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Model Predictive Control Employing Trajectory Sensitivities for Power Systems Applications

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Abstract— **Model Predictive Control (MPC) is a widely used method in process industry for control of multi-input, multi-output systems. It possesses some features, which make it attractive for applications in power systems. Power systems exhibit several features of complex systems, such as hybrid nature (mixed continuous and discrete dynamics), nonlinear dynamics and very large size. Since MPC involves optimization computations, it represents a big challenge to handle above listed properties in a reasonable time for large power systems. Therefore the reduction of the computational burden associated with MPC is a crucial factor.**

We describe in this paper a formulation of MPC for power systems based on trajectory sensitivities. Trajectory sensitivities are time varying sensitivities derived along the predicted nominal trajectory of the system. They refer to possible changes of initial conditions from which the nominal trajectory is predicted, or to a modification of system parameters including changes of discrete valued quantities, e. g. positions of transformers taps. Trajectory sensitivities allow an accurate reproduction of the nonlinear system behavior using a considerable reduced computational burden as compared with the full non-linear integration of the system trajectories.

I. INTRODUCTION

Model Predictive Control (MPC) is a control methodology, which allows explicit integration/inclusion of the constraints (imposed on the controlled system and/or employed controls) and explicit expression of the control quality criteria in the control objective. However, MPC drawbacks are: its open control loop nature and computational burden associated with the solution of the optimization problem. The first obstacle can be overcome by introducing an implicit feedback in form of repetitive computation of control laws in a receding horizon manner. (This has been proven for infinite horizon control.) The second obstacle has restricted the application of MPC mostly to control of slower processes, with the dynamics in order of minutes (e.g. chemical industry). This is also probably the factor limiting a wider spreading of MPC in power systems up to now.

Power systems comprise of many components; e.g. generators producing electrical energy, loads, consuming electrical energy and transmission elements connecting loads and generators. They interact with each other in various ways and in various time scales (i.e. dynamics speed ranging from milliseconds to years). Dynamics present in power systems may be very nonlinear and have a hybrid nature, i.e. both continuous and discrete state variables.

Several authors have addressed difficulties to apply MPC in power systems. Reference [1] has dealt with the hybrid dynamics by applying *Mixed Logic Dynamics* (MLD) framework. Among others, applications of tree search methods have been investigated in [2]. A significant acceleration of MPC computations has been reported in [3].

Our contribution aims at reduction of the modeling complexity and computational burden by employing the trajectory sensitivities concept. This paper continues along the line of work and ideas introduced first in [4], [5], where open-loop control focusing on emergency conditions has been introduced, and [6], where first investigations concerning closed loop MPC employing trajectory sensitivities have appeared. This paper presents closed loop MPC using trajectory sensitivities and covering both emergency and normal operation conditions in power systems.

The paper starts with a short overview of MPC. Then we introduce trajectory sensitivities concept and its application within the MPC framework for power systems applications. After that we outline some possible applications in power systems and provide simulation results for the voltage control of a realistically sized power system.

II. PROPOSED MODEL PREDICTIVE CONTROL

A. Nonlinear Model Predictive Control Formulation

In general, MPC computes an optimal sequence of manipulated inputs, which minimizes a tracking error (i.e., difference between the desired reference output and its real value) subject to constraints on inputs and outputs.

Similarly to approaches in [7] or [8], this can be formulated in the continuous time domain for a general case, applicable also for nonlinear systems, as follows:

$$\min_{u(t)} \Phi(x(t), y(t), u(t)) \tag{1}$$

subject to equality constraints (usually describing system dynamics):

$$\dot{x}(t) = f(x(t), y(t), u(t)) \tag{2}$$

$$0 = g(x(t), y(t), u(t)) \tag{3}$$

and inequality constraints:

$$0 \leq h(x(t), y(t), u(t)) \tag{4}$$

where $x(t)$ are differential variables, $y(t)$ are algebraic variables and $u(t)$ are manipulated inputs within the control time interval (horizon) of the length T_h, thus $t \in [t_0, t_0 + T_h]$. Furthermore, all types of variables may be nonlinearly bounded by spaces $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{U}$, respectively:

$$x(t) \in \mathcal{X} \quad y(t) \in \mathcal{Y} \quad u(t) \in \mathcal{U} \tag{5}$$

The solution of above problem at the time t_0 is an optimal input sequence $u^*(t)$. A feedback property is introduced into

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MPC by on-line repetitive applying of above computations in form of the sliding window moving in time, i.e. receding horizon control.

B. Trajectory Sensitivities

An efficient framework for modeling of nonlinear systems featuring discrete states (i.e. 2 and 3) has been presented in [9]. Its application on power systems modeling has been further shown in [10] and [11], where a natural flexible modular structure following power systems components classification has been adopted.

Omitting parameters λ from the original formulation from [9] and introducing control inputs u, we can write in compact form:

$$\dot{x} = f(x, y, z, u) \tag{6}$$

$$0 = g^0(x, y, z, u) \tag{7}$$

$$0 = \begin{cases} g^{i-}(x, y, z, u) & y_{d,i} < 0 \\ g^{i+}(x, y, z, u) & y_{d,i} > 0 \end{cases} \quad i = 1, \cdots, d \tag{8}$$

$$z^+ = h_j(x^-, y^-, z^-, u^-) \quad y_{e,j} = 0 \quad j \in \{1, \cdots, e\} \tag{9}$$

$$\dot{z} = 0 \quad y_{e,j} \neq 0 \quad j \in \{1, \cdots, e\} \tag{10}$$

where

$$\begin{array}{lll} y_d = Dy & x \in X \subseteq \mathbf{R}^n & y \in Y \subseteq \mathbf{R}^m \\ y_e = Ey & z \in Z \subseteq \mathbf{R}^k & u \in U \subseteq \mathbf{R}^l \end{array} \tag{11}$$

Differential variables are denoted x, algebraic variables y and discrete state variables z. Switching of the status of discrete variables is governed by the equation (9) when the corresponding auxiliary variables of y_e are equal to zero. Auxiliary variables y_d determine the region of validity of the equations (8). In the power systems context this may be explained on an example of a line, which changes its status. When the line is in service, equations linking the flowing current through the line and voltages at both ends of the line as well as line parameters (line impedance and shunt admittance) are valid. When the line is out of service (i.e., disconnected), current flowing through it is zero. The auxiliary variable is in that case the difference between the time and the instant when the line was tripped. Matrices D and E have normally a very sparse structure and their non-zero elements are equal to one on the positions aiming at the auxiliary variables.

Flow (i.e., time evolution) of the system from its initial point can be characterized by the time evolution of its variables, e.g. for algebraic variables we can write:

$$\phi_y(y_0, t) = y(t) \tag{12}$$

Note that the initial state y_0 is obtained by solving equations (7) - (8) by substituting initial values of $x(t), z(t)$ and $u(t)$ by x_0, z_0 and u_0, respectively.

An impact of small changes of initial conditions on the system flow can be investigated by trajectory sensitivities. So the impact of manipulated inputs (i.e., controls) on algebraic states time evolution can be obtained by a Taylor expansion of (12). When neglecting higher order terms:

$$\Delta y(t) = \frac{\partial y(t)}{\partial u_0} \Delta u_0 \tag{13}$$

Note that trajectory sensitivities are generally time varying quantities.

Note that a numerical approximation of trajectory sensitivities can be computed by solving (6) - (11) for an incremental change of each control input. But that would represent a large computation effort if many control inputs are considered. The methodology for computation of trajectory sensitivities described for example in [9] involves only minimal additional computations, since it uses parts of Jacobian evaluated when solving (6) - (11).

C. Proposed Formulation

Since the solution of (1) while satisfying (2) - (5) may be in case of nonlinear systems a very complex task, we propose a control procedure as follows.

Let us assume a discrete time control. The controller would then be employed (i.e., MPC optimization problem would be solved) in the regular time intervals of T_{cs} with the outlook of the prediction horizon (i.e., length of the sliding window) of T_h.

Assume that the control should be executed at the time t_0. Then the control method comprises of three stages:

1) A nominal trajectory of the system is predicted for the time interval $[t_0, t_0 + T_h]$ by employing discrete time form of (6) - (11) with the integration step of T_{ps}. This may be perceived as a time domain simulation. As a byproduct, trajectory sensitivities are computed with a little additional effort [9].
2) Controls are computed according to the equations (14) - (21) and the description in the next paragraphs.
3) The first set of computed controls is applied at the time t_0 (neglecting the time delay).

Note, that although we use a discrete time framework, we do not explicitly mention a particular number of samples, but rather times. This is due to possible discrete events. To capture them properly, two samples in the one time instant are necessary. Moreover, it may happen between two regular samples, thus the time of the discrete event occurrence has to be computed as well and the integration step T_{ps} is divided accordingly.

In most cases, the integration step has a relatively small size to capture dynamics properly. Therefore $T_{ps} \leq T_{cs} \leq T_h$.

Using a quadratic cost function, the control objective can be expressed by (14). The first term represents a cost of the deviation from the reference value, so the corresponding algebraic variable is defined in the model description in the equality constraints (15). The third term penalizes the control employment, whereas the second term the deviation of the control from its initial value in the time t_0. Note that all control deviations Δu refer to the initial point and not to the previous sample. However, a penalty on the deviation of the control from its previous sample value can be easily included by an appropriate choice of the non-diagonal elements of the matrix R_{du}.

$$\min_{\Delta U} \frac{1}{2} \sum_{k=0}^{N_{ps}-1} \{y_k^T Q y_k + \Delta u_k^T R_{du} \Delta u_k + u_k^T R u_k\} \quad (14)$$

$$y_k = y_k^{nom} + \frac{\partial y_k}{\partial u_k} \Delta u_k \quad (15)$$

$$u_k = u_k^{nom} + \Delta u_k \quad (16)$$

$$y_k^{min} \leq y_k \leq y_k^{max} \quad (17)$$

$$u_k^{min} \leq u_k \leq u_k^{max} \quad (18)$$

$$\Delta u_k^{min} \leq \Delta u_k \leq \Delta u_k^{max} \quad (19)$$

$$\Delta U = \{\Delta u_k\}_{k=0}^{N_{ps}-1} \quad (20)$$

$$u_k \in \mathbf{R}^{N_u} \quad \Delta u_k \in \mathbf{R}^{N_u} \quad \frac{\partial y_k}{\partial u_k} \in \mathbf{R}^{N_y \times N_u} \quad (21)$$

$$y_k \in \mathbf{R}^{N_y} \quad \forall k \in K \quad K = \{0, \cdots, N_{ps}-1\} \quad (22)$$

N_u is the number of available controls (manipulated inputs), N_y number of controlled variables (i.e., tracked outputs) and N_{ps} number of samples in the prediction horizon (as discussed earlier, it is not necessarily constant).

Unless employed controls change significantly, trajectory sensitivities reproduce the system behavior quite accurately, even considering nonlinear dynamics. However, it is very difficult to bound a region, in which the changes can be considered "reasonably small". Another possible source of errors may be certain type of discrete events. A discrepancy between the model and the actual system response can be corrected in the receding horizon manner.

Above defined controller uses a quadratic cost function and hard constraints. But this is not a limitation of the method. Soft constraints may be introduced as well as slack variables and the whole problem can be restated in a form of linear programming.

III. Illustrative Example

As discussed in the introduction section, power systems are very large complex systems. Therefore their control has basically hierarchical structure and the control of various phenomena is decoupled [12]. That means that the control of the frequency (i.e. keeping the balance between the consumed and produced active power) is done a different way and using a different tools/infrastructure as the control of voltages.

Voltage control (sometimes also referred to as a reactive power management) in power systems has several goals. They can be divided into two categories; economy and security oriented. From the long-term economical operation point of view, it is most desirable to avoid frequent under- or over- voltages, which increase aging of the equipment, and to keep the losses generated in the components low.

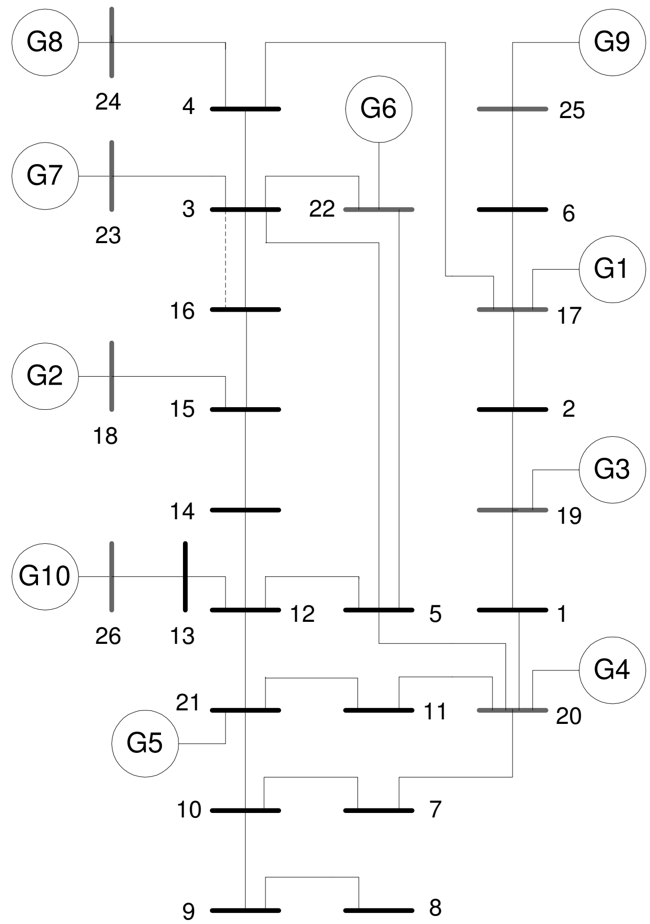

Fig. 1. Single line diagram of the test system. Loads are connected in the black buses. The dashed line between nodes 3 and 16 is the line tripped in the disturbance scenario.

This can be achieved by decreasing flowing current values (recall that most of the losses are resistive: $R.I^2$) by keeping the voltage profile high. From the security point of view, the main concern is a danger of encountering a voltage instability and resulting voltage collapse. Here the defense strategy is usually dependent on the system conditions. When the system is in emergency and voltages significantly drop, under-voltage load shedding relays trigger a disconnection of some loads. In normal operation conditions, the focus is on keeping system robust and to strengthen its ability to withstand a disturbance (e.g. a line or a generator trip), which could possibly cause a voltage collapse. So it is desirable to control the system in a way keeping large reactive power reserves of generators (in other words, keeping a low reactive power production of generators). Consequently there should be a sufficient amount of reactive power available to avoid voltage problems in the case of a disturbance (consult [12] or [13] for explanation of coupling between the reactive power and voltage).

Consider the system described in [14] and shown in the figure 1. The authors have used it for a study of power

TABLE I

THE TEST SYSTEM CHARACTERISTICS.

Component	Number/Amount
Buses	26
Lines	31
Loads	17
Generators	10
Governors	10
AVRs	10
PSSs	3

oscillations excited in Nordel (interconnected power system of Sweden, Norway, Finland and eastern part of Denmark). We have introduced small modifications (e.g. changing the load character to constant power load instead of impedance load) allowing us to focus on the voltage related issues. An overview of basic system parameters is provided in the table I. Basic control circuits of generators are modeled too, e.g., *governor* controlling the turbine supplying torque and thus the active power, *Automatic Voltage Regulator (AVR)* controlling excitation circuits of the generator and thus the generator's terminal voltage and reactive power output and *Power System Stabilizer (PSS)* modulating the reference signal for the AVR a way damping oscillations measured in the system frequency. All together, the system model has 822 variables of all types (i.e., differential, algebraic, and discrete variables, including controls). Tracked variables/outputs are voltages in all buses and reactive power production of all generators. Possible controls are reference voltages of all AVRs and load shedding of all loads. Load shedding factor of each load can have a value between 0 (no load shedding at all) and 1 (complete load is shed) and determines in which extend the load is disconnected. The voltage boundaries are 0.85 pu (per unit, referred to the nominal voltage equal to 1 pu) and 1.15 pu respectively. Limits imposed on the reference voltages (i.e. controls, manipulated inputs) are 0.9 pu and 1.1 pu. Note that voltage boundaries both of the outputs and inputs are usually narrower in reality.

The goal of the controller is to continuously supervise the power system and minimize the absolute value of reactive power production of all generators. Under normal conditions, this should be achieved only by selecting most suitable reference voltages for AVRs. Only when the system is seriously endangered, load shedding can be employed to recover the system into the acceptable operation region. The controller parameters have been chosen accordingly to serve this purpose. The nominal prediction sampling (i.e., integration step) T_{ps} is 0.5 second, prediction horizon T_h is 10 seconds and controls sampling T_{cs} is 5 seconds. The controller follows the form defined by (14) - (21). A possible way to implement this control scheme may be the platform introduced more in the detail in [15].

To show the controller's behavior, we have chosen two different scenarios. First the system is subjected to a severe disturbance, which is disconnection of the line between the buses 3 and 16. This is a common situation in power systems, for example when lightning hits the line. Short-

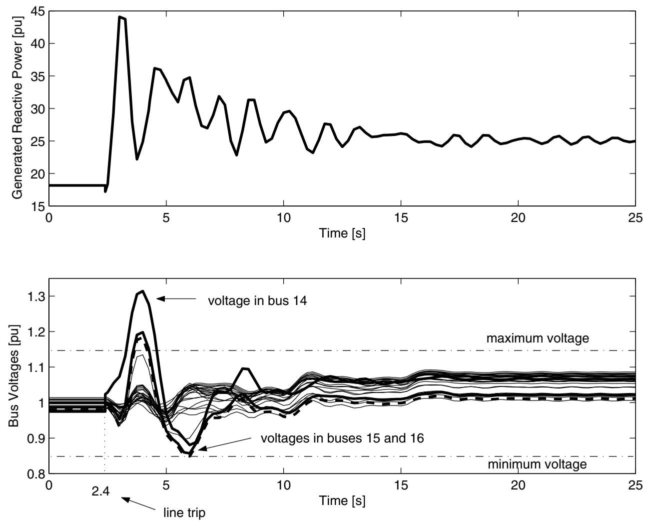

Fig. 2. Controlled system outputs. The test system is subjected to a severe disturbance. The upper plot represents the sum of absolute values of reactive powers produced by all generators. The lower plot shows buses voltage magnitudes.

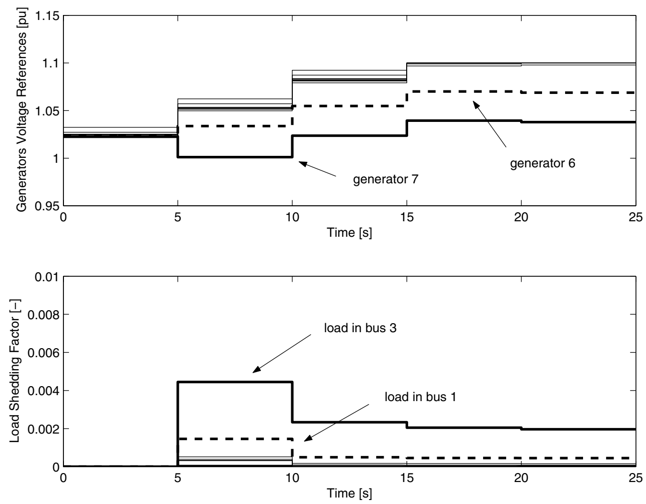

Fig. 3. Controls computed and executed by the controller. The test system is subjected to a severe disturbance. The upper plot shows reference voltages for generators' AVRs. The lower plot shows load shedding factors of all loads, closer explained in the text.

circuit protection relay reacts and disconnects the line. As it is shown in figure 2, the line outage triggers an oscillation driving voltages outside of the secure operation region.

Therefore, the controller orders load shedding at buses 1 and 3 in addition to the adjustment of generator voltage reference set points. The latter control action would be insufficient and too slow to save the system from more severe consequences, see figure 3.

In the second case, the system starts from the same initial conditions, but there is no disturbance. Therefore the employed controls shown in figure 5 aim at the decrease of the reactive power produced by generators as shown in figure

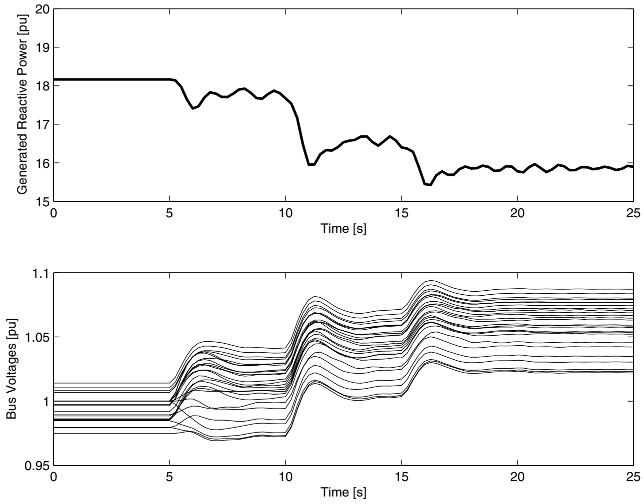

Fig. 4. Controlled system outputs. The test system is under normal conditions. The upper plot represents the sum of absolute values of reactive powers produced by all generators. The lower plot shows buses voltage magnitudes.

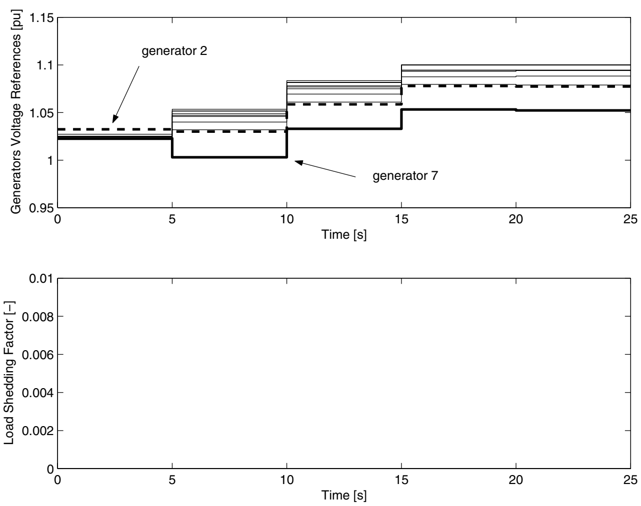

Fig. 5. Controls computed and executed by the controller. The test system is under normal conditions. The upper plot shows reference voltages for generators' AVRs. The lower plot shows load shedding factors of all loads, closer explained in the text.

4.

IV. Summary and Future Work Outline

We believe that continuing advances in implementation platforms suited for MPC and computational power (i.e. hardware), as well as research activities towards more efficient algorithms, make MPC attractive for applications in real power systems.

The paper describes an algorithm allowing relatively fast optimization computations, while keeping an accurate tracking/description of a nonlinear behavior. We show the principle and theoretical description of the proposed algorithm and demonstrate its application on the reactive power and voltage control example.

Possible applications on other power systems control problems can be investigated. A research towards most suitable implementation optimization algorithm (e.g. answering the question if linear programming with slack variables would be faster and numerically more robust) as well as introduction of explicit models of system wide objectives (e.g. one model of the entire system reactive power generation) may further significantly improve the performance of the algorithm.

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