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A Maximum Time Optimal Control Approach to Routing in Sensor Networks

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Abstract— **An optimal control approach is used to solve the problem of routing in sensor networks where the goal is to maximize the network's lifetime. We show that in a fixed topology case there exists an optimal policy consisting of fixed routing probabilities which may be obtained by solving a set of relatively simple Non-Linear Programming (NLP) problems. An alternative problem is also considered where, in addition to routing, we also allocate a total initial energy over the network nodes with the same network lifetime maximization objective. We prove that the solution to this problem is given by a policy that depletes all node energies at the same time and that the corresponding energy allocation and routing probabilities are obtained by solving a single NLP problem. Numerical examples are included to contrast the maximum lifetime we obtain to that resulting from alternative routing policies.**

Index Terms— **Sensor network, routing, power-limited system, optimal control**

I. Introduction

A sensor network consists of sensing devices that can perform large-scale real-time data gathering, processing, and wireless communication functions. Applications of such networks include exploration, surveillance, and environmental monitoring. Their design and management involve a variety of issues, such as deployment over a particular sensing region, routing from data sources to destinations, and security. In this paper, we focus on the problem of routing in a sensor network. Specifically, we are interested in routing schemes aimed at optimizing performance metrics that reflect the limited energy resources of the network while also preventing common security vulnerabilities. A routing protocol in a sensor network is responsible for finding routes between nodes and forwarding packets to their ultimate destinations. Unlike traditional routing protocols in wired networks which can rely on global information, routing protocols in sensor networks are constrained by limited node resources and computational capabilities. Consequently, they typically adopt local cost information (e.g., distance between two nodes) in order to make routing decisions. Such routing decisions can have major impact on the performance of the network.

The majority of the proposed routing protocols in mobile wireless networks are based on shortest path algorithms, e.g., [1],[2]. Such algorithms usually require each node to maintain a global cost (or state) information table, which is a significant burden for resource-constrained sensor networks. In order to deal with the issue of node failures, Ganesan et al. [3] proposed a multipath routing algorithm, so that a failure on the main path can be recovered without initiating a network-wide flooding process for path rediscovery. Since flooding consumes considerable energy, this routing method can extend the network's lifetime when there are failures. On the other hand, finding multiple paths and sending packets through them also consumes energy, thus adversely impacting the lifetime of the network if there are no failures.

The routing policies mentioned above may indirectly reduce energy usage in sensor networks but they do not explicitly use energy consumption models to address optimality of a routing policy with respect to energy-aware metrics. In recent years, such "energy awareness" has motivated a number of minimum-energy routing algorithms which typically seek paths minimizing the energy per packet consumed (or maximizing the residual node energy) to reach a destination (e.g., [4]). However, as also pointed out in [5], seeking a minimum energy (or maximum residual energy) path can rapidly deplete energy from some nodes and ultimately reduce the full network's lifetime by destroying its connectivity. Thus, an alternative performance metric is the network lifetime. Along these lines, Shah and Rabaey [6] proposed an Energy Aware Routing (EAR) policy which does not attempt to use a single optimal path, but rather a number of suboptimal paths that are probabilistically selected with the intent of extending the network lifetime by "spreading" the traffic and forcing nodes in the network to deplete their energies at the same time. In EAR, each node builds a cost information table and propagates local cost information to other nodes. Costs are determined by the residual energies of each node and by the distances between them. Each node also maintains a routing probability table determined by local cost information. In [5], routing with the goal of network lifetime maximization is formulated as a linear programming problem where the decision variables are source to destination path flows and a shortest cost path routing algorithm is proposed to efficiently approximate its solution; link costs are defined to combine energy consumption and residual energy at the end nodes of each link.

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>From a network security viewpoint, deterministic routing policies (i.e., policies where source nodes send data through one or more fixed paths) are highly vulnerable to attacks that can compromise a node and easily falsify cost information, leading to Denial of Service (DoS) attacks [7]. For example, a "sink-hole attack" compromises a node and broadcasts a fake low cost to neighboring nodes, thus enticing all such nodes to route packets to it. The neighboring nodes in turn broadcast the low cost of the compromised node to their neighbors and the end effect is that this node acts as a sink hole for all packets while also draining the energy of the network nodes. In order to reduce the effect of such attacks, probabilistic routing is an interesting alternative, since this makes it difficult for attackers to identify an "ideal" node to take over. In this sense, the EAR policy is attractive because of its probabilistic routing structure, even though it does not attempt to provide optimal routing probabilities for network lifetime maximization. Moreover, given the dynamic behavior of a sensor network in terms of changing topology, node failures, and energy consumption, one can expect optimal routing probabilities to be time-varying. Thus, an optimal control problem formulation is a natural setting.

In this paper, we adopt an optimal control setting with the goal of determining routing probabilities so as to maximize the lifetime of a sensor network subject to a dynamic energy consumption model for each node (Section II). In Section III, we show that in a fixed topology case there exists an optimal policy consisting of fixed routing probabilities. We subsequently show that the optimal control problem may be converted into a set of relatively simple Non-Linear Programming (NLP) problems. We also consider an alternative problem (Section IV) where, in addition to routing, we also allocate a total initial energy over the network nodes with the same network lifetime maximization objective; the idea here is that a proper allocation of energy can further increase the network lifetime. We prove that the solution to this problem is given by a policy that depletes all node energies at the same time and that the corresponding energy allocation and routing probabilities are obtained by solving a single NLP problem. In Section V we provide simulation examples. In Section VI we discuss the limitations of the proposed routing policy and related future work.

II. Problem formulation

In order to simplify our analysis, we will consider a sensor network with a single source node and one base station and will assume a fixed topology. It will become clear that the methodology we use can be extended to multiple sources and base stations and time-varying topologies.

A. Network model and dynamics

Consider a network with $N+1$ nodes where nodes 0 and N are the source and destination (base station) nodes respectively. Except for the base station whose energy supply is not constrained, a limited amount of energy is available to all other nodes. Let R_i be the initial energy allocated to node $i=0,\dots,N-1$, and $r_i(t)$ be its residual energy at time t. The vector $[r_0(t),\dots,r_{N-1}(t)]$ defines the state variables for our problem. The distance between nodes i and j at time t is denoted by $d_{ij}(t)$; since we assume a fixed topology, we will treat $d_{i,j}(t)$ as time-invariant in the sequel. The nodes in the network may be ordered according to their distance to the destination node N so that

$$d_{1,N} > d_{2,N} > \cdots > d_{i,N} > \cdots > d_{N-1,N}$$

and assume that $d_{0,N} > d_{i,N}$ for all $i=1,\dots,N-1$.

Let O_i denote the set of nodes to which node i can send packets. We assume full coverage of the network and define O_i as follows:

$$O_i = \{j : j > i,\ d_{i,j} < d_{i,N}\} \tag{1}$$

where $j > i$ implies that $d_{i,N} > d_{j,N}$, i.e., a node only sends packets to those nodes that are closer to the destination node, and $d_{i,j} < d_{i,N}$ means that a node cannot send packets to a node which is further away from it than the destination node. We will use the notation

$$i \prec j, \text{ if } j \in O_i \tag{2}$$

Let $w_{i,j}(t)$ be the routing probability of a packet from node i to node j at time t. The vector $w(t) = [w_{0,1}(t),\dots,w_{0,N-1}(t),\dots,w_{N-2,N-1}(t)]$ contains the control variables in our problem. We do not include $w_{0,N}(t),\dots,w_{i,N}(t),\dots,w_{N-1,N}(t)$ in the definition of $w(t)$, since it is clear that $w_{i,N}(t)$ is an implicit control variable given by

$$w_{i,N}(t) = 1 - \sum_{i\prec j,\ j<N} w_{i,j}(t), \quad i=0,\dots,N-2 \tag{3}$$

For simplicity, the sending rate of source node 0 is normalized to 1 and let $G_i(w)$ denote the inflow rate to node i. Given the definitions above, we can express $G_i(w)$ through the following recursive equation:

$$G_i(w) = \sum_{k\prec i} w_{k,i} G_k(w), \quad i=1,\dots,N \tag{4}$$

Next, we shall derive the dynamics of each node in the network based on a model for energy loss over time. The most common model that has been adopted (e.g., see [8], [9]) makes use of the following energy parameters: the energy needed to sense a bit (E_{sense}), the energy needed to receive a bit (E_{rx}), and the energy needed to transmit a bit (E_{tx}). If the distance between two nodes is d, these parameters are given by:

$$E_{tx} = C_f + C_s d^\beta, E_{rx} = C_r, E_{sense} = C_e \tag{5}$$

where C_f, C_s, C_r, C_e are given constants dependent on the communication and sensing characteristics of nodes, and β is a constant dependent on the medium involved (i.e., $1/d^\beta$ models the signal path loss). We shall use this energy model and ignore the sensing energy, i.e., set $C_e = 0$. Clearly, this is a relatively simple energy model that does not take into consideration the channel quality or the Shannon capacity of each wireless channel. The ensuing optimal control analysis is not critically dependent on the exact form of the energy consumption model used, although the ultimate optimal value of $w(t)$ obviously is.

Before proceeding, it is convenient to define the following constants:

$$\begin{aligned} k_{ij} &= C_s d_{i,j}^{\beta} - C_s d_{i,N}^{\beta}, \quad i < j < N & (6) \\ k_{0N} &= C_f + C_s d_{0,N}^{\beta} & (7) \\ k_{iN} &= C_f + C_r + C_s d_{0,N}^{\beta}, \quad i = 1 \dots N-1 & (8) \end{aligned}$$

We can now obtain the energy dynamics of the network as follows (all proofs are omitted but may be found in [10]).

Lemma 1: The dynamics of each node $i = 0, \dots, N-1$ in the network are given by:

$$\frac{dr_i}{dt} = -G_i(w) \left\{ \sum_{i \prec j,\, j<N} w_{i,j}(t) k_{ij} + k_{iN} \right\} \tag{9}$$

■

In order to formulate an optimal control problem under the dynamics given by (9) we need to give a clear definition of the optimization objective. In what follows, we identify the network performance metric of interest in our problem.

B. Sensor network performance metric

Our objective is to maximize the lifetime of a sensor network by controlling the routing probabilities $w_{i,j}(t)$. There are several definitions for the lifetime of a sensor network. The most common definition of lifetime, however, is the time when the first node dies (i.e., its energy level reaches zero) and we shall adopt this definition for our purposes:

Definition 2.1: The lifetime T of a sensor network is defined as

$$T = \min_{0 \le i < N} t_i(w)$$

where $t_i(w)$ is given by

$$t_i(w) = \inf\{t : r_i(t) = 0,\ t > 0\}$$

C. Optimal control problem formulation

Given the energy dynamics of the network and the definition of its lifetime, we can formulate an optimal control problem whose objective is

$$\min_{w} \int_0^T -1 dt \tag{10}$$

with state equations

$$\frac{dr_i}{dt} = -G_i(w) \left(\sum_{i \prec j, j<N} w_{i,j} k_{ij} + k_{i,N} \right), \tag{11}$$

where $i = 0, \dots, N-1$, and boundary conditions

$$\begin{aligned} r_i(0) &= R_i, \quad i = 0, \cdots, N-1 & (12) \\ \min r_i(T) &= 0,\ 0 \le i < N & (13) \end{aligned}$$

and control constraints

$$\sum_{i \prec j, j<N} w_{i,j}(t) \le 1 \tag{14}$$

$$0 \le w_{i,j}(t) \le 1 \tag{15}$$

The objective function (10) implies maximization of the lifetime, since (13) requires that the node with the least residual energy satisfy $r_i(T) = 0$ and $r_i(t)$ is monotonically nonincreasing.

This is a classic minimum (maximum) time optimal control problem except for two complicating factors: (i) The boundary condition (13) which involves the nondifferentiable $\min$ function, and (ii) The control constraints (14)-(15).

III. Solution of the optimal control problem

We begin by defining the Hamiltonian for this problem:

$$H(w, t, \lambda) = -1 + \sum_{i<N} \lambda_i \frac{dr_i}{dt} \tag{16}$$

where λ_i is the costate (multiplier function) that satisfies

$$\dot{\lambda}_i = \frac{\partial H}{\partial r_i}, \quad i = 0, \dots, N-1$$

Looking at (11), note that $\frac{dr_i}{dt}$ is not an explicit function of r_i, which implies that $H(w, t, \lambda)$ is also not an explicit function of r_i. It follows that $\frac{\partial H}{\partial r_i} = 0 = \dot{\lambda}_i$ and we have

$$\lambda_i = \text{constant}, \quad i = 0, \dots, N-1$$

In order to evaluate these constants, we need to make use of the transversality condition [11] as well as the boundary condition (13) at final time T . However, since the presence of the $\min$ function in (13) complicates matters, we proceed instead by considering different cases separately, one case for each possible node $i = 0, \dots, N-1$ dying first. We refer to S_i as the ith such scenario and analyze it next.

A. Analysis of scenario S_i

Under scenario S_i, node i dies first and at final time T we have

$$0 = r_i(T) \le r_j(T), \quad j \ne i$$

In this case, the boundary condition (13) becomes

$$r_i(T) = 0 \tag{17}$$

and all other j are unconstrained at T. The following lemma establishes the fact that, under S_i and assuming a fixed topology, there exists a time-invariant optimal routing policy.

Theorem 1: If $0 = r_i(T) \le r_j(T)$, $j \ne i$ for some i and the network topology is fixed, i.e., $d_{i,j}(t) = \text{constant}$, then there exists an optimal policy such that for all $t \in [0, T]$:

$$w^*(t) = w^*(T)$$

Note that there may exist multiple optimal control policies, including some that may be time-varying. The above result asserts that there is at least one which is time-invariant, i.e., $w_{ij}^*(t) = w_{ij}^*$, and it remains to obtain these values. From (16), we can see that in order to minimize H, we must obtain

w^* that maximizes $\frac{dr_i}{dt}$. Therefore, using (9) in Lemma 1, w^* must minimize

$$G_i(w)\left(\sum_{i\prec j,\ j<N} w_{i,j}k_{ij}+k_{iN}\right) \tag{18}$$

subject to (14)-(15). Thus, we convert a complex optimal control problem into a relatively simple Non-Linear Programming (NLP) problem. However, this NLP problem must also satisfy constraints implied by scenario S_i. In particular, node i must die before all other nodes. Let us define the lifetime of any node m under a time-invariant control policy w as

$$\begin{aligned} t_m(w) &= R_m\left|\frac{dr_m}{dt}\right|^{-1} \\ &= R_m\left[G_m(w)\left(\sum_{m\prec j,\ j<N} w_{m,j}k_{mj}+k_{mN}\right)\right]^{-1} \end{aligned} \tag{19}$$

Then, under scenario S_i, we must have

$$t_i(w)\le t_j(w) \quad \text{for all } j\neq i$$

in order to guarantee that node i dies first. To summarize, scenario S_i reduces to solving the following NLP problem:

$$\begin{aligned} \min_w\ & G_i(w)\left(\sum_{i\prec j, j<N} w_{i,j}k_{ij}+k_{iN}\right) \\ \text{s.t. } & t_i(w)\le t_j(w) \quad \text{for all } j\neq i \\ & \sum_{i\prec j,\ j<N} w_{i,j}\le 1 \\ & 0\le w_{i,j}\le 1 \end{aligned}$$

We will refer to this as problem $\mathbf{P}_i$. If $i\neq 0$, then $\mathbf{P}_i$ may have no feasible solution (when $i=0$, recall that $G_0(w)=1$). Before discussing the necessary condition that $\mathbf{P}_i$ has a feasible solution, let us obtain upper and lower bounds for $t_i(w)$ as defined in (19).

Looking at (7)-(8), we know that

$$k_{iN}>0, \text{ for all } i=0,\dots,N-1 \tag{20}$$

Based on (1) and (2), we also know that

$$d_{i,j}<d_{i,N}, \quad \text{if } i\prec j \text{ and } j<N$$

Then, if $j<N$ and $i\prec j$, we have in (6):

$$k_{ij}=C_s d_{i,j}^2-C_s d_{i,N}^2<0 \tag{21}$$

Let us also define the following index:

$$\gamma(i)=\arg\min_{j<N,\ i\prec j} k_{ij} \tag{22}$$

which, from the definition of k_{ij}, provides the nearest node in the output node set of i. Then, we can evaluate upper and lower bounds for any node lifetime $t_i(w)$ as follows.

Lemma 2: For all $i\neq 0$,

$$\frac{R_i}{k_{iN}}\le t_i(w)\le\infty \tag{23}$$

and

$$\frac{R_0}{k_{0N}}\le t_0(w)\le\frac{R_0}{k_{0\gamma(0)}+k_{0N}} \tag{24}$$

■

We now return to the issue of determining when $\mathbf{P}_i$ has a feasible solution. From Lemma 2, we know that if $i\neq 0$ and

$$\frac{R_i}{k_{iN}}>\frac{R_0}{k_{0\gamma(0)}+k_{0N}} \tag{25}$$

then $t_i(w)>t_0(w)$ and $\mathbf{P}_i$ has no feasible solution. Then, the necessary condition that $\mathbf{P}_i$ $(i>0)$ to have a feasible solution is

$$\frac{R_i}{k_{iN}}\le\frac{R_0}{k_{0\gamma(0)}+k_{0N}} \tag{26}$$

B. Algorithm for solving the optimal control problem

Based on the discussion above, if we focus on a fixed scenario S_i, the solution to the optimal control problem is just the solution to the NLP problem $\mathbf{P}_i$. However, since we do not know which node will die first under the actual optimal control policy, we need to solve all $\mathbf{P}_i$ problems and find the best policy among them. Since not all $\mathbf{P}_i$ problems have feasible solutions we can use (26) to check feasibility before solving this NLP problem.

The complete algorithm, referred to as **A1**, to solve this optimal control problem is:

1) Solve NLP problem $\mathbf{P}_0$ to obtain t_0^*.
2) For $0<i<N$, if $\frac{R_i}{k_{iN}}\le\frac{R_0}{k_{0\gamma(0)}+k_{0N}}$, solve NLP problem $\mathbf{P}_i$ and obtain t_i^*; otherwise, set $t_i^*=-1$.
3) The optimal lifetime is given by $\max_i\ \{t_i^*\}$ and the corresponding optimal policy w^* is the one obtained for the associated problem $\mathbf{P}_i$.

IV. A JOINT OPTIMAL ROUTING AND INITIAL ENERGY ALLOCATION PROBLEM

In addition to controlling routing in a sensor network, one may also control the allocation of initial energy over its nodes so as to achieve lifetime maximization. Let us define the total initial energy as $\bar{R}$ and let $R=[R_0,\dots,R_{N-1}]$. From Theorem 1, we know that the optimal routing policy is fixed unless the topology of the network changes. Then, we can formulate the following problem:

$$\begin{aligned} & \max_{w,\ R}\ T \\ \text{s.t. } & T\le t_i(w,R), \quad i=0,\dots,N-1 \\ & 0\le w_{i,j}\le 1,\ \ i,\ j=0,\dots,N,\ \ i\prec j \\ & \sum_{i\prec j,\ j<N} w_{i,j}\le 1 \\ & 0<R_i<\bar{R}, \qquad \sum_{i=0}^{N-1} R_i=\bar{R} \end{aligned} \tag{27}$$

This is a NLP problem where the control variables are the routing probabilities w_{ij} and initial energies R_i. We refer to this as the NLPMAX problem. One way to view this is as an *inverse of the deployment problem* in sensor networks, which considers how to deploy sensors in some area given an energy allocation over nodes. Here, we consider the problem of energy allocation given the locations of all sensors.

Let us define, for $i = 0, \ldots, N-1$, the functions

$$K_i(w) = \left| \frac{dr_i}{dt} \right|^{-1} = \left[G_i(w) \left(\sum_{i \prec j,\, j<N} w_{i,j} k_{ij} + k_{iN} \right) \right]^{-1} \quad (28)$$

Then, depending on w, R, the lifetime of node i, denoted by $t_i(w, R)$, is given by

$$t_i(w, R) = K_i(w) R_i \quad (29)$$

If the optimal solution for the NLPMAX problem is (w^*, R^*), then $t_i^* = t_i(w^*, R^*) = K_i(w^*) R_i^*$. This optimal solution can be shown to satisfy the following:

Theorem 2: The solution of problem NLPMAX satisfies

$$t_0^* = t_1^* = \cdots = t_{N-1}^* \quad (30)$$

Remark: In order to guarantee (30), we need to let $t_i(w, R) < \infty$. From (28), (29) this is equivalent to assuming that $G_i(w) > 0$, i.e., we assume no node is left unutilized.

Based on Theorem 2, we can simplify the NLPMAX problem (27). In particular, we solve this NLP problem in two steps. In Step 1, assuming a fixed routing policy w, we determine the corresponding optimal initial energy distribution policy by solving the following simple equations:

$$K_0(w) R_0 = \cdots = K_{N-1}(w) R_{N-1} \quad (31)$$

$$\sum_{i=0}^{N-1} R_i = \bar{R} \quad (32)$$

whose solution is defined to be $R^*(w)$ and the lifetime under this policy $(w, R^*(w))$ is $T^*(w)$. Then, in Step 2 we search over the feasible set of policies w to find the optimal $T^*(w)$. We further discuss this two-step process in what follows and show that it reduces to a single nonlinear minimization problem. Regarding the first step, the following lemma provides an expression for the network lifetime $T^*(w)$.

Lemma 3: Given the total energy $\bar{R}$ and a routing control policy w, the optimal initial energy allocation is

$$R_i = \frac{\bar{R}}{K_i(w)} \left[\sum_{j=0}^{N-1} \frac{1}{K_j(w)} \right]^{-1}, \quad i = 0, \ldots, N-1 \quad (33)$$

and the optimal lifetime is

$$T^*(w) = \bar{R} \left[\sum_{j=0}^{N-1} G_i(w) \left(\sum_{i \prec j,\, j<N} w_{i,j} k_{ij} + k_{i,N} \right) \right]^{-1} \quad (34)$$

Based on Theorem 2 and Lemma 3, it is clear that the solution of problem NLPMAX is the same as that of the following NLP problem:

$$\min_w \sum_{j=0}^{N-1} G_i(w) \left(\sum_{i \prec j,\, j<N} w_{i,j} k_{ij} + k_{i,N} \right)$$

$$0 \le w_{i,j} \le 1, \quad 0 \le i, j \le N \text{ and } i \prec j$$

$$\sum_{i \prec j,\, j<N} w_{i,j} \le 1$$

$$G_i(w) > 0$$

We refer to this as problem NLPMIN. We can find the solution of the original problem NLPMAX through the following two steps: (i) Solve problem NLPMIN and find the optimal routing policy w^*, (ii) Determine the optimal initial energy allocation R_i^*, $i = 0, \ldots, N-1$, through (33). Observe that the solution of NLPMIN only depends on the topology of the network and is independent of $\bar{R}$. Obviously, solving this problem by standard means can only guarantee a locally optimal routing policy w^*.

V. Numerical results

Table 1 gives the node coordinates of a randomly generated network with 7 nodes, in which node 0 is the source node, node 6 is the base station, and the remaining are relay nodes. In the following example, we set $C_s = 0.0001$, $C_f = C_r = 0.05$, and $\beta = 2$. The total initial energy is $\bar{R} = 100$.

i	0	1	2	3	4	5	6
x_i	0	9.14	35.80	69.08	94.53	131.35	150
y_i	0	5.69	50.47	76.25	110.94	145.80	150

Table 1: Node coordinates in a 7-node network

We consider the first case where the initial energy allocation is given as $R_i = 16.67$ for all $i = 0, \ldots, 5$. By using the **A1** algorithm, the optimal routing policies obtained are listed by Table 2. Observe that there is no obvious structure in this optimal routing policy. Since our optimization algorithm depends on the location of nodes, once the topology of the network changes due to some node failure or movement, we need to re-execute **A1** so as to obtain new optimal routing probabilities.

w_{ij}	0	1	2	3	4	5
0	N/A	0.498	0.426	0.073	0	0
1	N/A	N/A	0.923	0	0	0.077
2	N/A	N/A	N/A	0.957	0.001	0.042
3	N/A	N/A	N/A	N/A	0.931	0.069
4	N/A	N/A	N/A	N/A	N/A	1

Table 2: Optimal routing probabilities with fixed initial node energies

Next, we compare the lifetime under this optimal routing policy and the following alternative policies: (i) A local greedy policy, in which the routing probabilities are

$$w_{i,j} = \begin{cases} 1 & j = \gamma(i) \\ 0 & \text{otherwise} \end{cases} \quad (35)$$

where $\gamma(i)$ was defined in (22). In other words, every node always sends packets to the nearest node in its output set. (ii) A purely random policy, in which every node sends packets to all nodes in its output set with the same probability. (iii) The EAR policy mentioned in the introduction, in which the routing probabilities are set as follows (see [6]):

$$w_{i,j} = C_{ij}^{-1} \sum_{k \in O(i)} C_{ik}^{-1} \tag{36}$$

where C_{ij} is given by

$$C_{ij} = d_{i,j}^{k_1} r_j^{k_2} + C_j, \text{ for all } j \in O(i) \tag{37}$$

and C_i is given by

$$C_i = \sum_{j \in O(i)} w_{i,j} C_{ij} \tag{38}$$

In (37), r_j is the residual energy of node j and k_1 and k_2 are two parameters that need to be set. Based on (36)-(38), EAR uses a recursive way to set routing probabilities.

Table 3 compares the network lifetime under the optimal routing policy, T_{opt}, the local greedy policy, T_{greedy}, random routing, T_{ran}, and the EAR policy, T_{EAR}, with $k_1 = 3$ and $k_2 = 1$. We observe that the performance of the random routing and EAR policies are very far from optimal. Moreover, the EAR policy needs both location and residual energy information and its cost-based nature makes it highly vulnerable to DoS types of attack. The local greedy policy is considerably better but as a fixed routing policy it is also vulnerable to DoS attacks. The optimal routing policy we have developed provides significantly better lifetime performance. However, it depends on location information and requires a centralized solution of NLP problems.

T_{opt}	T_{greedy}	T_{ran}	T_{EAR}
54.3596	44.8513	8.1777	17.9615

Table 3: Network lifetime comparison under different routing policies

Finally, we consider the results of the joint optimal routing and initial energy allocation problem. By solving the NLPMIN problem, the optimal initial energy allocation is given in Table 4 and the optimal routing probabilities are given in Table 5. The optimal lifetime in this case is 67.1515, a significant improvement over the optimal lifetime under a fixed initial energy allocation. Thus, we can see that *adjusting the initial energy allocation can considerably extend the sensor network's lifetime*. Implementing such a policy may not always be feasible, since often the initial energy for each node is fixed. However, results such as those of Table 4 provide information regarding those areas where backup nodes should be allocated. It is also interesting to observe that in this case a greedy routing policy may be near-optimal, as Table 5 suggests.

i	0	1	2	3	4	5
R_i	4.1	24.9	18.6	19.1	24.0	9.2

Table 4: Optimal initial energy allocation

w_{ij}	0	1	2	3	4	5
0	N/A	0.9998	0.0002	0	0	0
1	N/A	N/A	1	0	0	0
2	N/A	N/A	N/A	1	0	0
3	N/A	N/A	N/A	N/A	1	0
4	N/A	N/A	N/A	N/A	N/A	1

Table 5: Optimal routing probabilities under optimal initial energy allocation

VI. Conclusions and Future Research

We have used an optimal control approach to model, analyze, and solve the problem of routing in sensor networks where the goal is to maximize the network's lifetime. For a fixed initial energy allocation, we have found that if the topology is fixed, there exists a time-invariant optimal routing policy and the associated routing probabilities can be determined by solving a set of NLP problems. This solution is centralized and requires global location information, so an obvious direction to pursue is one seeking distributed versions of the same approach if possible. We have also considered an interesting new problem, that of joint optimal routing and initial energy allocation, which can be solved through a simpler single NLP problem. The solutions we have obtained to both problems provide a baseline for comparing alternatives and assessing their difference from the optimal, as illustrated through the numerical examples we have presented.

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