Proceedings of the
44th IEEE Conference on Decision and Control, and
the European Control Conference 2005
Seville, Spain, December 12-15, 2005

A Nested Noncooperative OSNR Game in Optical Links with Dynamic Gain Filters

Lacra Pavel

***Abstract*— This paper considers the problem of channel optical signal-to-noise ratio (OSNR) optimization and extends the Nash game formulation in [11] to a more general configuration. The model in [11] considers optically amplified links and a lumped end-to-end network model, whereby channel powers can be adjusted independently only at the transmitter sites. In this paper a more general case is considered where channel powers are also adjustable at intermediary dynamic sites, specific to optical networks. For this inherent distributed configuration a new nested Nash game is formulated towards maximizing channel OSNR at receiver. Existence and uniqueness of the NE solution is shown and a recursive procedure for constructing it is given. Based on this, an iterative algorithm that is distributed with respect to both channels and spans is proposed.**

I. INTRODUCTION

There has been recent interest in optical wavelength-division multiplexed (WDM) communication networks and their dynamic and performance aspects [1]-[2]. Important questions address how to realize reconfigurable networks, while at the same time maintaining stability [3]-[4], and optimal channel performance after reconfiguration. Channel performance depends on optical signal to noise ratio (OSNR), dispersion and nonlinear effects, [5]. Typically in link optimization, OSNR is considered the performance parameter, with dispersion and nonlinearity being kept low by proper link design, [6], [7]. The dominant impairment affecting OSNR is noise accumulation in chains of optical amplifiers and its spectral dependence, [6]. By adjusting channel input power at transmitter (Tx), channel OSNR at receiver (Rx) can be equalized. Some static approaches have been developed for single-link OSNR optimization, [8], [6]. For reconfigurable optical networks, where different channels can travel via different optical paths, on-line decentralized algorithms are required.

Recently, noncooperative game theory [24], [17] has been used as an alternative to traditional system-wide optimization [9], [10], for network optimization and control [14]-[16]. In large-scale networks control decisions are often made by users independently, each according to its own performance objective [13], [14], and noncooperative game theory [24], [17] is suitable framework. This is appropriate for large-scale optical networks also, where a centralized system for transmitting real-time information between all channels is difficult to maintain and cooperation among channels is impractical.

This work was supported by the Natural Sciences and Engineering Research Council of Canada

L. Pavel is with Department of Electrical and Computer Engineering, University of Toronto, Toronto, ON M5S 3G4, Canada pavel@control.toronto.edu

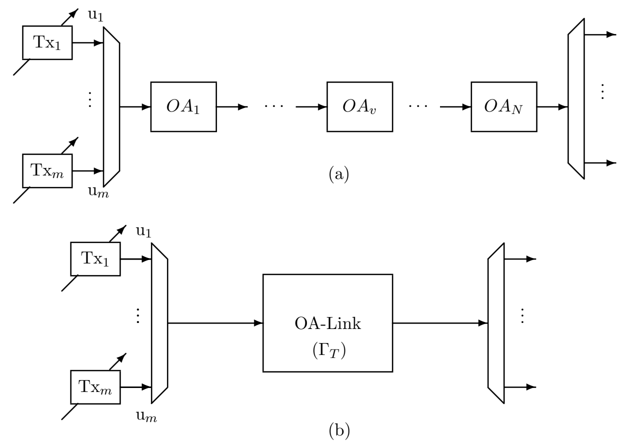

Fig. 1. Lumped OA-link model (Tx to Rx)

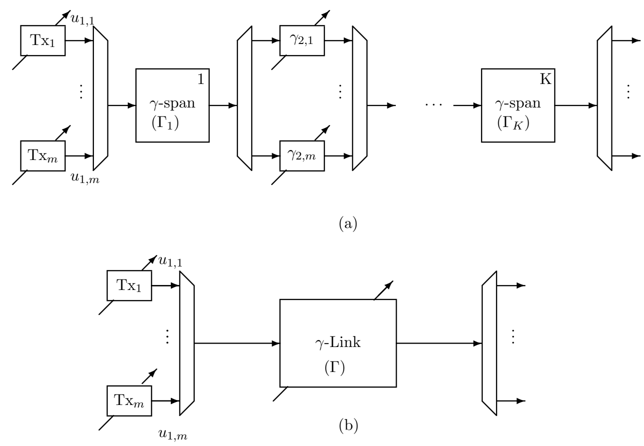

Fig. 2. Distributed γ-link model

This problem was considered recently in [11]-[12]. A general OSNR network model was developed as an end-to-end network model. The OSNR optimization problem was formulated as a centralized problem in [12], and as a noncooperative game between channels in [11]. Based on this game an iterative algorithm that is distributed with respect to channels was proposed. In a noncooperative Nash game players are self-interested, each player maximizing its own performance (utility) under the presence of all other players. The game settles at a Nash equilibrium (NE) if one exists, from which any player's deviation will result in degradation of other player's utility. The natural decoupling feature of a Nash game formulation lends itself to iterative algorithms that are decentralized with respect to players.

In this paper we extend the approach in [11] to a more gen-

0-7803-9568-9/05/$20.00 ©2005 IEEE

eral configuration motivated as follows. The model in [11]-[12] is a lumped end-to-end network model that considers that in an optical link between transmitter and receiver sites all intermediary sites are optical amplifiers (OA), (Figure 1). Optical amplifiers provide simultaneous channel amplification and maintain a target total output optical power. In such an OA-link channel powers can be adjusted independently only at the transmitter sites, which is similar to wireless links [14]-[16], as end-to-end systems. For the resulting lumped end-to-end OSNR model, from Tx to Rx, (Figure 1) a noncooperative game was formulated between channels towards maximizing OSNR at Rx [11].

However in optical networks that use dynamic optical filters, [19]-[22], there exists the flexibility of also adjusting individually channel powers at intermediary points (Figure 2), distributed across a link. Such a flexibility does not exist typically in wireless networks. This case of distributed optical spans is the more general configuration that we consider in this paper. For simplicity we consider a single link with multiple γ-spans, i.e., a distributed optical link. The following interesting question can be stated: how can we take advantage of this inherent distributed span structure ? Specifically how can we formulate a new meaningful game towards maximizing channel OSNR at Rx that is also distributed with respect to spans ?

We extend the non-cooperative game formulation in [11] to such a distributed γ-span configuration. We formulate a nested Nash game [17] with respect to both γ-spans and channels. Channel utility is related to maximizing its own OSNR at Rx, or to minimizing OSNR degradation on the path from Tx to Rx. At each γ-span channels are the non-cooperative players in the lower-level game. Between all γ-spans we formulate a higher-level Nash game that is naturally in a ladder-nested form, i.e., such that the actions of one player depend only on the actions of the preceding ones. For each γ-span the NE solution of the channel game is found by applying the results in [11]. At each γ-span, the channel power adjustment (player action) depends on previous γ-spans' actions and channel output powers. We take advantage of the ladder-nested form to develop a systematic recursive procedure for constructing an NE solution to the overall nested game

The paper is organized as follows. In Section II we review the OSNR model and the channel game in [11]. In Section III we extend the OSNR model to a γ- span optical link configuration. In Section IV we define a channel cost function naturally related to minimizing the OSNR degradation from one γ-span to another. Based on this we formulate a new nested noncooperative game with respect to both γ-spans and channels. In Section V we prove our main result (Theorem 2) for existence and uniqueness of the overall NE solution, and we give a recursive procedure for constructing it. We also propose an iterative algorithm that is distributed with respect to both channels and γ-spans. Conclusions are given in Section VI.

II. BACKGROUND

A. OA-Link / Network OSNR Model

In the following we review the network OSNR model, [11], specialized here for an optical link composed of N cascaded optical amplifiers (OAs), called an OA-link (Figure 1). OAs are used to amplify the optical power of all channels in a link simultaneously, at the expense of introducing amplified spontaneous emission (ASE) noise. Because of each k^{th} amplifier wavelength-dependent gain profile, each i^{th} channel experiences a different gain, $G_{k,i}$, and ASE noise power is also wavelength-dependent, $ASE_{k,i}$.

As in [6], [7], [11], the following assumptions are used: all spans in the OA-link have equal length, L, and all the amplifiers in an OA-link have the same spectral shape, G_i. These assumptions are representative for typical cases used in the industry but could be relaxed proceeding along the same lines. Amplifiers operate typically in automatic power control (APC) mode such that a specified target total power is launched into each of the following spans. This mode compensates variations in fiber-span loss across a link [6]. Moreover the target total power is selected to be bellow the threshold for nonlinear effects [7]. Since all spans have same length, this threshold power, and hence the total power target P_0, is the same for all spans in the OA-link.

There are m channels / wavelengths transmitted across a link, with $\mathcal{M} = \{1, ..., m\}$ denoting the channel set. For the i^{th} channel, let u_i and $n_{0,i}$ be the input signal and noise optical power (at Tx), respectively. Similarly, let $p_{N,i}$, $n_{N,i}^{out}$ be the output signal and noise optical power (at Rx), respectively. The i^{th} channel OSNR, is defined as $OSNR_i = \frac{p_{N,i}}{n_{N,i}^{out}}$. The following result, [11], restated here as Lemma 1, gives the OSNR model for an OA-link.

Lemma 1: The optical signal power and ASE noise power at the output of an OA-link are given as

$$\begin{aligned} p_{N,i} &= u_i \prod_{q=1}^{N} h_{q,i} \\ n_{N,i}^{out} &= n_{0,i} \prod_{q=1}^{N} h_{q,i} + \sum_{v=1}^{N} ASE_{v,i} \prod_{q=v+1}^{N} h_{q,i} \end{aligned}$$

where

$$\prod_{q=1}^{v} h_{q,i} = G_i^v \frac{P_0}{\sum_{j \in \mathcal{M}} G_j^v \, u_j}, \qquad \forall v = 1, \ldots, N$$

The channel OSNR at the output of the link is given as

$$OSNR_i = \frac{u_i}{n_{0,i} + \sum_{j \in \mathcal{M}} \Gamma_{i,j} \, u_j}$$

where

$$\Gamma_{i,j} = \sum_{v=1}^{N} \frac{G_j^v}{G_i^v} \, \frac{ASE_{v,i}}{P_0}, \qquad \forall \, i, j \in \mathcal{M}$$

and $ASE_{v,i}$ is ASE noise self-generated at the v^{th} optical amplifier, associated with the i^{th} channel.

B. End-to-end OSNR Game

Based on this end-to-end (or block) model for an OA-link, the OSNR optimization problem was formulated, [11], as a non-cooperative game, [17], between the m channels. We review here the main result.

Let $\mathbf{u} = [u_1, \ldots, u_i, \ldots, u_m]^T$, be the vector of channel powers at the Tx, and let $\mathbf{u}^{-\mathbf{i}}$, denote the vector obtained from $\mathbf{u}$ by deleting the i^{th} element, i.e., $\mathbf{u}^{-\mathbf{i}} = [u_1, \ldots, u_{i-1}, u_{i+1}, \ldots, u_m]^T$, with $u_i \in [u_{min}, u_{max}]$. Each channel minimizes its own cost function J_i,

$$J_i(u_i, \mathbf{u}^{-\mathbf{i}}) = \alpha_i\, u_i - \beta_i U_i(u_i, \mathbf{u}^{-\mathbf{i}})$$

as the difference between a linear pricing term and the channel utility function U_i,

$$U_i(u_i, \mathbf{u}^{-\mathbf{i}}) = \ln(1 + a_i\, \frac{u_i}{X^{-\mathbf{i}}}) \qquad a_i > 0 \qquad (1)$$

where $X^{-\mathbf{i}} = \sum_{j \neq i} \Gamma_{i,j}\, u_j + n_{0,i}$ and a_i is a channel specific parameter. This utility function U_i is twice continuously differentiable, monotone increasing and strictly concave in u_i. The game settles at an NE solution such that a channel cannot improve its performance by acting unilaterally without degrading the performance of other players. The following assumption guarantees that the NE solution is inner.

(A.1) $u_i = u_{min}$ is not a solution to the minimization of the cost function J_i, i.e., $J_i(u_{min}) > J_i(u_i)$, $\forall u_i \neq u_{min}$. Similarly, $u_i = u_{max}$ is not a solution to the minimization of the cost function J_i, i.e., $J_i(u_{max}) > J_i(u_i)$.

In the above, α_i, β_i are channel specific parameters, used to quantify the willingness to pay the price, and channel's desire to maximize its OSNR, selected such that (A.1) holds. Therefore the cost function to be minimized is

$$J_i(u_i, \mathbf{u}^{-\mathbf{i}}) = \alpha_i u_i - \beta_i \ln(1 + a_i\, \frac{u_i}{X^{-\mathbf{i}}}) \qquad (2)$$

Alternatively, U_i can be expressed as

$$U_i(u_i, \mathbf{u}^{-\mathbf{i}}) = \ln(1 + \frac{a_i}{\frac{1}{OSNR_i} - \Gamma_{i,i}})$$

which is monotone in OSNR, so that maximizing utility is related to maximizing channel OSNR. Then equivalently

$$J_i(u_i, \mathbf{u}^{-\mathbf{i}}) = \alpha_i u_i - \beta_i \ln(1 + \frac{a_i}{\frac{1}{OSNR_i} - \Gamma_{i,i}}) \qquad (3)$$

Conditions for existence and uniqueness of the NE solution are given in Theorem 1, [11], which is restated here.

Theorem 1: The m-player game problem with individual cost functions J_i, (2), admits a unique NE solution $\mathbf{u}^*$ if a_i are selected such that

$$\sum_{j \neq i} \Gamma_{i,j} < a_i, \qquad \forall i \in \mathcal{M}$$

The unique optimal NE solution $\mathbf{u}^*$ is given as

$$\mathbf{u}^* = \widetilde{\mathbf{\Gamma}}^{-1}\, \tilde{\mathbf{b}}$$

where $\widetilde{\mathbf{\Gamma}} = [\widetilde{\Gamma}_{i,j}]$ and $\tilde{\mathbf{b}} = [\tilde{b}_i]$ are defined as

$$\widetilde{\Gamma}_{i,j} = \begin{cases} a_i, & j = i \\ \Gamma_{i,j}, & j \neq i \end{cases} \quad \tilde{b}_i = \frac{a_i \beta_i}{\alpha_i} - n_{0,i}$$

and $\Gamma_{i,j}$ being defined in Lemma 1.

III. DISTRIBUTED γ-LINK OSNR MODEL

In this section we extend the OSNR model for a OA-link to the case of a more general optical link, called γ-link (Figure 2). A γ-link has K intermediary sites where a dynamic gain /adjustment element (DGE) exists. DGEs are optical filters with spectrally adjustable attenuation, such that wavelength (channel) powers can be individually adjusted, [19]-[22]. Depending on the technology, different resolutions, and even a decoupled spectral response can be achieved. This justifies a generic DGE model, that can be used independent of technology, [23], such that

$$p_{out,i} = \gamma_i\, p_{in,i}, \qquad \forall i \in \mathcal{M} \qquad (4)$$

where γ_i is the DGE filter adjustable attenuation per channel, and $p_{in,i}$, $p_{out,i}$ the input and output channel optical power, respectively. The attenuation factor $\gamma_i \in [\gamma_{min}, \gamma_{max}]$, $0 < \gamma_{min} < \gamma_{max} \leq 1$. Due to insertion loss and cost considerations, DGEs are not inserted at every optical amplifier (OA) site, but every few OA sites. We call a γ-span, an optical span with one DGE and R cascaded OAs. An optical link is composed of K such cascaded γ-spans and is called a γ-link. For this link channel powers are adjustable not only at the input (Tx), but also at each γ-span. Let $\mathcal{K} = \{1, ..., K\}$ denote the set of γ-spans in a link, and $\mathcal{M} = \{1, ..., m\}$ the set of channels. We denote by $u_{k,i}$, $n_{k,i}^{in}$, the signal and noise power, respectively, at the input of the k^{th} γ-span, $k \in \mathcal{K}$, for the ith channel, $i \in \mathcal{M}$. Similarly, let $p_{k,i}$, $n_{k,i}^{out}$, denote the signal and noise power, respectively, and $OSNR_{k,i}$,

$$OSNR_{k,i} = \frac{p_{k,i}}{n_{k,i}^{out}}$$

denote channel i^{th} OSNR at the output of k^{th} γ-span.

In what follows we let different γ-spans have optical fiber spans of different lengths L_k, in between their OAs. We also let OAs have different gain profiles $G_{k,i}$ dependent on the γ-span, $k \in \mathcal{K}$. As before, all cascaded OAs within a γ-span, are assumed to have the same spectral shape, $G_{k,i}$. The OAs are operated in APC mode, and have the same target total power within a γ-span, denoted by P_{0k}, $k \in \mathcal{K}$. At the input of the k^{th} γ-span, channel powers can be adjusted individually, so that from (4) we can write recursively, for $k \in \mathcal{K}$,

$$u_{k,i} = \gamma_{k,i}\, p_{k-1,i} \quad \forall k \in \mathcal{K}, \quad \forall i \in \mathcal{M} \qquad (5)$$

where $p_{k-1,i}$ is the signal power at the output of the previous γ-span, and $\gamma_{k,i}$ is the channel power adjustment. Similarly, the input noise $n_{k,i}^{in}$ to the kth γ-span is related to the output noise power $n_{k-1,i}^{out}$ of the previous span as

$$n_{k,i}^{in} = \gamma_{k,i}\; n_{k-1,i}^{out} \quad \forall k \in \mathcal{K}, \quad \forall i \in \mathcal{M} \qquad (6)$$

Note that (6) assumes $\gamma_{k,i}$ affects both signal and noise components similarly. This is a realistic assumption, since an actual DGE filter cannot separate ASE optical noise from signal.

For $k = 1$ in (5), $u_{1,i}$ is the Tx power (input to the first span), equal by convention to $\gamma_{1,i} p_{0,i}$, with $p_{0,i}$ as the initial condition. Note that in [11], only end-to-end adjustment was

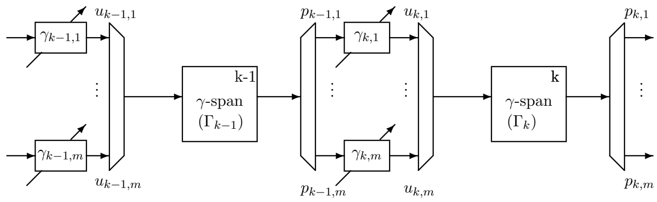

Fig. 3. Two consecutive γ-spans

considered, i.e., only Tx power $u_{1,i}$, or, $\gamma_{1,i}$ is adjustable and $\gamma_{k,i} = 1$ for all $k = 2, \dots, K$.

We can use the OA-link model in Lemma 1 together with the connection relations (5) to obtain a recursive γ-span model, for both output signal power and channel OSNR.

Lemma 2: Consider K cascaded γ-spans (Figure 3), each being composed of R cascaded OAs. The following recursive relations hold for any $k \in \mathcal{K}$ and $i \in \mathcal{M}$:

(i) The signal power of the i^{th} channel at the output of the k^{th} γ-span is given recursively as

$$p_{k,i} = \frac{P_{0k}\, G^R_{k,i}}{\sum_{j\in\mathcal{M}} G^R_{k,j}\, \gamma_{k,j}\, p_{k-1,j}}\, \gamma_{k,i} p_{k-1,i} \quad \forall k \in \mathcal{K}$$

(ii) The OSNR of the i^{th} channel at the output of the kth γ-span is given recursively as

$$\frac{1}{OSNR_{k,i}} = \frac{1}{OSNR_{k-1,i}} + \sum_{j\in\mathcal{M}} \Gamma_{k_{i,j}} \frac{\gamma_{k,j}\, p_{k-1,j}}{\gamma_{k,i}\, p_{k-1,i}}$$

where $\mathbf{\Gamma_k} = [\Gamma_{k_{i,j}}]$ is the k^{th} γ-span matrix, defined as

$$\Gamma_{k_{i,j}} = \sum_{r=1}^{R} \frac{G^r_{k,j}}{G^r_{k,i}} \frac{ASE_{r,i}}{P_{0k}}, \qquad \forall i,j \in \mathcal{M} \tag{7}$$

Proof:

(i) Applying Lemma 1 to the k^{th} γ-span, we have

$$n^{out}_{k,i} \;=\; n^{in}_{k,i} \prod_{q=1}^{R} h_{q,i} + \sum_{r=1}^{R} ASE_{r,i} \prod_{q=r+1}^{R} h_{q,i}$$

where $p_{k,i} = u_{k,i} \prod_{q=1}^{R} h_{q,i}$ and

$$\prod_{q=1}^{r} h_{q,i} = G^r_{k,i} \frac{P_{0k}}{\sum_{j\in\mathcal{M}} G^r_{k,j}\ u_{k,j}}, \qquad \forall r = 1, \dots, R$$

Using the γ-span connection relation (5) yields (i).

(ii) For the channel OSNR, from Lemma 1 applied to the k^{th} γ-span we have

$$OSNR_{k,i} = \frac{u_{k,i}}{n^{in}_{k,i} + \sum_{j\in\mathcal{M}} \Gamma_{k_{i,j}}\, u_{k,j}} \qquad \forall\, i \in \mathcal{M}$$

where $\mathbf{\Gamma_k} = [\Gamma_{k_{i,j}}]$ is given in (7). Then

$$\frac{1}{OSNR_{k,i}} = \frac{n^{in}_{k,i}}{u_{k,i}} + \sum_{j} \Gamma_{k_{i,j}} \frac{u_{k,j}}{u_{k,i}} \qquad \forall\, i \in \mathcal{M}$$

Using now (5, 6) we have

$$\frac{u_{k,i}}{n^{in}_{k,i}} = \frac{\gamma_{k,i} u_{k-1,i}}{\gamma_{k,i} n^{out}_{k-1,i}} = OSNR_{k-1,i}$$

so that from the foregoing we obtain

$$\frac{1}{OSNR_{k,i}} = \frac{1}{OSNR_{k-1,i}} + \sum_{j} \Gamma_{k_{i,j}} \frac{u_{k,j}}{u_{k,i}}$$

Then (ii) follows immediately by using (5) again. ■

Remark 1: Note that signal power $p_{k,i}$, for the i^{th} channel at the output of the k^{th} γ-span, depends nonlinearly on the corresponding channel power at the output of the $(k-1)^{th}$ span $p_{k-1,i}$ and on the adjustable factor $\gamma_{k,i}$. Also $p_{k,i}$ is coupled to all other channels' powers $p_{k-1,j}$, $j \neq i$.

Let $\mathbf{p_k} = [p_{k,1}, \dots, p_{k,m}]^T$ and $\boldsymbol{\gamma}_\mathbf{k} = [\gamma_{k,1}, \dots, \gamma_{k,m}]^T$, so we can write compactly in vector notation,

$$\mathbf{p_k} = \mathbf{F_k}(\mathbf{p_{k-1}}, \boldsymbol{\gamma}_\mathbf{k}) \tag{8}$$

where the nonlinear vector-valued function $\mathbf{F_k}$ is defined component-wise by the right-hand side of (i), Lemma 2.

In the following we use the recursive γ-span model in Lemma 2 to obtain the end-to-end γ-link OSNR model (Figure 2) and relate it to the OA-link OSNR model in Lemma 1.

Lemma 3: Consider a γ-link with K cascaded γ-spans. The following end-to-end relation holds

$$OSNR_{K,i} = \frac{u_{1,i}}{n_{0,i} + \sum_{j=1}^{m} \Gamma_{\gamma_{i,j}} u_{1,j}}$$

where $\mathbf{\Gamma}_\gamma = [\Gamma_{\gamma_{i,j}}]$, is given as

$$\Gamma_{\gamma_{i,j}} = \sum_{k=1}^{K} \Gamma_{k_{i,j}} \left(\prod_{q=1}^{k-1} \frac{G^R_{q,j}}{G^R_{q,i}} \right) \left(\prod_{r=2}^{k} \frac{\gamma_{r,j}}{\gamma_{r,i}} \right)$$

$u_{1,i} = \gamma_{1,i}\ p_{0,i}$ and $\Gamma_{k_{i,j}}$ as in Lemma 2, (ii).

Proof:

Using Lemma 2, (ii), recursively after k, we have for the end-to-end output OSNR, for all $i \in \mathcal{M}$,

$$\frac{1}{OSNR_{K,i}} = \frac{1}{OSNR_{0,i}} + \sum_{k=1}^{K} \sum_{j=1}^{m} \Gamma_{k_{i,j}} \frac{\gamma_{k,j}}{\gamma_{k,i}} \frac{p_{k-1,j}}{p_{k-1,i}} \tag{9}$$

Note that, since the output of span 0 is actually the input to span 1, $OSNR_{0,i}$ is the OSNR at the Tx site, i.e.,

$$OSNR_{0,i} = \frac{u_{1,i}}{n_{0,i}}$$

At the Tx site the signal power is directly adjustable, since there is virtually no ASE noise, and hence $n_{0,i}$ is usually negligible at the Tx site. From (i) in Lemma 2 we can write

$$\frac{p_{k,i}}{p_{k,j}} = \frac{G^R_{k,i}\, \gamma_{k,i}}{G^R_{k,j}\, \gamma_{k,j}} \frac{p_{k-1,i}}{p_{k-1,j}} \tag{10}$$

which shows that the ratio of any two channel powers at the output of the k^{th} γ-span is linearly related to the corresponding ratio at the previous γ-span's output.

Using now (10) on the right-hand-side of (9), we obtain after recursive manipulation

$$\frac{1}{OSNR_{K,i}} = \frac{n_{0,i}}{u_{1,i}} + \sum_{k=1}^{K} \sum_{j=1}^{m} \Gamma_{k_{i,j}} \left(\prod_{q=1}^{k-1} \frac{G^R_{q,j}}{G^R_{q,i}} \right) \left(\prod_{r=2}^{k} \frac{\gamma_{r,j}}{\gamma_{r,i}} \right) \frac{u_{1,j}}{u_{1,i}}$$

which completes the proof.

■

Remark 2: The first relation in (ii), (9), gives the end-to-end OSNR at the output of the link as a function of the input power at the beginning of the link (Tx) and all the adjustable factors $\gamma_{k,i}$, $k \geq 2$. Note that for the case when only the Tx powers are adjustable, $\gamma_{k,i} = 1$, $k \geq 2$, it can be shown that, for $N = KR$, we can recover the end-to end block results in Lemma 1.

IV. NESTED NONCOOPERATIVE GAME FORMULATION

In this section we extend the noncooperative OSNR game formulation in Section II.B to a distributed γ-span configuration (γ-link) as in Section III. We define a two-level noncooperative Nash game with respect to both K γ-spans and m channels. Starting from each channel's objective of optimizing its own OSNR (along its path from Tx to Rx), we give firstly an alternative interpretation of the channel game. Taking this interpretation in the context of a γ-link, we define a channel cost function related to minimizing the OSNR degradation from one γ-span to another.

A. γ-Link Channel OSNR Game Interpretation

Recall that in the noncooperative game in Section II.B, each channel maximizes its own utility U_i, which is related to channel OSNR at Rx. This is done by minimizing its own channel cost function J_i, (3), in response to the other channels' actions, and hence minimizing the OSNR degradation, $\frac{1}{OSNR_{K,i}}$, from Tx to Rx. For notational convenience we denote this relationship as

$$J_i(u_i, \mathbf{u^{-i}}) \approx \frac{1}{OSNR_{K,i}} \qquad (11)$$

The game can be equivalently defined in terms of a system-like cost, J_t, defined in an NE sense, [17], as a two-argument function

$$J_t(\mathbf{u}, \mathbf{u}) = \sum_i J_i(u_i, \mathbf{u^{-i}}) \qquad (12)$$

The same channel cost formulation is used for a γ-link. We neglect the low noise at Tx, $n_{0,i}$, so that from (9) we can write

$$\frac{1}{OSNR_{K,i}} = \sum_{k=1}^{K} \left(\frac{1}{\delta Q_{k,i}} \right), \quad \forall i \in \mathcal{M} \qquad (13)$$

where

$$\frac{1}{\delta Q_{k,i}} = \sum_j \Gamma_{k_{i,j}} \frac{\gamma_{k,j}}{\gamma_{k,i}} \frac{p_{k-1,j}}{p_{k-1,i}}, \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{M}$$

Using (ii) Lemma 2, we can write $1/\delta Q_{k,i}$, (13), as

$$\frac{1}{\delta Q_{k,i}} = \frac{1}{OSNR_{k,i}} - \frac{1}{OSNR_{k-1,i}} \qquad (14)$$

Hence $1/\delta Q_{k,i}$ can be interpreted as a measure of the OSNR degradation from one γ-span to another. The additive relationship in (13) with respect to γ-spans will be used to define an appropriate γ-span cost function.

From (13) we see that maximizing output OSNR, $OSNR_{K,i}$, could be formulated by minimizing each of $1/\delta Q_{k,i}$. Then, for a γ-link from (11, 13), J_i is the end-to-end channel cost function

$$J_i \approx \sum_{k=1}^{K} \left(\frac{1}{\delta Q_{k,i}} \right) \qquad (15)$$

Also, equivalently we use the same system-like cost interpretation as in (12), so that using J_i as in (15). for a γ-link we have

$$J_t = \sum_i \sum_k J_{k,i} \qquad (16)$$

with $J_{k,i}$ given as

$$J_{k,i} \approx \frac{1}{\delta Q_{k,i}}$$

After changing the summation order in (16), we write

$$J_t = \sum_k \mathbf{J}_k, \quad \text{with} \quad \mathbf{J}_k = \sum_i J_{k,i} \qquad (17)$$

and $J_{k,i}$ as in (16).

Based on these observations we will use the cost function of each k^{th} γ-span as in (17) with the sum being taken also in an NE sense.

Note that $J_{k,i}$ (16) used in (17) is defined similarly to J_i (3), but with respect to $\delta Q_{k,i}$. Therefore, let

$$J_{k,i}(\gamma_k) = \alpha_{k,i}\gamma_{k,i}p_{k-1,i} - \beta_{k,i} \ln(1 + \frac{a_{k,i}}{\frac{1}{\delta Q_{k,i}} - \Gamma_{k_{i,i}}}) \qquad (18)$$

where from (14)

$$\frac{1}{\delta Q_{k,i}} = \frac{1}{OSNR_{k,i}} - \frac{1}{OSNR_{k-1,i}}$$

and $\alpha_{k,i}$, $\beta_{k,i}$ are channel and γ-span specific parameters selected such that (A.1) holds. Therefore minimizing $J_{k,i}$ is related to minimizing $1/\delta Q_{k,i}$. Equivalently $J_{k,i}$ is related minimizing the degradation in OSNR for a span to another.

Now, from (13) we can express $1/\delta Q_{k,i}$ as

$$\frac{1}{\delta Q_{k,i}} = \sum_{j \neq i} \Gamma_{k_{i,j}} \frac{\gamma_{k,j}}{\gamma_{k,i}} \frac{p_{k-1,j}}{p_{k-1,i}} + \Gamma_{k_{i,i}}$$

Using this into (18) yields, for all k, i

$$J_{k,i}(\gamma_k) = \alpha_{k,i}\, \gamma_{k,i}\, p_{k-1,i} - \beta_{k,i} \ln(1 + a_{k,i} \frac{\gamma_{k,i}}{\widetilde{X}_{0k}^{-\mathbf{i}}}) \qquad (19)$$

where $\widetilde{X}_{0k}^{-\mathbf{i}} = \sum_{j \neq i} \Gamma_{k_{i,j}}\, \gamma_{k,j} \frac{p_{k-1,j}}{p_{k-1,i}}$. Since $p_{k-1,i}$ is given as in Lemma 2 (i),

$$p_{k-1,i} = \frac{P_{0k-1}\, G_{k-1,i}^R}{\sum_{j \in \mathcal{M}} G_{k-1,j}^R \gamma_{k-1,j}\, p_{k-2,j}} \gamma_{k-1,i}\, p_{k-2,i}$$

we see that $J_{k,i}$ (19) depends on $\gamma_{k,i}$, the adjustable parameters (actions) for each channel at the k^{th} γ-span, and also implicitly on previous γ-span's actions such as $\gamma_{k-1,i}$. This cost function $J_{k,i}$ (18, 19) will be used next.

B. Nested Game Formulation

In the following we define a two-level noncooperative Nash game with respect to both K γ-spans and m channels. We use the channel cost function $J_{k,i}$ as defined in (18, 19).

The following notations are used. For each $k \in \mathcal{K}$, let $\mathbf{u}_k$ denote the vector of channel input powers into the k^{th} γ-span, $\mathbf{u}_k = [u_{k,1}, \ldots, u_{k,m}]^T$. From (5) we have

$$\mathbf{u}_k = Diag(\gamma_k)\,\mathbf{p}_{k-1}, \quad \text{or} \quad \mathbf{u}_k = Diag(\mathbf{p}_{k-1})\,\gamma_k \quad (20)$$

where $\mathbf{p}_{k-1}$ (8) is the vector of channel output powers from the $(k-1)^{th}$ γ-span, given as in Lemma 2. γ_k is the vector of channel actions (adjustments) at the k^{th} γ-span, $\gamma_k \in U^m$, $U^m = U \times \ldots \times U$, $U = [\gamma_{min}, \gamma_{max}]$. Let $\gamma_k^{-\mathbf{i}}$ be obtained by deleting the i^{th} element, $\gamma_{k,i}$, from γ_k

$$\gamma_k^{-\mathbf{i}} = (\gamma_{k,1}, \ldots, \gamma_{k,i-1}, \gamma_{k,i+1}, \ldots, \gamma_{k,m})$$

Then γ_k can be written as $\gamma_k = (\gamma_{k,i}, \gamma_k^{-\mathbf{i}}) \in U^m$. Let $\widehat{\gamma}$ denote the K-tuple of all γ_k, i.e., $\widehat{\gamma} \in U^{mK}$, and $\widehat{\gamma}^{-\mathbf{k}}$ the $(K-1)$-tuple obtained by deleting the k^{th} element,

$$\begin{aligned} \widehat{\gamma} &= (\gamma_1, \ldots\ldots, \gamma_k, \ldots\ldots \gamma_K), \quad \gamma_k \in U^m \\ \widehat{\gamma}^{-\mathbf{k}} &= (\gamma_1, \ldots, \gamma_{k-1}, \gamma_{k+1}, \ldots, \gamma_K), \quad \forall k \in \mathcal{K} \end{aligned}$$

Then $\widehat{\gamma}$ can be written as $\widehat{\gamma} = (\gamma_k, \widehat{\gamma}^{-\mathbf{k}}) \in U^{mK}$.

We consider a game with K players $\mathbf{P}_k$, represented by the γ-spans, each player having the action γ_k, the adjustments of all channels' powers at the k^{th} γ-span. Each player $\mathbf{P}_k$ has a cost function $\mathbf{J}_k$ whose value depends not only on its own action, but also on the actions of other players, i.e., on $\widehat{\gamma}$. Each γ-span attempts to minimizes its own cost function, in response to others γ-spans' (players') actions. Then the relevant concept is the noncooperative Nash equilibrium (NE), [24], [17].

Definition 1: Consider an K-player game between γ-spans. Each player minimizes the cost function $\mathbf{J}_k$, $\mathbf{J}_k : U^{mK} \to R$, over $\gamma_k \in U^m$. A vector $\widehat{\gamma}^* = (\gamma_1^*, \ldots, \gamma_K^*)$, $\widehat{\gamma}^* \in U^{mK}$, is called a Nash equilibrium (NE) solution of this game if

$$\mathbf{J}_k(\widehat{\gamma}^*) \le \inf_{\gamma_k \in U^m} \mathbf{J}_k(\gamma_k, \widehat{\gamma}^{-\mathbf{k}*}) \qquad \forall k \in \mathcal{K}$$

or, equivalently, if, for all $k \in \mathcal{K}$ and any given $\widehat{\gamma}^{-\mathbf{k}*}$,

$$\mathbf{J}_k(\gamma_k^*, \widehat{\gamma}^{-\mathbf{k}*}) \le \mathbf{J}_k(\gamma_k, \widehat{\gamma}^{-\mathbf{k}*}), \quad \forall \gamma_k \in U^m$$

Definition 1 specifies that $\widehat{\gamma}^*$ is an NE when γ_k^* is the solution to the individual optimization problem $\mathbf{J}_k$ for γ-span k, given all γ-spans in its link have equilibrium power levels, $\widehat{\gamma}^{-\mathbf{k}*}$. Existence of an NE solution depends on existence of a common intersection point for the reaction curves of all players, [17]. It can be seen that the K-player game is a nonzero-sum finite game with a fixed order of play (order of precedence of the γ-spans), in ladder-nested form, [17]. Each player $\mathbf{P}_k$ has access to the information acquired by all his precedents, and the difference between $\mathbf{P}_k$'s information and his immediate precedent, $\mathbf{P}_{k-1}$'s information, involves only actions of $\mathbf{P}_{k-1}$. Games in ladder-nested form can be recursively decomposed into simpler structures, and Nash equilibria can be obtained recursively. We use this in the following.

Note that correspondingly, each γ-span's cost function needs to be minimized in an NE sense, between all channels that share the γ-span. For each $k \in \mathcal{K}$, we define an m-player game between channels, with individual cost function $J_{k,i}$, (19), so that in effect we formulate a nested two-level (K x m) noncooperative game.

Definition 2: For each $k \in \mathcal{K}$, consider an m-player game between channels, with each channel being a player that minimizes the cost $J_{k,i}$, $\mathbf{J}_{k,i} : U^m \to R$, over $\gamma_{k,i} \in U$. Then a vector (m-tuple) $\gamma_k^* = (\gamma_{k,1}^*, \ldots, \gamma_{k,m}^*)$ is called a NE solution of this game if, for each $k \in \mathcal{K}$,

$$J_{k,i}(\gamma_k^*) \le \inf_{\gamma_{k,i} \in U} J_{k,i}(\gamma_{k,i}, \gamma_k^{-\mathbf{i}*}), \qquad \forall i \in \mathcal{M}$$

or, equivalently, for all $i \in \mathcal{M}$

$$J_{k,i}(\gamma_{k,i}^*, \gamma_k^{-\mathbf{i}*}) \le J_{k,i}(\gamma_{k,i}, \gamma_k^{-\mathbf{i}*}), \quad \forall \gamma_{k,i} \in U$$

Recall that the defined cost $J_{k,i}$, (19), which is related to maximizing OSNR, depends implicitly on the actions taken at previous spans, so that we need to write

$$J_{k,i}(\gamma_{k,i}, \gamma_k^{-\mathbf{i}*}, \widehat{\gamma}^{-\mathbf{k}}), \qquad J_{k,i} : U^{mK} \to R \qquad (21)$$

For simplicity of notation, we can drop the last argument.

Next we will define an appropriate kth γ-span cost function, $\mathbf{J}_k$, that is related to the m channels' cost functions, $J_{k,i}$, (19), and has the natural interpretation as in (17). We will make use of the following "system-like" cost function interpretation (see (4.10) in [17]). Definition 2 involves a set of m inequalities that have to be satisfied simultaneously. It can be equivalently formulated by a two-argument cost function, $\widehat{J}_k(\cdot;\cdot)$, $\widehat{J}_k : U^m \times U^m \to R$, defined as

$$\widehat{J}_k(\gamma_k; \gamma_k^*) := \sum_{i=1}^{m} J_{k,i}(\gamma_{k,i}, \gamma_k^{-\mathbf{i}*}), \quad \forall k \in \mathcal{K} \qquad (22)$$

with $J_{k,i}$ defined as in (19). From (21), it follows that

$$\widehat{J}_k(\gamma_k; \gamma_k^*, \widehat{\gamma}^{-\mathbf{k}*}) := \sum_{i=1}^{m} J_{k,i}(\gamma_{k,i}, \gamma_k^{-\mathbf{i}*}, \widehat{\gamma}^{-\mathbf{k}*}) \qquad (23)$$

Each γ-span cost function is taken as in (23) with $J_{k,i}$, (19).

V. EXISTENCE AND UNIQUENESS OF THE NE SOLUTION

Next we give conditions for existence and uniqueness of a Nash equilibrium (NE) for the overall game between γ-spans and channels.

Theorem 2: Assume that $J_{k,i}$, $k \in \mathcal{K}$, is defined as in (19), and (A.1) holds. Then the K-player game between γ-spans, with cost function $\widehat{J}_k$, (23), admits an NE solution, $\widehat{\gamma}^*$. If $a_{k,i}$ are selected such that

$$\sum_{j \ne i} \Gamma_{k_{i,j}} < a_{k,i} \qquad \forall i \in \mathcal{M} \qquad (24)$$

with $\Gamma_{k_{i,j}}$ as in Lemma 2, then the NE solution is unique.

Proof: We prove the result by applying twice Theorem 4.3 in [17]. Consider the cost $\widehat{J}_k$ (23) with $J_{k,i}$ as in (19). It can be seen that $\widehat{J}_k$ is continuous in all its arguments, and is separable in $\gamma_{k,i}$, for every given $\widehat{\gamma}^{-\mathbf{k}}$ and γ_k^*. Its gradient with respect to γ_k, is given component-wise as

$$\frac{\partial \widehat{J}_k}{\partial \gamma_{k,i}} = \frac{\partial J_{k,i}}{\partial \gamma_{k,i}} \tag{25}$$

The Hessian, $\frac{\partial^2 \widehat{J}_k}{\partial \gamma_k^2} = [\frac{\partial^2 \widehat{J}_k}{\partial \gamma_{k,j} \partial \gamma_{k,i}}]$ is given as

$$\frac{\partial^2 \widehat{J}_k}{\partial \gamma_{k,j} \partial \gamma_{k,i}} = \begin{cases} 0, & j \neq i \\ \frac{\partial^2 J_{k,i}}{\partial \gamma_{k,i}^2}, & j = i \end{cases}$$

From (19), for any given $\gamma_{k,j}$ and $p_{k-1,i} \neq 0$, we see that $\frac{\partial^2 J_{k,i}}{\partial \gamma_{k,i}{}^2} > 0$, i.e., $J_{k,i}$ is strictly convex in $\gamma_{k,i}$. Hence the Hessian is positive definite and $\widehat{J}_k$ is strictly convex in γ_k, for every given $\widehat{\gamma}^{-\mathbf{k}}$ and γ_k^*. Then, for each $k \in \mathcal{K}$, there exists a minimizing $\gamma_{k,i}^*$ on the closed and bounded (compact) set U such that,

$$J_{k,i}(\gamma_{k,i}^*, \gamma_k^{-\mathbf{i}}) < J_{k,i}(\gamma_{k,i}, \gamma_k^{-\mathbf{i}}), \quad \forall \gamma_{k,i} \neq \gamma_{k,i}^*, \quad \forall i \in \mathcal{M}$$

for every given $\gamma_k^{-\mathbf{i}}$. By Theorem 4.3 in [17], for each k there exists a vector solution γ_k^* to the set of m foregoing inequalities, which is an NE solution to the m-player game. Furthermore by (A.1) $\gamma_{k,i}^*$ is inner. With the notation in (21), for any given $\widehat{\gamma}^{-\mathbf{k}}$

$$J_{k,i}(\gamma_{k,i}^*, \gamma_k^{-\mathbf{i}}, \widehat{\gamma}^{-\mathbf{k}}) < J_{k,i}(\gamma_{k,i}, \gamma_k^{-\mathbf{i}}, \widehat{\gamma}^{-\mathbf{k}}), \forall \gamma_{k,i} \neq \gamma_{k,i}^* \tag{26}$$

For the K-player game, since for every given $\widehat{\gamma}^{-\mathbf{k}}$ and γ_k^*, $\widehat{J}_k$, the cost (23), is strictly convex in γ_k on the compact and convex set U^m, there exists a unique mapping T_k that uniquely minimizes $\widehat{J}_k$. This mapping is the reaction function of the k^{th} player defined as

$$\begin{aligned} T_k(\gamma_k^*, \widehat{\gamma}^{-\mathbf{k}}) = \{ & \gamma_k \in U^m | \\ & \widehat{J}_k(\gamma_k; \gamma_k^*, \widehat{\gamma}^{-\mathbf{k}}) < \widehat{J}_k(\mathbf{v}_k; \gamma_k^*, \widehat{\gamma}^{-\mathbf{k}}), \forall \mathbf{v}_k \in U^m \} \end{aligned}$$

for any given γ_k^*, and $\widehat{\gamma}^{-\mathbf{k}}$. Then applying again Theorem 4.3 in [17], there exists a vector solution, $\widehat{\gamma}^*$, to the set of K foregoing inequalities, which is an NE solution to the K-player game. Such an NE is given by the intersection of all reaction functions, so that $\widehat{\gamma}^*$ is a fixed point of T,

$$\widehat{\gamma}^* = T(\widehat{\gamma}^*) \tag{27}$$

where $\widehat{\gamma}^* = [\gamma_k^*]$, $T = [T_k]$ in vector notation. Note that using (23, 26) yields,

$$\widehat{J}_k(\gamma_k^*; \gamma_k^*, \widehat{\gamma}^{-\mathbf{k}*}) < \widehat{J}_k(\gamma_k; \gamma_k^*, \widehat{\gamma}^{-\mathbf{k}*}), \quad \forall \gamma_k \in U^m \tag{28}$$

for any given $\widehat{\gamma}^{-\mathbf{k}}$, for $k \in \mathcal{K}$. Therefore, given a $\gamma_{k,i}^*$ that minimizes $J_{k,i}$, (19), as in (26), we see that the vector $\gamma_k^* = [\gamma_{k,i}^*]$, $i \in \mathcal{M}$, minimizes $\widehat{J}_k$ as in 28. Hence, γ_k^* are the individual components of $\widehat{\gamma}^*$ and constitute an NE solution to the K-player game.

In the following we prove the uniqueness of this NE solution, which by (A.1) is also inner. To find $\widehat{\gamma}^*$, or its components γ_k^* we solve the necessary conditions

$$\frac{\partial \widehat{J}_k}{\partial \gamma_k} = 0, \qquad \forall k \in \mathcal{K}$$

which defines the k^{th} player's reaction curve, T_k.

The vector solution of this set of equations is an NE solution to the K-player game. Recalling (25), this reduces to

$$\frac{\partial J_{k,i}}{\partial \gamma_{k,i}} = 0 \qquad \forall k \in \mathcal{K}, \qquad \forall i \in \mathcal{M} \tag{29}$$

We show next that (29) admits a unique solution. For each $k \in \mathcal{K}$, the solution of (29), is an NE solution for the m-player game with cost functions $J_{k,i}$, (19). Using (5), $J_{k,i}$ (19) is equivalently written for each k as

$$J_{k,i}(u_{k,i}, \mathbf{u_k}^{-\mathbf{i}}) = \alpha_{k,i}\, u_{k,i} - \beta_{k,i} \ln \left(1 + a_{k,i} \frac{u_{k,i}}{X_{0k}^{-\mathbf{i}}} \right) \tag{30}$$

with $u_{k,i} = \gamma_{k,i}\, p_{k-1,i}$, and $X_{0k}^{-\mathbf{i}} = \sum_{j \neq i} \Gamma_{k_{i,j}}\, u_{k,j}$. Then $J_{k,i}$ (30) is similar to J_i (2). For each k we use Theorem 1 to characterize the NE solution to the m-player game with costs $J_{k,i}$ (30). We will express this solution in terms of $\gamma_{k,i}$. Using (5), the necessary conditions become

$$\frac{\partial J_{k,i}}{\partial \gamma_{k,i}} = \frac{\partial J_{k,i}}{\partial u_{k,i}} p_{k-1,i} = 0$$

which leads to $\frac{\partial J_{k,i}}{\partial u_{k,i}} = 0$ since $p_{k-1,i} > 0$. Then using (30) yields,

$$a_{k,i}\, u_{k,i}^* + X_{0k}^{-\mathbf{i}*} = \frac{a_{k,i} \beta_{k,i}}{\alpha_{k,i}} \qquad \forall i \tag{31}$$

By (24), from Theorem 1 it follows that for each k, this game admits a unique NE solution in terms of $\mathbf{u}_k$,

$$\mathbf{u}_k^* = \widetilde{\mathbf{\Gamma}}_k^{-1}\, \tilde{\mathbf{b}}_k$$

or, equivalently using (20) for each given $\mathbf{p}_{k-1}$, in terms of γ_k given as

$$\gamma_k^* = Diag(\mathbf{v}_{k-1}) \widetilde{\mathbf{\Gamma}}_k^{-1}\, \tilde{\mathbf{b}}_k \tag{32}$$

where $\mathbf{v}_{k-1} = [1/p_{k-1,1}, \ldots, 1/p_{k-1,m}]$, also denoted as $\mathbf{v}_{k-1} = 1./\mathbf{p}_{k-1}$. In the above $\widetilde{\mathbf{\Gamma}}_\mathbf{k} = [\widetilde{\Gamma}_{i,j}]$ and $\tilde{\mathbf{b}}_\mathbf{k} = [\tilde{b}_{k,i}]$ are defined as

$$\widetilde{\Gamma}_{k_{i,j}} = \begin{cases} a_{k,i}, & j = i \\ \Gamma_{k_{i,j}}, & j \neq i \end{cases} \quad \tilde{b}_{k,i} = \frac{a_{k,i} \beta_{k,i}}{\alpha_{k,i}}$$

Recall that $\mathbf{p}_{k-1}$ is the output of the previous span, given recursively as in Lemma 2, (i), or compactly, as in (8),

$$\mathbf{p_{k-1}} = \mathbf{F_{k-1}}(\mathbf{p_{k-2}}, \gamma_{\mathbf{k-1}}) \tag{33}$$

Therefore the optimal γ_k^* (32) depends recursively on γ_{k-1}^* via $\mathbf{p}_{k-1}$ (33) and hence depends on $\widehat{\gamma}^{-\mathbf{k}}$. Then the full NE solution will be the mK vector solution of the set of equations (32, 33) for all k. It can be seen that (32, 33) has a triangular structure, and therefore, for any given $\mathbf{p}_0$, the unique solution $\widehat{\gamma}^*$ can be easily found component-wise, by forward substitution. ■

Remark 3: The formulation developed here corresponds to a ladder-nested structure of the game between the γ-spans, where k^{th} player decision is taken after $(k-1)^{th}$ player's action. This enabled us to obtain a decoupled existence condition, while the overall coupled NE solution can be computed recursively.

VI. ITERATIVE ALGORITHM

Based on (31), consider the recursive algorithm

$$u_{k,i}(n+1) = \frac{\beta_{k,i}}{\alpha_{k,i}} - \frac{X_{0k}^{-\mathbf{i}}(n)}{a_i} \qquad \forall i \in \mathcal{M} \tag{34}$$

for updating channel power level. Note that (34) corresponds to a parallel adjustment scheme, [17], whereby each player responds optimally to the previously selected action of the other players. From the definition (30),

$$X_{0k}^{-\mathbf{i}} = \sum_{j \neq i} \Gamma_{k_{i,j}} u_{k,j}$$

where $u_{k,j} = \gamma_{k,j} p_{k-1,j}$ from (5). $X_{0k}^{-\mathbf{i}}$ needs measurements of all input channel powers, $u_{k,j}$, of γ-span k and all channel gains, hence centralized information. However, using Lemma 2, (ii) and (5) we can write

$$X_{0k}^{-\mathbf{i}} = u_{k,i} \left(\frac{1}{OSNR_{k,i}} - \frac{1}{OSNR_{k-1,i}} - \Gamma_{k_{i,i}} \right)$$

and $X_{0k}^{-\mathbf{i}}$ is expressed in terms of OSNR at the output of k^{th} γ-span and $(k-1)^{th}$ γ-span (which can be measured in real-time). Using this and

$$u_{k,i}(n) = \gamma_{k,i}(n)\, p_{k-1,i}(n-1)$$

into (34), we obtain the following algorithm for channel adjustment $\gamma_{k,i}$ at the input of the kth span

$$\begin{aligned} \gamma_{k,i}(n+1) \quad &= \tfrac{\beta_i}{\alpha_i} \tfrac{1}{p_{k-1,i}(n)} \\ -\tfrac{\gamma_{k,i}(n)}{a_i} &\left(\tfrac{1}{OSNR_{k,i}(n)} - \Gamma_{k_{i,i}} \right) \tfrac{p_{k-1,i}(n-1)}{p_{k-1,i}(n)} \\ +\tfrac{\gamma_{k,i}(n)}{a_i} &\tfrac{1}{OSNR_{k-1,i}(n)} \tfrac{p_{k-1,i}(n-1)}{p_{k-1,i}(n)} \end{aligned} \tag{35}$$

The next result can be proved using arguments as in [11].

Lemma 4: If (24) holds, then algorithm (35) converges to the NE solution.

Remark 4: Algorithm (35) is distributed with respect to both channels and γ-spans. The only information fedback is local: individual channel $OSNR$ and power from the current γ-span and the (neighboring) previous γ-span, and the channel "gain" $\Gamma_{k_{i,i}}$.

VII. CONCLUSIONS

In this paper we extended the approach in [11], for optimization of optical signal to noise ratio (OSNR) in optical networks, to a more general configuration. Instead of a lumped end-to-end model, whereby channel powers can be adjusted independently only at the transmitter sites, we considered a more flexible model with adjustable channel powers at intermediary dynamic sites. For this inherent distributed configuration specific to optical networks, a nested Nash game was formulated, towards maximizing channel OSNR at Rx that is distributed with respect of spans and channels. Existence and uniqueness of the NE solution was shown and a recursive procedure for constructing the NE solution was given. We proposed an iterative algorithm, that is distributed with respect to both channles and γ-spans, based on local feedback of channel parameters from neighboring spans. Interesting future directions are adjusting the pricing parameters for capacity constraints.

REFERENCES

[1] B. Mukherjee, 'WDM optical communication networks: progress and challenges', *IEEE J. Sel. Areas Comm.*, *vol. 18*, 1810-1824, 2000.

[2] B. Ramamurthy, R. Ratta, H. Feng, J. P. Heritage and B. Mukherjee, 'Impact of transmission impairments on the teletraffic performance of wavelength-routed optical networks', *IEEE J. Lightwave Tech.*, *vol. 17*, 1713-1723, 1999.

[3] L. Pavel, 'A μ-Analysis Application to Stability of Optical Networks', in *Proc. American Control Conf.*, 3956-3961, July 2004.

[4] L. Pavel, 'Dynamics and stability in optical communication networks: A system theoretic framework', *Automatica*, *vol. 40*(8), 1361-1370, Aug 2004.

[5] G. P. Agrawal, *Fiber-optic communication systems*, John Wiley & Sons, 2002.

[6] F. Forghieri, R.W. Tkach, R.L. Favin, 'Simple model of optical amplifier chains to evaluate penalties in WDM systems', *IEEE J. Lightwave Tech.*, *vol. 16*, 1570-1576, 1998.

[7] A. Mecozzi, 'On the optimization of the gain distribution of transmission lines with unequal amplifier spacing', *IEEE Phot. T. Lett.*, *vol. 10*(7), 1033-1035, 1998.

[8] A. R. Chraplyvy, J.A. Nagel, R.W. Tkach, 'Equalization in amplified WDM lightwave trasnmission systems', *IEEE Phot. T. Lett*, *vol. 4*, 920-922, 1992.

[9] J. Zander, 'Performance of optimum transmitter power control in cellular radio systems', *IEEE Trans. Veh. Techn.*, *vol. 41*, 305-311, 1992.

[10] Grandhi, S.A, and R. Vijayan, R.J. Goodman, J. Zander, 'Centralized power control in cellular radio systems', *IEEE Trans. on Vehicular Technology*, vol. *42*, 466-468, 1993.

[11] L. Pavel, 'Power control for OSNR optimization in optical networks: a noncooperative game approach', in *Proc. 43rd IEEE Conf. Decision and Control*, 3033-3038, Dec. 2004.

[12] L. Pavel, 'Power control for OSNR optimization in optical networks: a distributed algorithm via a central cost approach', in *Proc. IEEE INFOCOM 2005*, 1095-1105, Miami, March 2005.

[13] L. Libman and A. Orda, The designer's perspective to atomic noncooperative networks', *IEEE Trans. Networking*, *vol. 7 (6)*, 875-884, Dec. 1999.

[14] C. Saraydar, N. B. Mandayam, and D.J. Goodman, 'Efficient power control via pricing in wireless data networks', *IEEE Trans. Communication*, *vol. 50 (2)*, 291-303, Feb 2002.

[15] D. Falomari, N. Mandayam and D. Goodman, 'A new framework for power control in wireless data networks: Games, utility and pricing', *Proc. Allerton Conf. on Communication, Control and Computing*, Illinois, 546-555, 1998.

[16] T. Alpcan, T. Basar, R. Srikant, E. Altman, 'CDMA uplink power control as a noncooperative game', in *Proc. 40th IEEE Conf. Decision and Control*, 197-202, 2001.

[17] T. Basar and G. J. Olsder, *Dynamic Noncooperative Game Theory*, 2nd ed, SIAM Series Classics in Applied mathematics, PA, 1999.

[18] D. P. Bertsekas, *Nonlinear Programming*, 2nd ed, Athena Scientific, Belmont, MA, 1999.

[19] J.E. Ford, J. A. Walker, 'Dynamic spectral power equalization using micro-optomechanics', *IEEE Phot. T. Lett.*, vol. **10**, 1440-1442, 1998.

[20] C. K. Madsen, J. H. Zhao, *'Optical Filter Design and analysis - a signal processing approach'*, John Wiley & Sons, 1999.

[21] Doerr, C.R., and L. W. Stultz, et al., 'An automatic 40-wavelength channelized equalizer', *IEEE Phot. T. Lett*, vol. **12**, 1195-1197, 2000.

[22] L. Pavel, 'Control design for transient power and spectral control in optical communication networks', *Proc. IEEE Conference for Control Applications*, Turkey, 415-422, June 2003.

[23] L. Pavel, 'Modeling and dynamics in large-scale optical communication systems', in *Proc. 10th IFAC Large-Scale Systems Symposium*, Osaka, 566-575, July 2004.

[24] J. Nash, ' Equilibrium points in n-person games', *Proc. Nat. Acad. Sci. USA*, *36*, 1950.