

Pricing for Rate Allocation in Unicast Service Provisioning is Informationally Efficient

Tudor Mihai Stoenescu and Demosthenis Teneketzis

Abstract—We establish the informational efficiency of the pricing mechanism for unicast service provisioning, a class of decentralized resource allocation problems that arise in communication networks. We also present instances of network resource allocation problems where users are price takers and pricing mechanisms are not informationally efficient.

I. INTRODUCTION

Integrated services networks support the delivery of a variety of services to their users. One of the main challenges in integrated services networks is the design of resource allocation strategies which guarantee the delivery of different services and maximize some performance criterion (e.g. the network's utility to its users). The challenge in determining such resource allocation strategies comes from the fact that the network is an informationally decentralized system. Decentralized resource allocation problems have been explored in great detail by mathematical economists in the context of mechanism design. Mechanism design consists of two components: realization theory and implementation theory.

In this paper we use the formalism and the methodologies of realization theory to derive new results on the informational efficiency of the pricing mechanism for unicast service provisioning, a class of decentralized resource allocation problems that arise in communication networks. Results on the informational efficiency of the pricing mechanism already exist for classical environments [29] where the utility functions are either quadratic [27, 28], or of the Douglas-Cobb form (see [30]). The environments in unicast service provisioning are different from those of [27, 28, 30]. A discussion of the differences between the problem investigated in this paper and those of [27, 28, 30] is presented in [43].

The rest of the paper is organized as follows. Since the development of our results relies on the formalism and methodologies of realization theory, we briefly present the salient features and key results of this theory in Section II. In Section III we prove the informational efficiency of the pricing mechanism for unicast service provisioning. We formulate the rate allocation problem in unicast service provisioning with fixed routing in Section III-A. We present a market mechanism for its solution in Section III-B. We

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T. Stoenescu is a Postdoctoral Scholar in the Social and Information Science Laboratory, California Institute of Technology, 1200 E. California Boulevard, Pasadena Ca 91101 tudor@caltech.edu

D. Teneketzis is with the Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, Mi 48105 teneketzis@eecs.umich.edu

embed the rate allocation problem within the framework of realization theory (Hurwicz's model) in Section III-C; and we prove the informational efficiency of the market (pricing) mechanism in Section III-D. In Section IV we present instances of network resource allocation problems where pricing mechanisms are not informationally efficient. We conclude in Section V.

II. REALIZATION THEORY

Formally, resource allocation problems can be described by the following triple: environment, action space, and goal correspondence. We define the *environment* \mathbf{E} of such problems to be the set of individual endowments, the technology, and preferences, taken together. More generally, the environment is defined as the set of circumstances that cannot be changed either by the designer of the mechanism or by the agents. The *action space* \mathbf{A} of the problem is considered to be the set of all possible actions, (e.g. resource exchanges) conducted by the various agents. Finally, the *goal correspondence* π is the map from \mathbf{E} to \mathbf{A} which assigns for every $e \in \mathbf{E}$ the set of actions in \mathbf{A} which are solutions to the resource allocation problem.

The setup described above corresponds to the case in which one of the agents has enough information about the environment so as to determine the actions that would satisfy the goal correspondence (i.e. the information in the systems is centralized). Generally this is not the case. Usually, different agents have different information about the environment (i.e. we have an informationally decentralized system). For this reason it is desired/necessary to devise a message exchange process among the various agents that eventually enables them to jointly take an action which corresponds to a solution of the centralized problem. We call such a process of communication, decisions and actions a *resource allocation mechanism*.

The first effort to formally study resource allocation mechanisms can be traced back to Hurwicz's work [11–14]. Formally, the mechanism model proposed by Hurwicz can be described by the triple (\mathcal{M}, μ, h) : a message space \mathcal{M} , an equilibrium message correspondence μ , and an outcome correspondence h . The *message space* is the set of messages that may be exchanged by the agents. The *equilibrium message correspondence* describes the sets of messages that the agents “agree” upon given any particular environment. The *outcome function* describes the set of actions that are taken based on a particular set of “equilibrium” messages. The above formulation is depicted graphically in Figure 1 (cf. [39]).

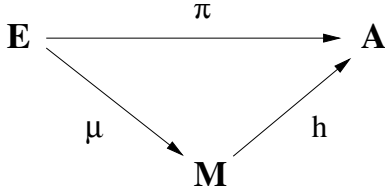


Fig. 1. Message exchange for a decentralized system.

Realization theory is concerned with existence and design of mechanisms (\mathcal{M}, μ, h) such that the diagram in Figure 1 commutes. Hurwicz's setup is quite general and can incorporate many types of mechanisms.

The following requirements are generally imposed on Hurwicz's model:

R1. For each element of the environment $e \in \mathbf{E}$ there exists a non-empty set of possible *feasible* actions. The notion of feasibility can usually be split into two categories *individual feasibility* and *compatibility*. In particular:

- 1) Within the context of the network problems considered in Section III a user's individual feasible actions are formed by the set of non-negative demand vectors. On the other hand, the network's individual feasible actions are formed by the set of amounts of services that are delivered and satisfy the network's capacity constraints.
- 2) Within the context of the network problems considered in Section III an action is incompatible if in equilibrium a user requests an amount of service which differs from the amount of service the network intends to supply.

R2. For each element of the environment the set of feasible actions that satisfy the goal correspondence π is non-empty.

R3. The actions generated by π must satisfy some sort of optimality criterion. Examples of such criteria appear in [2, 3, 6, 21, 34, 40].

R4. For any environment $e \in \mathbf{E}$, $\mu(e) \neq \emptyset$; that is, for any environment there exists a set of messages to which all agents "agree".

R5. The maps π, μ and h satisfy the following relationship:

$$h(\mu(e)) \subseteq \pi(e) \quad \forall e \in \mathbf{E}. \quad (\text{II.1})$$

Mechanisms satisfying this assumption, when π is a Pareto optimal correspondence, are also referred to as *non-wasteful*.

R6. The non-wasteful criterion established above can sometimes be inadequate. For example we can have the case where the equilibria of a given process always favors one group of participants at the expense of others. We call such mechanisms *biased*. To avoid biased mechanisms we require *unbiasedness*.

A formal test for unbiasedness can be viewed as follows: Suppose that we think of our process as being formed of two stages. In the first stage the process "distributes parameters" (e.g. resources, information etc.) to the various agents, while in the second stage we have a tâtonnement process (cf. [4]). If for any environment $e \in \mathbf{E}$ and any goal realizing action

$a \in \pi(e)$ there exists a set of distributional parameters such that at the end of the tâtonnement process the agents take action a , then the process is called unbiased.

R7. For any environment, the rules of the process lead the system to a uniquely determined allocation. This requirement may be difficult to satisfy. We call such processes, where equilibrium indeterminacies are trivial in nature, *essential single-valued*.

R8. There are two types of information regarding the environment agents have access to: direct and indirect (see [43]). We assume that an agent's direct information is information pertaining only to himself and not to other agents. We will refer to processes satisfying this assumption as *informationally consistent*.

When considering the equilibrium messages generated by agents, we call a process to be *privacy preserving* when all the agents generate their messages based only on their own information about the environment. Hurwicz's model restricts attention to *privacy preserving* resource allocation mechanisms.

R9. Assume that agents communicate with one another through a *communication alphabet* which permits them to communicate in one shot their direct information (profile) to the other agents. Such a language in most cases is too complex for consideration and hence is undesirable. Restricting the language may also not be enough to alleviate this problem. Agents may be able to encode in a relatively "simple" language their full profile. This type of encoding though may be done by the use of equilibrium message and outcome correspondences which are highly discontinuous and resemble such functions as the Peano space filling curves [1]. Such mechanisms are generally highly unstable (hence undesirable) since minor perturbations/errors in communication will lead to drastically different/nonoptimal actions. In order to alleviate problems such as the above, we impose extra conditions, such as spot threadiness, on the correspondences μ and h .

Definition 2.1: A correspondence $F : E \rightrightarrows M$ is *spot threaded* if for every $e \in E$ there exists an open set $U_e \subseteq E$, and a continuous function $f : U_e \rightarrow M$ such that $f(e') \in F(e')$ for all $e' \in U_e$.

We note that the first three requirements **R1-R3** are constraints on the type of problems considered, and they are defined independently of the mechanism. The next four requirements **R4-R7** are imposed on the mechanisms to be considered and are generally referred to as (*Pareto*) *satisfactoriness*¹. Mechanisms that satisfy **R8-R9** are called *regular*.

Realization theory was developed using subsets of the above requirements. To proceed with a more formal description of the main results on communication and information processing requirements we need the following definitions:

¹In many models requirement **R7** is omitted in the definition of Pareto satisfactoriness.

Definition 2.2: We say a mechanism (\mathcal{M}, μ, h) is *goal realizing* if it satisfies requirements **R4** and **R5**.

Definition 2.3: We say that a mechanism (\mathcal{M}, μ, h) is *informationally efficient* if it is goal realizing and regular and it has a message space of a dimension which is minimal among all the other goal realizing and regular mechanisms.

Most of the research on realization theory has dealt with the discovery of goal realizing mechanisms that have a message space of minimum dimension (cf. [22]) among the message spaces of goal realizing and regular mechanisms. Some of the key results are: (i) the competitive process is Pareto satisfactory for classical environments [30,31]; (ii) for classical environments, the competitive mechanisms are goal realizing and have a message space of minimum dimension among the message spaces of goal realizing and regular mechanisms [11, 15, 27, 28, 30]; (iii) competitive mechanisms are informationally efficient for classical environments where the utility functions are of the Douglas Cobb form [30], or of the quadratic form [27,28]; (iv) for environments with public goods the Lindahl mechanism is goal-realizing and has a message space of minimum dimension among the message spaces of all goal-realizing and regular mechanisms [14, and references therein].

Work in [18, 37, 38] addressed issues related to the complexity of information processing of goal-realizing mechanisms that have message spaces of minimum dimension among the message spaces of goal-realizing and regular mechanisms.

In this paper we show that the pricing mechanism is informational efficient for a class of environments that are different from those of [27, 28, 30]. This class of environments arises in decentralized resource allocation problems in communication networks. We present these network problems in the sequel.

III. RATE ALLOCATION IN UNICAST NETWORKS

This section is organized as follows. We formulate the rate allocation problem in unicast service provisioning in Section III-A. We present a decentralized resource allocation mechanism for rate allocation in Section III-B. We embed the rate allocation problem within the framework of Hurwicz's model in Section III-C. We prove the proposed mechanism's informational efficiency in Section III-D.

A. Problem Formulation

We consider a set of users/agents, denoted by $\mathbf{N} = \{1, 2, \dots, N\}$, requesting services from a network. We assume that each user $i \in \mathbf{N}$ requests a single service from the network, and the user's preference over the service rate received is summarized by a quasi linear utility function (cf. [29]) of form $u_i(x^i) + x_0^i$ where $x^i \in \mathbb{R}_+$. We define $x \triangleq (x^1, x^2, \dots, x^N)$ to be the vector of demands requested by the N users. We denote a user and its service by the same index.

We consider the network to be the $(N + 1)^{th}$ agent. The network is formed by a set of links \mathbf{L} . For every $l \in \mathbf{L}$ we denote by c_l the amount of resource/bandwidth available on

link l . We denote by $c_{\mathbf{L}}$ the vector denoting the amount of resources available on all the links of the network.

The goal of the network is to allocate resources to the various services in order to maximize a social welfare function described by the sum of the user utilities. Hence, the goal of the network is:

$$\max_x \sum_{i \in \mathbf{N}} u_i(x^i) \quad \mathbf{P}$$

subject to:

$$x^i \geq 0, \quad \forall i \in \mathbf{N} \quad \mathbf{P.a}$$

$$\sum_{i \in \mathbf{N}} x^i r_l^i \leq c_l, \quad \forall l \in \mathbf{L} \quad \mathbf{P.b}$$

where

$$r_l^i = \begin{cases} 1, & \text{if user } i\text{'s request utilizes link } l \\ 0, & \text{otherwise.} \end{cases} \quad \mathbf{P.c}$$

We further assume that:

P.d The users' utility functions are strictly concave, continuously differentiable and have second derivatives that are bounded in absolute value by some $M < \infty$.

Definition 3.1: We define by \mathcal{D}_M the set of user utility functions satisfying constraint **P.d**.

In addition, the following informational constraints are present:

P.e The network has no information about the users' utility functions.

P.f Each user's preferences over the particular services is his private information. The users are unaware of the topology of the network and the amount of resources available on each link. The users are also unaware of the number of other users requesting services from the network, or their utility functions.

For a detailed discussion of the assumptions made on the problem formulation the reader is referred to [43].

The goal in unicast is to determine a mechanism that allocates resources in order to generate services for individual users, is social welfare maximizing², and satisfies the aforementioned informational constraints. To achieve this goal we present a market mechanism, which results in a solution of the centralized optimization problem **P** - **P.d** and satisfies the informational constraints **P.e** - **P.f**.

Unicast service provisioning has received significant attention. Most of the results on decentralized resource allocation in unicast service provisioning, currently available in the literature are based on pricing mechanisms [7–10, 16, 17, 19, 20, 23, 25, 26, 32, 33, 35, 36, 44, 46]. These publications have addressed, either by analysis [8–10, 16, 19, 20, 23, 25, 33, 44, 46], or simulation and analysis [7, 10, 32, 35, 36], issues associated with unicast service provisioning, but have not investigated the informational efficiency of the proposed mechanisms.

²The social welfare function in this work is characterized by the sum of individual user utility functions.

B. Market Mechanism

We present: (i) a market economy consisting of a service provider, users, and an auctioneer; and (ii) a tâtonnement process that describes how the market economy works.

1) *Description of the market:* In our market, for conceptual clarity, we assume that the network consists of a service provider and an auctioneer. Under this assumption, the economy consists of the following three types of agents: a service provider, users and an auctioneer. The auctioneer sets the prices per unit of rate at each link. We denote the price for the unit of rate on link $l \in \mathbf{L}$ by λ_l . The service provider and the users are price takers. They act as if their behavior has no effect on the equilibrium prices reached by the market allocation process. The service provider uses the network's resources and the prices λ_l , specified by the auctioneer, to set up services and the corresponding prices for each unit of these services. Then, it announces the price per unit of service to the users. Based on the announced price, each user decides on the amount of service it should request.

The price taking assumption and the fact that we try to maximize the sum of the users' utilities imply that: (i) the service providers will not attempt to make a profit; (ii) the service prices are directly derived from the resource prices.

Below we describe each type of agent in more detail.

Service provider:

The users request service from the service provider. Each user's request is described by the origin and the destination of the service. For each $i \in \mathbf{N}$ denote by \mathbf{V}_i the set of links forming the route on which i 's service is delivered.

Since the service provider is not a profit maker, it computes the price per unit of service according to the following formula:

$$p^i(\lambda) = \sum_{l \in \mathbf{V}_i} \lambda_l. \quad (\text{III.1})$$

Users:

Users request one way connections from the service provider. Each user's demand is determined by

$$x^i(p^i(\lambda)) = \operatorname{argmax}_{x^i} \left[u_i(x^i) - x^i p^i(\lambda) \right]. \quad (\text{III.2})$$

Auctioneer:

The role of the auctioneer is to regulate the prices of the resources. He does this based on the aggregate excess demand vector $z(\lambda)$:

$$z_l(\lambda) \triangleq \sum_{i \in \mathbf{N}} x^i(p^i(\lambda)) r_l^i(\lambda) - c_l. \quad (\text{III.3})$$

2) *The tâtonnement process:* We present a tâtonnement process, specified by an algorithm, called **Algorithm 1**, that describes how the market works. The algorithm proceeds iteratively as follows:

Step 1: The auctioneer announces prices $\lambda := (\lambda_l, l \in \mathbf{L})$ for the resource at each node of the networks.

Step 2: Based on the auctioneer's announcement, the service provider computes for each user $i \in \{1, 2, \dots, N\}$ the price per unit of service $p^i(\lambda)$ according to (III.1). Then, the

service provider announces to each user i the service price $p^i(\lambda)$.

Step 3: Based on the prices $p(\lambda) := (p^i(\lambda), i = 1, 2, \dots, N)$ announced by the service provider, each user i requests an amount of service $x^i(p^i(\lambda))$, $i = 1, 2, \dots, N$, satisfying (III.2).

Step 4: Based on the service demand vector $x(p(\lambda)) := (x^i(p^i(\lambda)), i = 1, 2, \dots, N)$ the auctioneer computes the excess demand vector $z(\lambda) := (z_l(\lambda), l \in \mathbf{L})$ using (III.3).

Step 5: If $z(\lambda) \leq 0$ the process ends. Otherwise the auctioneer changes the prices λ of resources according to a specific mechanism based on Scarf's algorithm which is described in detail in [41, 42, 45], announces new prices, say λ' , and the process is repeated from Step 2 on.

C. Embedding into Hurwicz's framework

We embed the unicast routing problem, formulated in Section III-A, within the framework of Hurwicz's model described in Section II.

1) *The resource allocation problem:* As presented in Section II, a resource allocation problem can be described by the following triple: environment, action space, and goal correspondence.

Environment:

"Characteristics" of a particular agent i , say e^i , is called the *local environment* of i . The set of all possible environments of i is denoted by E^i . The (*system*) *environment* is a tuple consisting of the local environments of all agents and is denoted by e . The set of possible system environments is denoted by $E := (\otimes_{i \in \mathbf{N}} E^i) \otimes E^{N+1}$.

For the network Problem **P** the local environment E^i of each user i is the set of differentiable strictly concave functions on \mathbb{R}_+ . The local environment of the network is the set $E^{N+1} \triangleq \{\{\mathbf{L}\} \times \{\mathbf{V}\} \times \{\mathbf{c}_L\}\}$, where $\mathbf{V} \triangleq \otimes_{i \in \mathbf{N}} \mathbf{V}_i$. *Action Space:*

The set of possible actions taken by the system is called the *action space* of the system, and is denoted by \mathbf{A} . For Problem **P** the action space is the feasible region of **P**.

Goal Correspondence:

The relation between the environment and the (desired) actions of the system is represented by a point-to-set map, called the *goal correspondence / social choice rule / social welfare maximizing rule*, and is denoted by π . For Problem **P**, $\pi : E \rightarrow \mathbf{A}$ is defined as follows:

$$\pi(e) := \operatorname{argmax} P(e), \quad \forall e \in E \quad (\text{III.4})$$

2) *Mechanism specification:* A mechanism in equilibrium correspondence form is characterized by the triple (\mathcal{M}, μ, h) , where \mathcal{M} is the message space, μ is the equilibrium correspondence, and h is the outcome function.

Message Space:

The set of messages chosen for communication by the designer is called the *message space*, and is denoted by \mathcal{M} . The *size of a finite dimension message space* \mathcal{M} is defined to be the dimension [22] of the smallest real vector space in which there is an open set W such that $\mathcal{M} \subseteq W$. The size of \mathcal{M} is denoted by $\dim \mathcal{M}$.

In the allocation mechanism described above (see [42] for details), two types of messages are exchanged among agents:

- The price p^i per unit of service is communicated by the network to the user i , $i = 1, 2, \dots, N$.
- The demand x^i , $i = 1, 2, \dots, N$, is communicated by user i to the network.

Thus, the message space for Problem **P** has dimension equal to $2 \times N$.

Equilibrium Correspondence:

The relation between the environment and a mechanism's equilibrium messages is represented by a point-to-set map, called *equilibrium correspondence*, and denoted by $\mu: E \rightarrow \mathcal{M}$. The *individual equilibrium correspondence* of participant i , denoted by $\mu^i: E^i \rightarrow \mathcal{M}$, represents a relationship between the local environment of i and the terminal messages emitted by i .

To capture the private nature of the initial distribution of information, we require the following *privacy-preserving* property to be satisfied:

$$\mu(e) = \cap_i \mu^i(e^i). \quad (\text{III.5})$$

To determine the equilibrium correspondence for the first N agents (the users) we define the Lagrangian function $\Lambda(x, \lambda)$ (see [5]) of Problem **P**:

$$\Lambda(x, \lambda) := \sum_{i \in \mathbf{N}} u_i(x^i) + \lambda_l (c_l - \sum_{i \in \mathbf{N}} x_j^i r_l^i) \quad (\text{III.6})$$

with λ_l being the Lagrangian multiplier corresponding to the l^{th} link, and $x := (x^1, x^2, \dots, x^N)$.

Using the first-order optimality conditions, we define the equilibrium correspondence of the first N agents (the users) as follows:

$$\mu^i(u_i(x^i)) := \{(p, x) \in \mathcal{M} \mid \begin{aligned} &\frac{\partial}{\partial x^i} u_i(x^i) - p_i \leq 0; \\ &x^i \left(\frac{\partial}{\partial x^i} u_i(x^i) - p_i \right) = 0 \} \quad (\text{III.7}) \end{aligned}$$

where $p := (p^1, p^2, \dots, p^N)$.

From equation (III.6) and the Karush-Kuhn-Tucker (KKT) conditions, the equilibrium correspondence for agent $N + 1$ (i.e. the network) is:

$$\begin{aligned} \mu^{N+1}(\mathbf{L} \times \mathbf{V} \times c_{\mathbf{L}}) := & \{(p, x) \in \mathcal{M} \mid \sum_{i \in \mathbf{N}} x^i r_l^i \leq c_l, \\ & \lambda_l (c_l - \sum_{i \in \mathbf{N}} x^i r_l^i) = 0, \\ & p_i = \sum_{l \in \mathbf{V}_i} \lambda_l \}. \quad (\text{III.8}) \end{aligned}$$

Outcome Function:

A function which translates messages into actions is called an *outcome function*, and it is denoted by $h: \mathcal{M} \rightarrow \mathbf{A}$. In Problem **P** the outcome function is $h(p, x) := x$.

D. Informational Efficiency of the Pricing Mechanism of Section III-B

The main result of this section is:

Theorem 3.1: The pricing mechanism of Section III-B is informationally efficient for the class of environments described by Problem **P**.

Proof: To establish the assertion of the theorem we must prove that the pricing mechanism of Section III-B is goal realizing, regular, and has a message space of minimum dimension among all goal realizing and regular mechanisms for Problem **P**.

Minimality of dimension

Lemma 3.1: For the network Problem **P** any goal realizing and regular mechanism must have a message space of dimension at least $2 \times N$.

Proof: See [43]. ■

The goal-realizing property

In [42, 45] the authors show that the pricing mechanism proposed in Section III-B satisfies requirements **R1-R3** (see Section II). Since for any environment $e \in \mathbf{E}$, equations (III.7) and (III.8) guarantee that $(p, x) \in \mu(e)$ satisfies the KKT optimality conditions of problem **P**, requirements **R4** and **R5** (see Section II) are also satisfied. This shows that the pricing mechanism proposed in Section III-B is goal realizing for Problem **P**.

Regularity

By construction, the pricing mechanism of Section III-B is privacy preserving. Thus, to establish regularity we must prove that the mechanism is spot threaded, that is, it has an equilibrium correspondence μ and an outcome correspondence h that are spot threaded on their entire domain.

Proof of spot-threadedness. First we note that the outcome function h is spot threaded by construction.

Lemma 3.2: For Problem **P** the pricing mechanism (\mathcal{M}, μ, h) has a μ which is spot threaded.

Proof: See [43]. ■

Lemma 3.2 along with the spot threadedness of h and the privacy preserving property of the proposed mechanism establish the regularity of the mechanism.

The proof of the assertion of Theorem 3.1 is now complete. ■

IV. DISCUSSION

In this paper we have established that pricing mechanisms for rate allocation in networks with unicast service provisioning (Problem **P**) are informationally efficient. In [41] the author also shows that pricing mechanisms are informationally efficient in rate allocation network problems for multirate multicast. Now we present two instances of network problems for which there exists no informationally efficient mechanisms.

I: When the goal correspondence π is not spot threaded there exists no informationally efficient mechanism for the corresponding resource allocation problem. In particular there exists no informationally efficient mechanism for rate allocation in unicast service provisioning with routing.

II: If the utility functions do not belong to class \mathcal{D}_M (cf. Definition 3.1) there exists no informationally efficient mechanism for the corresponding resource allocation problem.

A detailed discussion of the above instances of problems appears in [43].

V. SUMMARY

We used the formalism and methodologies of realization theory to derive a new result on the informational efficiency of the pricing mechanism for unicast service provisioning with fixed routing, a class of resource allocation problems that arise in communication networks. We demonstrated that pricing mechanisms are not informationally efficient for certain instances of decentralized network resource allocation problems including unicast service provisioning with routing.

REFERENCES

- [1] T. Apostol. *Mathematical Analysis*. Addison-Wesley, 1974.
- [2] K. Arrow. A difficulty in the concept of social welfare. *Journal of Political Economy*, 58:328–346, 1950.
- [3] K. Arrow. *Social Choice and Individual Values*. Yale University Press, 1951.
- [4] K. Arrow and F. Hann. *General Competitive Analysis*. North Holland Elsevier Science Publishers, 1991.
- [5] M. Bazaraa, H. Sherali, and C. Shetty. *Nonlinear Programming Theory and Algorithms*. John Wiley, New York, 1993.
- [6] A. Bergson. A reformulation of certain aspects of welfare economics. *Quarterly Journal of Economics*, 52:310–334, 1938.
- [7] R. Cocchi, D. Estrin, S. Shenker, and L. Zhang. Pricing in computer networks: motivation, formulation, and example. *IEEE/ACM Transactions on Networking*, 1(6):614–627, Dec 1993.
- [8] C. Courcoubetis, F. Kelly, and R. Weber. Measurement-based usage charges in communications networks. *Operation Research*, 48(4):535–548, 2000.
- [9] G. de Veciana and R. Baldick. Resource allocation in multiservice networks via pricing. *Computer Networks and ISDN Systems*, 30:951–962, 1998.
- [10] A. Gupta, D. Stahl, and A. Whinston. A stochastic equilibrium model of internet pricing. *Journal of Economic Dynamics and Control*, 21:697–672, 1997.
- [11] L. Hurwicz. On informationally decentralized systems. In B. McGuire and R. Radner, editors, *Decision and Organization*, Volume in Honor of Jacob Marschak, pages 297 – 336. North Holland, 1972.
- [12] L. Hurwicz. The design of mechanisms for resource allocation. *American Economic Review*, 63(2):1–30, 1973.
- [13] L. Hurwicz. On the dimensional requirements of informationally decentralized Pareto satisfactory processes. *Studies in Resource Allocation Processes*, 1977.
- [14] L. Hurwicz. On informational decentralization and efficiency in resource allocation mechanisms. In S. Reiter, editor, *MAA Studies in Mathematical Economics*, volume 25, pages 238–350. Mathematical Association of America, 1986.
- [15] L. Hurwicz, S. Reiter, and D. Saari. On constructing mechanisms with message spaces of minimal dimension for smooth performance functions. mimeo. Northwestern University, 1985.
- [16] H. Jiang and S. Jordan. The role of price in the connection establishment process. *European Transactions on Telecommunications*, 6(4):421–429, July-Aug. 1995.
- [17] S. Jordan and H. Jiang. Connection establishment in high speed networks. *IEEE Selected Areas in Communications*, 13(7):1150–1161, 1995.
- [18] H. Kanemitsu. Informational efficiency and decentralization in optimal resource allocation. *The Economic Studies Quarterly*, 16:22–40, 1966.
- [19] F. Kelly. On tariffs, policing and admission control for multiservice networks. *Operations Research Letters*, 15:1–9, 1994.
- [20] F. Kelly, A. Maulloo, and D. Tan. Rate control for communication networks: shadow prices, proportional fairness and stability. *Operational Research Society*, 49:237–252, 1998.
- [21] T. Koopmans. Analysis of production as an efficient combination of activities. In T. Koopmans, editor, *Activity Analysis of Production and Allocation*, number 13, pages 33–37. Cowles Commission Monograph, New York, 1951.
- [22] S. Lang. *Linear Algebra*. Springer-Verlag, New York, 1991.
- [23] S. Low and P. Varaiya. A new approach to service provisioning in ATM networks. *IEEE/ACM Transactions on Networking*, 1:547–553, 1993. See also [24] for corrections.
- [24] S. Low and P. Varaiya. Corrections to: A new approach to service provisioning in ATM networks. *IEEE/ACM Transactions on Networking*, 2:312, 1994.
- [25] J. MacKie-Mason and H. Varian. Pricing congestible network resources. *Journal of Selected Areas in Communications*, 13(7):1141–1149, 1995.
- [26] J. MacKie-Mason and H. Varian. Some FAQs about usage-based pricing. *Computer Networks and ISDN Systems*, 28:257–265, 1995.
- [27] T. Marschak and S. Reichelstein. *The economics of informational decentralization: Complexity, Efficiency, and Stability*, chapter Communication Requirements for Individual Agents in Networks and Hierarchies, pages 311–346. Kluwer Academic Publishers, 1995.
- [28] T. Marschak and S. Reichelstein. Network mechanisms, informational efficiency, and hierarchies. *Journal of Economic Theory*, 79:106–141, 1998.
- [29] A. Mas-Colell, M. Whinston, and J. Green. *Microeconomic Theory*. Oxford University Press, New York, 1995.
- [30] K. Mount and S. Reiter. The informational size of message spaces. *Journal of Economic Theory*, 8:161–192, 1974.
- [31] K. Mount and S. Reiter. Economic environments for which there are Pareto satisfactory mechanisms. *Econometrica*, 45(4):821–842, 1977.
- [32] L. Murphy and J. Murphy. Bandwidth allocation by pricing in ATM networks. In *Proceedings of the IFIP Broadband Communications*, pages 333 – 351, 1994.
- [33] L. Murphy, J. Murphy, and E. Posner. Distributed pricing for embedded ATM networks. In *Proceedings of the International Teletraffic Congress ITC-14*, 1994.
- [34] V. Pareto. The new theories of economics. *Journal of Political Economics*, 5:485–502, 1896.
- [35] C. Parris and D. Ferrari. A resource based pricing policy for real-time channels in a packet-switching network. Technical Report TR-92-018, International Computer Science Institute, Berkeley, CA, 1992.
- [36] C. Parris, S. Keshav, and D. Ferrari. A framework for the study of pricing in integrated networks. Technical Report TR-92-016, International Computer Science Institute, Berkeley, CA, 1992.
- [37] S. Reiter. Informational efficiency of iterative processes and the size of message spaces. Discussion Paper 11, Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1972.
- [38] S. Reiter. The knowledge revealed by an allocation process and the informational size of the message space. Discussion Paper 6, Center for Mathematical Studies in Economics and Management Science, Northwestern University, 1972.
- [39] S. Reiter. Information and performance in the (new) welfare economics. *The American Economic Review*, 67(1):226–234, 1977. Papers and Proceedings of the Eighty-ninth Annual Meeting of the American Economic Association.
- [40] P. Samuelson. *Foundations of Economic Analysis*. 1947.
- [41] T. Stoenescu. *Decentralized Resource Allocation in Networks*. PhD thesis, University of Michigan, 2004.
- [42] T. Stoenescu and D. Teneketzis. A pricing methodology for resource allocation and routing in integrated-service networks with quality of service requirements. *Mathematical Methods of Operations Research (MMOR)*, 56(2), 2002.
- [43] T. Stoenescu and D. Teneketzis. Informational efficiency of pricing mechanisms in unicast service provisioning. CGR 05-03, EECS Department, University of Michigan, 2005.
- [44] P. Thomas and D. Teneketzis. An approach to service provisioning with quality of service requirements in ATM Networks. *Journal of High Speed Networks*, 6(4):263–291, 1997.
- [45] P. Thomas, D. Teneketzis, and J. MacKie-Mason. A market-based approach to optimal resource allocation in integrated-services connection-oriented networks. *Operations Research*, 50(5):603–616, 2002.
- [46] Q. Wang, J. Peha, and M. Sirbu. Optimal pricing for integrated-services networks. In L. W. McKnight and J. P. Bailey, editors, *Internet Economics*, pages 353–376. MIT Press, 3 edition, 1997.