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Adaptive Control of Nonlinear Systems with Uncertain Dead-zone Nonlinearity

Jing Zhou[a], Meng Joo Er[b] and Changyun Wen[c]

Abstract— **In this paper, we present a new scheme to design an adaptive controller for uncertain nonlinear systems with unknown dead-zone nonlinearity. The control design is achieved by introducing smooth approximate dead-zone model and certain well defined functions and by using backstepping technique. For the design and implementation of the controller, no knowledge is assumed on the uncertain system parameters and dead-zone parameters. It is shown that the proposed controller not only can guarantee global stability, but also can achieve excellent transient performance.**

Keywords: Adaptive control, backstepping, dead-zone, nonlinear systems, stability

I. INTRODUCTION

Adaptive control is becoming popular in many fields of engineering and science as concepts of adaptive systems are becoming more attractive in developing advanced applications. However, it still faces many important challenges, especially in nontraditional applications, such as nonsmooth nonlinearity (dead-zone, backlash, and hysteresis) [1]-[4]. The problems of controlling such systems are particularly important when the expected accuracy of the motion system is high. Several adaptive control schemes have recently been proposed, see for example [5]- [9].

Dead-zone, which can severely limit system performances, is one of the most important nonsmooth nonlinearities, such as valves, DC servo motors and other devices. In most practical motion systems, the dead-zone nonlinearity is poorly known and may vary with time. It often limits system performance. Therefore, the effect of dead-zone should be taken into consideration in the design and analysis of control systems. The study of adaptive control for systems with dead-zone nonlinearity at the input was initiated in [10], where an adaptive scheme was proposed with full state measurement. Intelligent control using neural networks to handle dead-zone was presented in [11], while fuzzy logic is used in [12]. An alternative approach based on sliding mode control was proposed in [13]. The model reference control was considered in [14]-[16], where the method for handling dead-zone is to construct an adaptive dead-zone inverse. The above approaches assume that the term multiplying the control and uncertain parameters of the system and actuator nonlinearity must be within known intervals. The inverse technique cannot be used in the recursive backstepping design because the backstepping technique requires system functions to be continuous differentiable. In [17]-[20], state feedback control was considered for a class of special nonlinear systems with dead-zone or hysteresis. In [17] and [18], it was assumed that the dead-zone slopes in positive and negative must be same, so that the dead-zone can be modelled as a combination of a line and a disturbance-like term. The characteristics of the dead-zone nonlinearity were not considered in the controller design, so the performance was not very good.

This paper will address a general class of nonlinear systems as in [21]. The dead-zone exists in each differential equation of the system. The existence of unknown dead-zone nonlinearities imposes a great challenge for the controller development. To address such a challenge, a new smooth dead-zone model is adopted and the controller is constructed based on the recursive backstepping design. In the dead-zone approximation, the new indicator function is designed, which is continuous and differentiable, while in [12] and [22] the indication function in the nonlinearity is non-smooth and cannot be used in the backstepping design. In this paper, we take the dead-zone into account in the controller design, instead of only considering its effect like bounded disturbances in [17] and [18]. The specific treatment of the dead-zone can bring more performance improvement. A new parametrization update law is proposed, which involves two sets of the parameters: one from the dead-zone nonlinearity and the other from the plants. Owing to the approximation of dead-zone nonlinearity, the new signal function and the backstepping technique, a priori knowledge on system parameters and dead-zone nonlinearities is no longer needed. Besides showing global stability of the system, the transient performance in terms of L_2 norm of the tracking error is derived to be an explicit function of design parameters and thus our scheme allows designers to obtain closed-loop behavior by tuning design parameters in an explicit way.

II. PROBLEM STATEMENT

Consider a class of nonlinear systems with unknown dead-zone nonlinearity:

$$\begin{aligned}
\dot{x}_1 &= x_2 + \psi_{01}(x_1) + \sum_{j=1}^{r} a_j Y_{1j}(x_1) + DZ_1(x_1) \\
\dot{x}_i &= x_{i+1} + \psi_{0i}(\bar{x}_i) + \sum_{j=1}^{r} a_j Y_{ij}(\bar{x}_i) + DZ_i(x_i) \\
& \quad i = 2, \ldots, n-1 \qquad (1) \\
\dot{x}_n &= bu + \psi_{0n}(x) + \sum_{j=1}^{r} a_j Y_{nj}(x) + DZ_n(x_n) \\
y &= x_1
\end{aligned}$$

[a,b]Intelligent Systems Center, Research TechnoPlaza, Nanyang Technological University, Singapore 637533
[c]School of EEE, Nanyang Technological University, Singapore 639798
[a] Corresponding Author. E-mail: zhoujing@pmail.ntu.edu.sg; Tel: +65-6790-6962; Fax: +65-6790-6961

where $\bar{x}_i = [x_1, \ldots, x_i]^T,\ x = [x_1, \ldots, x_n]^T \in R^n$, Y_{ij} and ψ_{0i}, $i = 1, \ldots, n, j = 1, \ldots, r$ are known continuous linear or nonlinear functions, parameters $a_i, i = 1, \ldots, r$ are unknown constants and the control gain b is unknown constant with known sign, $u(t)$ is the control input, and $y(t)$ is the output. The nonlinearity $DZ_i(x_i)$, $i = 1, \ldots, n$ is described as dead-zone characteristics.

The control objective is to design an output feedback control signal $u(t)$ for the plant (1) to ensure that all closed-loop signals are bounded and the plant output $y(t)$ tracks a given reference signal $y_r(t)$ under the following assumption:

Assumption 1: The reference signal $y_r(t)$ and its first nth derivatives are known and bounded.

III. Dead-zone Characteristic

Dead-zone, backlash, hysteresis and piecewise-linear characteristics are typical examples of the actuator nonlinearity. These nonlinear characteristics have break-points so that they are non-differentiable (nonsmooth) but can be parameterized as in [22]-[24].

The parameterized model of the dead-zone characteristic $DZ_i(.)$ can be unified as

$$DZ_i(x_i) = \begin{cases} m_{ri}(x_i - b_{ri}) & x_i \geq b_{ri} \\ 0 & b_{li} < x_i < b_{ri} \\ m_{li}(x_i - b_{li}) & v(t) \leq b_{li} \end{cases} \tag{2}$$

where $b_{ri} \geq 0, b_{li} \leq 0$ and $m_{ri} > 0, m_{li} > 0$ are constants and x_i is the input. In general, neither the break-points b_{ri}, b_{li} nor the slopes m_{ri}, m_{li} are equal. A graphical representation of the dead-zone is shown in the following Fig. 1.

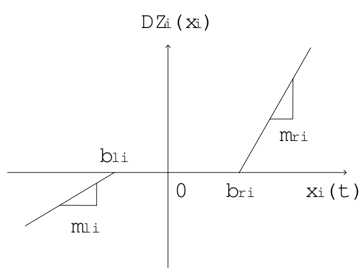

Fig. 1. Dead-zone model

To parameterize the dead-zone, we introduce the following new smooth continuous functions

$$\phi_{ri}(x_i) = \frac{e^{x_i/e_0}}{e^{x_i/e_0} + e^{-x_i/e_0}} \tag{3}$$

$$\phi_{li}(x_i) = \frac{e^{-x_i/e_0}}{e^{x_i/e_0} + e^{-x_i/e_0}} \tag{4}$$

where e_0 is any positive constant. As $e_0 \to 0$, ϕ_{ri} approaches a step transition from 0 at $x_i = -\infty$ to 1 at $x_i = +\infty$ continuously and ϕ_{li} approaches a step transition from 0 at $x_i = +\infty$ to 1 at $x_i = -\infty$ continuously.

Remark 1: Note that the functions ϕ_{ri} and ϕ_{li} are continuous and differentiable. Compared with the nonlinearity functions in [22]-[24], the inverse indicator functions are not differentiable (non-smooth) so they cannot be used in the recursive backstepping design which requires the function to be continuous differentiable.

The dead-zone (2) can be expressed as

$$\begin{aligned} DZ_i(x_i) &= \overline{DZ}_i(x_i) + d_i(t) && (5) \\ \overline{DZ}_i(x_i) &= m_{ri}(x_i - b_{ri})\phi_{ri}(x_i) + m_{li}(x_i - b_{li})\phi_{li}(x_i) \\ &= \theta_i^T \omega_i(x_i) && (6) \end{aligned}$$

where

$$\begin{aligned} \theta_i &= [m_{ri}, m_{ri}b_{ri}, m_{li}, m_{li}b_{li}]^T && (7) \\ \omega_i(t) &= [\phi_{ri}x_i, -\phi_{ri}, \phi_{li}x_i, -\phi_{li}]^T && (8) \end{aligned}$$

The bound of the error term $d_i(t)$ in (5) can be obtained as follows

$$d_i(t) = DZ_i - \theta_i^T \omega_i(x_i) \tag{9}$$

$$|d_i(t)| \leq \begin{cases} \frac{|m_{ri} - m_{li}|)e_0}{2e} + \frac{|m_{ri}b_{ri} - m_{li}b_{li}|}{e^{2b_{ri}/e_0} + 1} & x_i(t) \geq b_{ri} \\ max\{m_{ri}, m_{li}\}|b_{ri} - b_{li}| & b_{li} < x_i(t) < b_{ri} \\ \frac{|m_{ri} - m_{li}|)e_0}{2e} + \frac{|m_{ri}b_{ri} - m_{li}b_{li}|}{e^{-2b_{li}/e_0} + 1} & x_i(t) \leq b_{li} \end{cases} \tag{10}$$

where $|x_i|e^{-|x_i|} \leq e^{-1}$ was used. Note that in the section $[b_{li},\ b_{ri}]$ the bound of $d_i(t)$ decreases as e_0 increases, while outside this section, the bound of $d_i(t)$ increases as e_0 increases. It has the desired properties that $d_i(t)$ is bounded for all $t \geq 0$ and $d_i(t)$ approaches to 0 as $e_0 \to 0$. Figure 2 shows the approximation of the dead-zone.

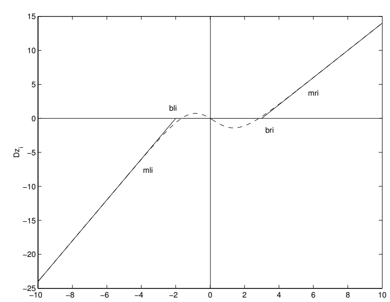

Fig. 2. dashed line: smooth approximate dead-zone $\overline{DZ}_i$; solid line: nonsmooth dead-zone DZ_i

Remark 2: Note that in [17] and [18], the dead-zone nonlinearity was treated as a combination of a straight line and a disturbance-like term and an assumption was needed that the dead-zone slopes in positive and negative must be same. Furthermore, the characteristics of the dead-zone, such as the parameters m_r, m_l, b_r, b_l, are not considered in the control design. In this paper, we do not need to assume that slopes are the same and we will take the dead-zone into account in the controller design, instead of only considering its effect like bounded disturbances in [17], [18]. The specific treatment of the nonsmooth nonlinearities can bring more performance improvement.

In order to develop a controller, we rewrite the plant (1) as shown

$$\begin{aligned}\dot{x}_i &= x_{i+1} + \psi_{0i}(\bar{x}_i) + a^T Y_i(\bar{x}_i) + \theta^T \Omega_i + e_i D(t) \\ & \quad i = 1, \ldots, n-1 \qquad (11) \\ \dot{x}_n &= bu + \psi_{0n}(x) + a^T Y_n(x) + \theta^T \Omega_n + e_n D(t) \\ y &= x_1\end{aligned}$$

where

$$\begin{aligned}Y_i &= [Y_{i1}, \ldots, Y_{ir}]^T, \ a = [a_1, \ldots, a_r]^T, \\ \theta &= [\theta_1^T, \ldots, \theta_n^T]^T, \ \Omega_i = [0, \ldots, 0, \omega_i^T, 0, \ldots, 0]^T, \\ D(t) &= [d_1, \ldots, d_n]^T, \ e_i = [0, \ldots, 0, 1, 0, \ldots, 0].\end{aligned}$$

Remark 3: Note that the known functions Ω_i $(i = 1, \ldots, n)$ are smooth and differentiable from the definitions (3) and (4). The parameters $m_{ri}, m_{li}, b_{ri}, b_{li}$ $(i = 1, \ldots, n)$ of the dead-zone nonlinearities will be estimated by using the update laws in the backstepping design. From (10), we know that the term $D(t)$ is bounded. Hence, in the control design, we will use the estimator to estimate the bound of $D(t)$ to cancel the effect of the dead-zone nonlinearities.

IV. Design of Adaptive Controllers

As usual, in the backstepping approach, the following change of coordinates is made

$$z_1 = y - y_r \qquad (12)$$

$$z_i = x_i - y_r^{(i-1)} - \alpha_{i-1}, \ i = 2, 3, \ldots, \rho \qquad (13)$$

where α_{i-1} is the virtual control at the *ith step* and will be determined in later discussions.
In order to avoid using the discontinuous sign function in the backstepping design which requires the function is differentiable, we firstly define a continuous sign function $sg_i(z_i)$ as follows

$$sg_i(z_i) = \begin{cases} \frac{z_i}{|z_i|} & |z_i| \geq \delta_i \\ \frac{z_i^{(2q+1)}}{(\delta_i^2 - z_i^2)^{n-i+2} + |z_i^{(2q+1)}|} & |z_i| < \delta_i \end{cases}$$

where $\delta_i (i = 1, \ldots, n)$ is a positive design parameter, and $q = round\{(n-i+2)/2\}$, where $round\{x\}$ means the element of x to the nearest integer. Clearly $2q+1 \geq (n-i+2)$. It can be shown that $sg_i(z_i)$ is $(n-i+1)th$-order differentiable. We also design a function $f_i(z_i)$ as follows

$$f_i(z_i) = \begin{cases} 1 & |z_i| \geq \delta_i \\ 0 & |z_i| < \delta_i \end{cases} \qquad (14)$$

As a consequence, we have

$$sg_i(z_i) f_i(z_i) = \begin{cases} 1 & z_i \geq \delta_i \\ 0 & |z_i| < \delta_i \\ -1 & z_i \leq \delta_i \end{cases} \qquad (15)$$

To illustrate the backstepping procedures, only the first two steps and the last step of the design, i.e. *steps 1, 2 and* n below, are elaborated in details.

- *Step* 1: We start with the equation for the tracking error and z_1 is obtained from (11) and (12) as follows

$$\begin{aligned}\dot{z}_1 &= x_2 + \psi_{01}(x_1) + a^T Y_1(x_1) + \theta^T \Omega_1(x_1) \\ & \quad + e_1 D(t) - \dot{y}_r \\ &= z_2 + \alpha_1 + \psi_{01}(x_1) + a^T Y_1(x_1) + \theta^T \Omega_1 \\ & \quad + e_1 D(t) \qquad (16)\end{aligned}$$

Selecting the first virtual control law α_1 as

$$\begin{aligned}\alpha_1 &= -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1 - \psi_{01} - \hat{a}^T Y_1 \\ & \quad - \hat{\theta}^T \Omega_1 - \hat{D}_1 sg_1 - (\delta_2 + 1) sg_1 \qquad (17)\end{aligned}$$

where c_1 is a design positive parameter, $\hat{a}, \hat{\theta}$ and $\hat{D}_1$ are the estimates of a, θ and $\bar{D}$ respectively, $\bar{D}$ is the unknown bound of $D(t)$. Then, the choice results in the following system

$$\begin{aligned}\dot{z}_1 &= -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^n sg_1(z_1) + z_2 + \tilde{a}^T Y_1 \\ & \quad + \tilde{\theta}^T \Omega_1 + e_1 D(t) - \hat{D}_1 sg_1 - (\delta_2 + 1) sg_1 \quad (18)\end{aligned}$$

where $\tilde{a} = a - \hat{a}$ and $\tilde{\theta}_1 = \theta - \hat{\theta}$.
We define the following positive definite function

$$\begin{aligned}V_1 &= \frac{1}{n+1}(|z_1| - \delta_1)^{n+1} f_1 + \frac{1}{2}\tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \\ & \quad + \frac{1}{2}\tilde{a}^T \Gamma_a \tilde{a} + \frac{1}{2\gamma_{d1}} \tilde{D}_1^2 \qquad (19)\end{aligned}$$

where Γ and Γ_a are positive constant matrix, γ_{d1} is a positive constant, $\tilde{D}_1 = \bar{D} - \hat{D}_1$.We select an adaptive update law for $\hat{D}_1$ as follows:

$$\dot{\hat{D}}_1 = \gamma_{d1}(|z_1| - \delta_1)^n f_1 \qquad (20)$$

Then from (18) to (20), we obtain the time derivative of V_1 as follows:

$$\begin{aligned}\dot{V}_1 &= -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^{2n} f_1 + \tilde{a}^T(\tau_{a1} - \Gamma_a^{-1}\dot{\hat{a}}) \\ & \quad + \tilde{\theta}_1^T(\tau_{\theta 1} - \Gamma^{-1}\dot{\hat{\theta}}_1) + z_2(|z_1| - \delta_1)^n f_1 sg_1 \\ & \quad - (\delta_2 + 1)(|z_1| - \delta_1)^n f_1 \\ &\leq -(c_1 + \frac{1}{4})(|z_1| - \delta_1)^{2n} f_1 + \tilde{a}^T(\tau_{a1} - \Gamma_a^{-1}\dot{\hat{a}}) \\ & \quad + \tilde{\theta}_1^T(\tau_{\theta 1} - \Gamma^{-1}\dot{\hat{\theta}}_1) + (|z_2| - \delta_2 - 1)(|z_1| - \delta_1)^n f_1 \\ \tau_{a1} &= Y_1(|z_1| - \delta_1)^n f_1 sg_1 \qquad (21) \\ \tau_{\theta 1} &= \Omega_1(|z_1| - \delta_1)^n f_1 sg_1 \qquad (22)\end{aligned}$$

- *Step* 2: Using ((5), 11) and (13), we have

$$\begin{aligned}\dot{z}_2 &= \dot{x}_2 - \dot{\alpha}_1 - \ddot{y}_r \\ &= z_3 + \alpha_2 + \psi_{02}(\bar{x}_2) + a^T Y_2(\bar{x}_2) + \theta^T \Omega_2(x_2) \\ & \quad + e_2 D(t) - \frac{\partial \alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial \alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial \alpha_1}{\partial \hat{a}}\dot{\hat{a}} \\ & \quad - \frac{\partial \alpha_1}{\partial \hat{D}_1}\dot{\hat{D}}_1 - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} \\ &= z_3 + \alpha_2 + \beta_2 + a^T Y_2(\bar{x}_2) + \theta^T \Omega_2(x_2) + e_2 D(t) \\ & \quad - \frac{\partial \alpha_1}{\partial x_1}(a^T Y_1 + \theta^T \Omega_1 + e_1 D) - \frac{\partial \alpha_1}{\partial \hat{a}}\dot{\hat{a}} - \frac{\partial \alpha_1}{\partial \hat{\theta}}\dot{\hat{\theta}} \quad (23)\end{aligned}$$

$$\beta_2 = \psi_{02} - \frac{\partial\alpha_1}{\partial x_1}(x_2+\psi_{01}) - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\hat{D}_1}\dot{\hat{D}}_1 \quad (24)$$

We choose the virtual control law for α_2 as follows:

$$\begin{aligned}\alpha_2 &= -(c_2+\frac{5}{4})(|z_2|-\delta_2)^{n-1}sg_2 - \beta_2 - (\delta_3+1)sg_2 \\ &- \hat{a}^T(Y_2 - \frac{\partial\alpha_1}{\partial x_1}Y_1) - \hat{\theta}^T(\Omega_2 - \frac{\partial\alpha_1}{\partial x_1}\Omega_1) + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_{\theta 2} \\ &- (1+\sqrt{\| \frac{\partial\alpha_1}{\partial x_1}\|^2+\delta_0})\cdot\hat{D}_2 sg_2 + \frac{\partial\alpha_1}{\partial\hat{a}}\Gamma_a\tau_{a2} \quad (25)\end{aligned}$$

$$\dot{\hat{D}}_2 = \gamma_{d2}\big(1+\sqrt{\| \frac{\partial\alpha_1}{\partial y}\|^2+\delta_0}\ \big)\cdot(|z_2|-\delta_2)^{n-1}f_2 \quad (26)$$

$$\tau_{a2} = \tau_{a1} + (Y_2 - \frac{\partial\alpha_1}{\partial x_1}Y_1)(|z_2|-\delta_2)^{n-1}f_2 sg_2 \quad (27)$$

$$\tau_{\theta 2} = \tau_{\theta 1} + (\Omega_2 - \frac{\partial\alpha_1}{\partial x_1}\Omega_1)(|z_2|-\delta_2)^{n-1}f_2 sg_2 \quad (28)$$

where δ_0 is a small positive constant, c_2 and γ_{d2} are positive constants, $\hat{D}_2$ is an estimate of $\bar{D}$. Defining the positive Lyapunov function as

$$V_2 = V_1 + \frac{1}{n}(|z_2|-\delta_2)^n f_2 + \frac{1}{2\gamma_{d2}}\tilde{D}_2^2 \quad (29)$$

where $\tilde{D}_2 = \bar{D} - \hat{D}_2$. Then the derivative of V_2 is

$$\begin{aligned}\dot{V}_2 \le& -\sum_{i=1}^{2} c_i(|z_i|-\delta_i)^{2(n-i+1)}f_i + \tilde{a}^T(\tau_{a2} - \Gamma_a^{-1}\dot{\hat{a}}) \\ &+\theta^T(\tau_{\theta 2} - \Gamma^{-1}\dot{\hat{\theta}}) + \big[\frac{\partial\alpha_1}{\partial\hat{a}}(\Gamma_a\tau_{a2} - \dot{\hat{a}}) \\ &+\frac{\partial\alpha_1}{\partial\hat{\theta}}(\Gamma\tau_{\theta 2} - \dot{\hat{\theta}})\big](|z_2|-\delta_2)^{n-1}f_2 sg_2 \\ &+(|z_2|-\delta_2)^{n-1}(|z_3|-\delta_3-1)f_2 \\ &-\frac{1}{4}(|z_2|-\delta_2)^{2(n-1)}f_2 + M_2 \quad (30)\end{aligned}$$

where $M_2 = -\frac{1}{4}(|z_1|-\delta_1)^{2n}f_1 + (|z_2|-\delta_2-1)(|z_1|-\delta_1)^n f_1 - (|z_2|-\delta_2)^{2(n-1)}f_2$. Now, we show that $M_2 < 0$. It is clear that $M_2 \le 0$ for $|z_2| < \delta_2+1$. For $|z_2| \ge \delta_2+1$

$$\begin{aligned}M_2 \le& -\frac{1}{4}(|z_1|-\delta_1)^{2n}f_1 + \frac{1}{4}(|z_1|-\delta_1)^{2n}f_1^2 \\ &+(|z_2|-\delta_2-1)^2 - (|z_2|-\delta_2)^{2(n-1)} \\ <& (|z_2|-\delta_2)^2 - (|z_2|-\delta_2)^{2(n-1)} \\ =& (|z_2|-\delta_2)^2(1-(|z_2|-\delta_2)^{2(n-2)}) \\ \le& 0 \quad (31)\end{aligned}$$

- *Step* i, $i = 3, \ \ldots, \ n$: Choose

$$\begin{aligned}\alpha_i =& -(c_i+\frac{5}{4})(|z_i|-\delta_i)^{n-i+1}sg_i - (\delta_{i+1}+1)sg_i \\ &- \hat{a}^T Y_i - \hat{\theta}^T\Omega_i - \hat{D}_i sg_i + \sum_{j=1}^{i-1}\big[\frac{\partial\alpha_{i-1}}{\partial x_j}(\hat{a}^T Y_j + \hat{\theta}^T\Omega_j) \\ &+ \sqrt{\| \frac{\partial\alpha_{i-1}}{\partial x_j}\|^2+\delta_0}\cdot\hat{D}_i sg_i\big] + \frac{\partial\alpha_{i-1}}{\partial\hat{a}}\Gamma_a\tau_{ai} - \beta_i \\ &+ \sum_{k=1}^{i-2}(|z_k|-\delta_k)^{n-k+1}f_k sg_k\big[-\frac{\partial\alpha_{k-1}}{\partial\hat{a}}\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}Y_j \\ &- \frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Omega_j\big] + \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\Gamma\tau_{\theta i} \quad (32)\end{aligned}$$

where $\tilde{D}_i = \bar{D} - \hat{D}_i, \ i = 3, \ldots, n$. The update laws are designed as

$$\dot{\hat{D}}_i = \gamma_{di}\big(1+\sqrt{\| \frac{\partial\alpha_{i-1}}{\partial y}\|^2+\delta_0}\ \big)(|z_i|-\delta_i)^{n-i+1}f_i \quad (33)$$

$$\tau_{ai} = \tau_{ai-1} + \big(Y_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}Y_j\big)(|z_i|-\delta_i)^{n-i+1}f_i sg_i$$

$$\tau_{\theta i} = \tau_{\theta i-1} + \big(\Omega_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\Omega_j\big)(|z_i|-\delta_i)^{n-i+1}f_i sg_i$$

$$V_i = V_{i-1} + \frac{1}{n-i+2}(|z_i|-\delta_i)^{n-i+2}f_i + \frac{1}{2\gamma_{di}}\tilde{D}_i^2 \quad (34)$$

where β_i contains all known terms and c_i and $\gamma_{di}, i = 3, \ldots, n$ are positive constants.

In the last *step* n: The derivative of $z_n = x_n - y_r^{(n-1)} - \alpha_{n-1}$ is given by

$$\begin{aligned}\dot{z}_n =& bu + \psi_{0n} + a^T Y_n + \theta^T\Omega_n + e_n D + \beta_n \\ &-\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(a^T Y_j + \theta^T\Omega_j + e_j D_j) \\ &-\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \frac{\partial\alpha_{n-1}}{\partial\hat{a}}\dot{\hat{a}} - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{D}_j}\dot{\hat{D}}_j \quad (35)\end{aligned}$$

where β_n contains all known terms. Defining the positive definite Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}(|z_n|-\delta_n)^2 f_n + \frac{1}{2\gamma_{dn}}\tilde{D}_n^2 + \frac{|b|}{2\gamma_e}\tilde{e}^2 \quad (36)$$

where γ_e is a positive constant, $\hat{e}$ is the estimate of $e = 1/b$, and $\tilde{e} = e - \hat{e}$. We choose the update laws for $\hat{a}$, $\hat{D}_n$, $\hat{\theta}$ and $\hat{e}$ as follows:

$$\dot{\hat{a}} = \Gamma_a\tau_{an} \quad (37)$$

$$\dot{\hat{\theta}} = \Gamma\tau_{\theta n} \quad (38)$$

$$\dot{\hat{D}}_n = \gamma_{dn}\big(1+\sqrt{\| \frac{\partial\alpha_{n-1}}{\partial y}\|^2+\delta_0}\big)(|z_n|-\delta_n)f_n \quad (39)$$

$$\dot{\hat{e}} = -\gamma_e sgn(b)\alpha_n \quad (40)$$

The control law is designed as

$$u = \hat{e}\alpha_n \quad (41)$$

With this choice, the derivative of V_n becomes

$$\dot{V}_n \le -\sum_{i=1}^{n} c_i(|z_i|-\delta_i)^{2(n-i+1)}f_i + \tilde{a}^T(\tau_{an} - \Gamma_a^{-1}\dot{\hat{a}})$$

$$
\begin{aligned}
&+\tilde{\theta}^T(\tau_{\theta n}+\Gamma^{-1}\dot{\hat{\theta}})-\frac{|b|}{\gamma_e}\tilde{e}(\dot{\hat{e}}+\gamma_e sgn(b)\alpha_n)\\
&+\sum_{k=2}^{n}(|z_k|-\delta_k)^{n-k+1}f_k\big[\frac{\partial\alpha_{k-1}}{\partial\hat{a}}(\Gamma_a\tau_{an}-\dot{\hat{a}})\\
&+\frac{\partial\alpha_{k-1}}{\partial\theta}(\tau_{\theta n}-\dot{\hat{\theta}})\big]\\
= \quad & -\sum_{i=1}^{n}c_i(|z_i|-\delta_i)^{2(n-i+1)}f_i \qquad (42)
\end{aligned}
$$

Therefore boundedness of all signals in the closed loop system is ensured as stated in the following theorem.

Theorem 1: The adaptive backstepping control design with the adaptive feedback law (6) and the parameter update laws (37)-(40), applied to the plant (1) with a dead-zone nonlinearity (2), ensures that all closed-loop signals are bounded.

- $z_1,\ldots,z_n,\hat{\theta},\hat{a},\hat{e},\hat{D}_i$ are all bounded.
- The tracking error approaches δ_1 asymptotically, i.e.,

$$
\lim_{t\to\infty}|y(t)-y_r(t)|=\delta_1 \qquad (43)
$$

- The transient tracking error performance is given by

$$
\begin{aligned}
&\parallel y(t)-y_r(t)\parallel_2\leq \delta_1+c_1^{-1/2n}\big(\frac{1}{2}\tilde{a}(0)^T\Gamma_a^{-1}\tilde{a}(0)\\
&+\frac{|b|}{2\gamma_e}\tilde{e}(0)^2+\frac{1}{2\Gamma}\tilde{\theta}(0)^2+\sum_{i=1}^{n}\frac{1}{2\gamma_{di}}\tilde{D}_i(0)^2\big)^{1/2n} \qquad (44)
\end{aligned}
$$

with $z_i(0)=\delta_i, i=1,\ldots,n,$

Proof: From (42), we establish that V is non-increasing. Hence, $z_i, i=1,\ldots,n, \hat{\theta},\hat{a},\hat{e},\hat{D}_i$ are bounded. By applying the LaSalle-Yoshizawa theorem in [25] to (42), it further follows that $z_i(t)\to 0, i=1,\ldots,n$ as $t\to\infty$, which implies that $\lim_{t\to\infty}|y(t)-y_r(t)|=\delta_1$.
From (42), we also have

$$
\begin{aligned}
\parallel |z_1|-\delta_1\parallel_2^{2n} &= \int_0^\infty(|z_1(\tau)|-\delta_1)^{2n}d\tau\\
&\leq \frac{1}{c_1}(V(0)-V(\infty))\\
&\leq \frac{1}{c_1}V(0) \qquad (45)
\end{aligned}
$$

Thus, by setting $z_i(0)=0, i=1,\ldots,n$, we obtain

$$
\begin{aligned}
V(0) &= \frac{1}{2}\tilde{a}(0)^T\Gamma_a^{-1}\tilde{a}(0)+\frac{1}{2}\tilde{\theta}(0)^T\Gamma^{-1}\tilde{\theta}(0)\\
&+\frac{|b|}{2\gamma_e}\tilde{e}(0)^2+\sum_{i=1}^{n}\frac{1}{2\gamma_{di}}\tilde{D}_i(0)^2 \qquad (46)
\end{aligned}
$$

a decreasing function of Γ_a,Γ,γ_e and γ_{di}, independent of c_1. This means that the bound resulting from (45) and (46) is

$$
\begin{aligned}
\parallel z_1\parallel_2 &\leq c_1^{-1/2n}\big(\frac{1}{2}\tilde{a}(0)^T\Gamma_a^{-1}\tilde{a}(0)+\frac{1}{2}\tilde{\theta}(0)^T\Gamma^{-1}\tilde{\theta}(0)\\
&+\frac{|b|}{2\gamma_e}\tilde{e}(0)^2+\sum_{i=1}^{n}\frac{1}{2\gamma_{di}}\tilde{D}_i(0)^2\big)^{1/2n} \qquad (47)
\end{aligned}
$$

△△△

Remark 4: From Theorem 1, the following conclusions can be obtained:

- The transient performance depends on the initial estimate errors $\tilde{\theta}(0)$, $\tilde{a}(0)$, $\tilde{e}(0)$, $\tilde{D}_i(0)$ and the explicit design parameters. The closer the initial estimates $\tilde{\theta}(0),\tilde{a}(0),\tilde{e}(0)$ and $\hat{D}_i(0)$ to the true values θ, a, e and D, the better the transient performance.
- The bound for $\parallel y(t)-y_r(t)\parallel_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains Γ_a,Γ,γ_e and γ_{di}.
- To improve the tracking error performance, we can also increase the gain c_1.

△△△

V. Simulation Studies

In this section, we illustrate the above methodology on a nonlinear uncertain system with unknown dead-zone, which is described as follows:

$$
\dot{x} = a\frac{1-e^{-x(t)}}{1+e^{-x(t)}}+bu(t)+DZ(x) \qquad (48)
$$

where u is the control input. The actual parameter values are $b=1$ and $a=1$, and the dead-zone parameter values are $m_r=1.05; m_l=1.05; b_r=0.3; b_l=-0.5$. The objective is to control the system state x to follow a desired trajectory $y_r(t)=8.5sin(2.5t)$.
In the simulations, we take $c_1=4,\Gamma_a=0.1,\gamma_{d1}=0.3,\gamma_e=0.2,\Gamma=[0.1,0.1,0.1,0.1]^T, e_0=1,\delta_1=0.02$ and the initial parameters are set as $\hat{e}(0)=\hat{D}(0)=0.3,\hat{\theta}(0)=[1,1,0.2,-0.3]^T$. The initial state is chosen as $x(0)=-0.2$. Fig. 3 and Fig. 4 show the tracking error with proposed scheme and scheme in [17]. Fig. 5 and Fig. 6 show the parameter estimate $\hat{a}$ and the control input $u(t)$. Clearly, the simulation results verify our theoretical design and demonstrate the effectiveness of our proposed scheme.

VI. Conclusion

This paper presents an adaptive backstepping design for a class of uncertain nonlinear systems with uncertain dead-zone nonlinearities. We propose a new approximation to model the non-smooth dead-zone nonlinearity and use a backstepping smooth control law to avoid possible chattering caused by the discontinuous function. The tracking error is ensured to approach a prescribed bound. It is the new parametrization that leads to parameterized systems and suitable for developing adaptive parameter update laws. Besides guaranteeing global stability, we also give an explicit bound on the L_2 performance of the tracking error in terms of design parameters.

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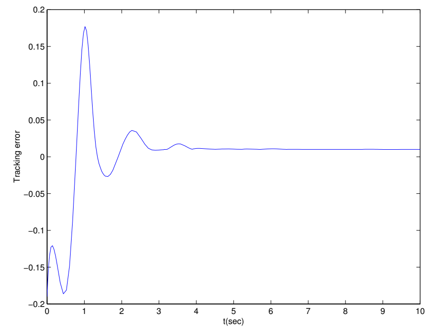

Fig. 3. Tracking error with proposed scheme

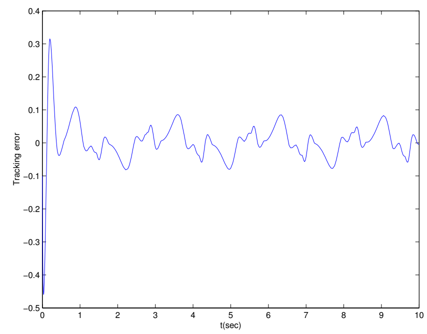

Fig. 4. Tracking error with scheme in [17]

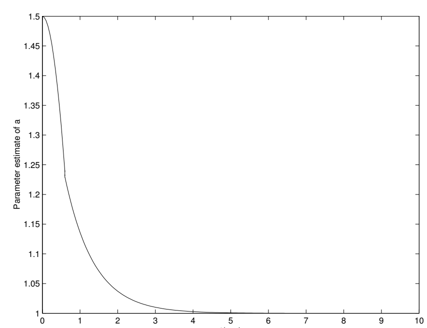

Fig. 5. Parameter estimate $\hat{a}$

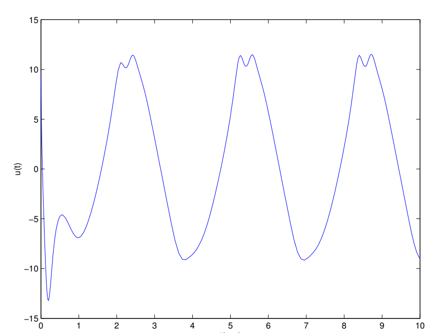

Fig. 6. Control signal $u(t)$