac{a_{1s}^2}{20} + \frac{b_{1s}^2}{20}\right)dt \qquad (1)$$

B. *Outer Loop Design*

The pitch-roll loops are low gain and are used to handle the low-frequency behaviour, which determines the pole placement of the phugoid modes. They make use of integrators to give the control system attitude hold. H_∞ loop-shaping technique was used to design this outer loop. Only the body angular measurements were used. Simple first-order weights used for pre-compensator in the roll and pitch channels.

C. *Simulation results*

The resulting controllers were simulated on a linear 28-state Bell 412 model at the hover flight condition. The results are shown in Fig. 8 for a step demand on lateral cyclic and in Fig. 9 for a step demand on longitudinal cyclic. It can be observed that the rise time is very good and very little cross coupling exists between the channels.The achievable bandwidth was found to be 5.5 rad/s in the roll channel and about 3.4 rad/s in the pitch channel. These bandwidths easily satisfy the Level 1 requirements set by the

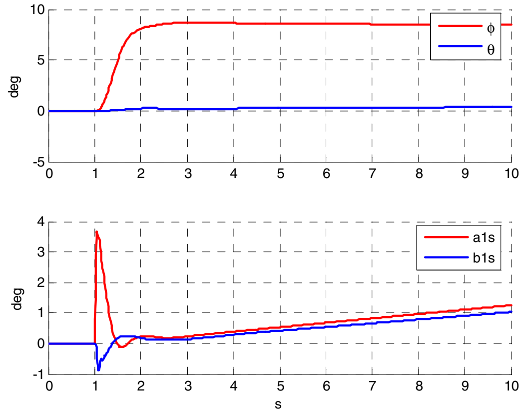

Fig. 8. Responses to a 1-inch step demand on lateral stick

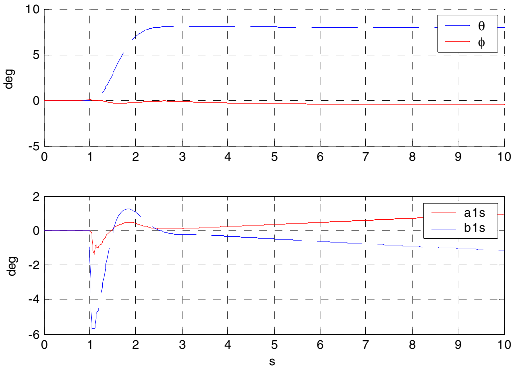

Fig. 9. Responses to a 1-inch step demand on longitudinal stick

ADS-33 specifications. In an earlier study carried out on the same helicopter model, it was found that, without rotor state feedback, the achievable bandwidths were about 3.8 rad/s and 2.6 rad/s in the roll and pitch channels respectively. Thus, including rotor state feedback in the design allowed higher feedback gains to be used and increased the bandwidth of the system.

VII. CONCLUSIONS

This paper has provided an assessment of the baseline FGR SCAS and a new attitude command-attitude hold decoupled control law designed using H_∞ loop-shaping techniques. It was observed that the H_∞ controller performed generally better the baseline FGR SCAS, achieving more Level 1 handling qualities for the ADS-33 criteria tested. Its performance was worse in a few cases, and this will be the subject of further investigations. Frequency domain and time domain analysis also indicate that the H_∞ controller has better robust stability and performance properties than the baseline SCAS. This could be important in situations where the aircraft has to perform tasks such as carrying under-slung loads, something which changes its dynamics considerable and which is known to "upset" the basic SCAS.

The paper also discussed the merits of using rotor state feedback in the design of control laws for the Bell 412 helicopter. It was possible to use high feedback gains in the design of a coupled pitch-roll controller, thus allowing larger bandwidth to be achieved in the system.

REFERENCES

[1] D.J. Walker, "Multivariable control of the longitudinal and lateral dynamics of a fly-by-wire helicopter", *Control Engineering Practice*, vol. 11(7), 2003.

[2] Anonymous, *ADS-33E-PRF Aeronautical Design Standard, Performance Specification, Handling Qualities Requirements for Military Rotorcraft,* US Army Aviation and Missile Command, Aviation Engineering Directorate, Redstone Arsenal, Alabama, 2001.

[3] P. Apkarian, C. Samblancat, L. Le Letty and R. Patton, "A two feedback loop robust helicopter control based on eigenspace techniques and H_∞ synthesis", *Proc. 29th IEEE Conf. Decision and Control*, vol. 6, 1990, pp. 3337-3339.

[4] S.W. Baillie, J.M. Morgan and K.R. Goheen, "Practical experiences in control system design using the NRC Bell 205 airborne simulator", *AGARD Flight Mechanics Panel Symposium on Active Control*, Italy, 1994, pp. 27.1-27.12.

[5] R.D. Holdbridge, W.S. Hindson and A.E. Bryson, "LQG design and flight test of a velocity command system for a helicopter", *Proc. AIAA Guidance, Navigation and Control*, 1985.

[6] J. Leitner, A. Calise and J.V.R Prasad, "Analysis of adaptive neural networks for helicopter flight control", *AIAA Journal of Guidance, Control and Dynamics*, vol. 20(5), 1997, pp. 972-979.

[7] J. Liccaga-Castro, C. Verde, J. O'Reilly and W.E. Leithead, "Helicopter flight control using individual channel design", *Proc. IEE Conf. Control Theory and Applications*, UK, vol. 142(1), 1995, pp. 58-72.

[8] C. Philips, C.L. Karr and G.W. Walker, "Helicopter flight control with fuzzy logic and genetic algorithms", *Engineering Applications of Artificial Intelligence*, vol. 9(2), 1996, pp. 175-184.

[9] R.L. Wade and G.W. Walker, "Flight test results of the fuzzy logic adaptive controller-helicopter (FLAC-H)", *Proc. Of SPIE – the international society for optical engineering*, vol. 2738, 1996, pp. 200-208.

[10] M.S. Tombs, *Robust control system design with application to high performance helicopters*, Ph.D. Thesis, University of Oxford, 1987.

[11] D.J. Walker and I. Postlethwaite, "Full authority active control system design for a high performance helicopter", *Proc. XV1 European Rotorcraft Forum*, 1990, pp. III.3.2.1-III.3.2.14.

[12] D.J. Walker, M.C. Turner, A. Smerlas, A.W. Gubbels and M.E. Strange, "Robust control of the longitudinal and lateral dynamics of the Bell 205 helicopter", *Proc. American Control Conference*, USA, vol. 4, 1999, pp. 2742-2726.

[13] G.D. Padfield and M.D. White, "Flight simulation in academia: HELIFLIGHT in its first year of operation", *Journal of the Royal Aeronautical Society*, vol. 107(1075), 2003.

[14] D.C. McFarlane and K. Glover, *Robust controller design using normalized coprime factor plant descriptions,* in *Lecture notes in control and information sciences*, vol. 138, Berlin Springer, 1990.

[15] G. Ferreres, *A practical approach to robustness analysis*, Kluwer Academic/Plenum Publishers, 1999.

[16] D.G. Bates, R. Kureemun, M.J. Hayes and I. Postlethwaite, "μ-analysis of linear stability criteria" in *Advanced techniques for clearance of flight control laws*, Fielding, Varga, Bennani, Selier (Eds.), Springler Verlaag, 2002.

[17] R. Kureemun, *μ-analysis tools for the flight clearance of highly augmented aircraft*, Ph.D. Thesis, University of Leicester, 2003.

[18] M. D. Takahashi, "Rotor-state feedback in the design of flight control laws for a hovering helicopter", *Journal of the American Helicopter Society*, vol. 39(1), 1994, pp. 50-62.

[19] R. Kureemun, "A preliminary study in the design of high bandwidth controllers using rotor state feedback", *Internal report,* University of Liverpool, 2005.