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Control of hysteretic base-isolated structures: an adaptive backstepping approach

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Abstract— **A hybrid seismic control system for building structures is considered, which combines a class of passive nonlinear base isolator with an active control system. The objective of the active control component applied to the structural base is to keep the base displacement relative to the ground, the interstory drift and the absolute acceleration within a reasonable range according to the design of the base isolator. We use the adaptive backstepping approach for the control design. The base isolator device exhibits a hysteretic nonlinear behavior which is described analytically by the Bouc-Wen model. The control problem is formulated representing the system dynamics in two alternative coordinates: absolute (with respect to an inertial frame) and relative to the ground. A comparison between both strategies is presented by means of numerical simulations.**

I. INTRODUCTION

For the purpose of maintaining the seismic response of structures within safety, service and comfort limits, the combination of passive base isolators and feedback controllers (applying forces to the base) has been proposed in the last years. Some works have proposed active feedback systems, like for instance [1], [5] and [6]. More recently, semiactive controllers have been proposed in the same setting trying to get advantages of their easier implementation (see for instance [8] and [10]).

The basic concept of base isolation is to ideally make the structure behave like a rigid body through a certain degree of decoupling from the ground motion. In this way it is possible to absorb part of the energy induced by the earthquake, by reducing simultaneously the relative displacements of the structure with respect to the base (damage source) and the absolute accelerations (endangering human comfort and safety of installations). The feasibility of adding a feedback control is based on the premise that only a control action is to be applied at the base with force magnitudes which are not excessive due to the high flexibility of the isolators. The benefits of the inclusion of the control lie mainly in that the "cooperation" of such a force can avoid large displacements of the base isolator, which could endanger the scheme integrity; and it may also introduce an additional resistant scheme not dependable of the interstory drifts, which are already small due to the effect of the isolator. This may be useful, particularly for structures having sensitive installations, like hospitals, public services, computer facilities, etc.

From a theoretical point of view, the development of a control law to calculate the active forces involves difficulties associated with the nonlinear behavior of the base isolators and with the uncertainties in the model describing the structure-base-isolator system and in the seismic excitation. Recently a new tool has been proposed in the control theory to design nonlinear schemes for uncertain systems [7]. Previous works have proposed nonlinear controllers for active base isolation, like [6], [1], [5] and [8]. The backstepping is an appealing alternative since it gives computable explicit bounds for the tracking error as a function of the size of the uncertainties.

An important issue in considering the model for the control formulation is the coordinates adopted to represent the motion. In the vein of the most common practice in the earthquake engineering design of base isolation systems, some authors have used models based on coordinates relative to the ground for the control design (see for example [5] and [10]). Other authors have approached the problem by using absolute coordinates with respect to an inertial frame of reference, like in [6], [1] and [8].

In this work, we design a backstepping control system for a simple prototype base isolated uncertain structure by using models in both coordinates trying to give some insight on their advantages and drawbacks.

II. STRUCTURAL MODELS

Consider a base isolated structure with an active controller as illustrated in Figure 1. The passive component consists of a hysteretic base isolator.

The whole system can be described by a model composed of two coupled systems: Σ_s (the structure) and Σ_b (the base). It can be represented in two different coordinate systems:

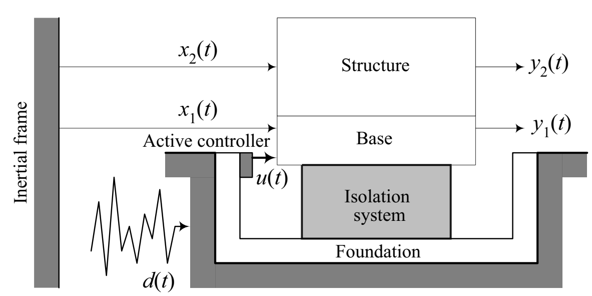

Fig. 1. Building structure with hybrid control system.

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absolute (with respect to an inertial frame) and relative to the ground.

The relative equations of motion are

$$\begin{aligned}\Sigma_b^r : m_1\ddot{y}_1 + (\bar{c}_1+\bar{c}_2)\dot{y}_1 + (\bar{k}_1+\bar{k}_2)y_1 &= \\ = \bar{k}_2 y_2 + \bar{c}_2\dot{y}_2 - \Phi(y_1,t) - m_1\ddot{d} + u & \quad (1)\\ \Sigma_s^r : m_2\ddot{y}_2 + \bar{c}_2\dot{y}_2 + \bar{k}_2 y_2 - \bar{c}_2\dot{y}_1 - \bar{k}_2 y_1 = -m_2\ddot{d} & \quad (2)\end{aligned}$$

The absolute equations of motion are the following

$$\begin{aligned}\Sigma_b^a : m_1\ddot{x}_1 + (\bar{c}_1+\bar{c}_2)\dot{x}_1 + (\bar{k}_1+\bar{k}_2)x_1 &= \\ = \bar{k}_2 x_2 + \bar{c}_2\dot{x}_2 - \Phi(x_1-d,t) + \bar{c}_1\dot{d} + \bar{k}_1 d + u & \quad (3)\\ \Sigma_s^a : m_2\ddot{x}_2 + \bar{c}_2\dot{x}_2 + \bar{k}_2 x_2 - \bar{c}_2\dot{x}_1 - \bar{k}_2 x_1 = 0 & \quad (4)\end{aligned}$$

where m_1 and m_2 are the mass of the base and the structure, respectively; $\bar{c}_1$ and $\bar{c}_2$ are the damping coefficients; $\bar{k}_1$ and $\bar{k}_2$ are the stiffness coefficients; x_1 and x_2 are the absolute displacement of the base and the structure, respectively, which can be measured by using some recently developed technique ([2]); the excitation is produced by a horizontal seismic ground motion characterized by an inertial displacement $d(t)$, velocity $\dot{d}(t)$ and acceleration $\ddot{d}(t)$; the base displacement relative to the ground is $y_1 = x_1 - d$, while $y_2 = x_2 - d$ is the relative structure displacement; Φ is the restoring force characterizing the hysteretic behavior of the isolator material, which is usually made with inelastic rubber bearings; and u is the control force supplied by an appropriate actuator.

The hysteretic force Φ is described by the Bouc-Wen model ([11]) in the following form:

$$\Phi(x) = \alpha k_0 x + (1-\alpha) D k_0 z \quad (5)$$

$$\dot{z} = D^{-1}\left[\bar{A}\dot{x} - \beta|\dot{x}||z|^{n-1}z - \bar{\gamma}\dot{x}|z|^n\right] \quad (6)$$

where $\Phi(x,t)$ can be considered as the superposition of an elastic component $\alpha k_0 x(t)$ and a hysteretic component $(1-\alpha)Dk_0 z(t)$, in which $D > 0$ is the yield constant displacement and $\alpha \in [0,1]$ is the post to pre-yielding stiffness ratio. The hysteretic part involves a dimensionless auxiliary variable z which is the solution of the nonlinear first order differential equation (6). In this equation, $\bar{A}, \beta$ and $\bar{\gamma}$ are dimensionless parameters which control the shape and the size of the hysteresis loop, while n is an scalar that governs the smoothness of the transition from elastic to plastic response.

III. Control strategies

Looking at equation (1), it is clear that a feedback control law can be designed to supply a force u able to control the relative displacement of the base against the earthquake excitation, which is the ground acceleration. However, this excitation enters also in equation (2), which has no control. This means that the relative motion of the structure is subjected to the seismic acceleration but no feedback control is directly exerted to mitigate the effect of this excitation. In fact, the use of relative coordinates arises from the desire to keep the motion (displacement and velocity) of each floor relative to the ground small (and hence, of a given floor relative to those below and above it). Roughly speaking, feedback control is employed to achieve that end attempting "to move the whole structure" so as to follow the motion of the ground ([6]).

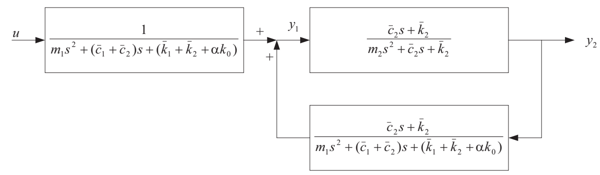

Fig. 2. Block diagram representation.

Looking at equation (3), it is clear that a feedback control law can be designed to supply a force u able to control the absolute displacement of the base against the earthquake excitation, which is now a linear combination of the ground displacement and velocity. We may observe that this excitation does not enter in equation (4). Then, the control of the base motion leads to the control of the structure's motion. Also, it is expectable that the effort to control the system (3)-(4) be smoother than in the case of controlling system (1)-(2) since the ground displacement and velocity are smoother than the ground acceleration. The origin of the use of absolute coordinates can be found in [6] based on the idea of keeping the whole structure stationary relative to its initial configuration (i.e., relative to an inertial frame of reference) and, roughly speaking, "letting the ground move under it".

In order to establish a comparison between both alternatives we consider two strategies for the feedback control design:

(a) when using the relative coordinates: measure y_1 and regulate y_1;
(b) when using the absolute coordinates: measure x_1 and regulate x_1.

In Section V, a comparison between the strategies is presented by means of numerical simulations.

IV. Controller design

In order to use the adaptive backstepping approach for the control design, we need to describe the models (1)-(2) and (3)-(4), respectively, along with equations (5)-(6), in the transfer function form.

A. Model description

We consider $d(t)$ and $z(t)$ as external disturbances. In the particular case $d(t) = 0$ and $z(t) = 0$, we can write the equations (7)-(8) (see Figure 3) in a block diagram representation depicted in Figure 2.

In this case it can be shown, using the Nyquist stability criterion [9], that the system represented in Figure 2 is always stable, that is, the transfer function between the signals y_2 and u is stable. The explicit expression of this transfer function is

$$y_2(s) = \frac{\dfrac{c_2}{m_1 m_2}s + \dfrac{k_2}{m_1 m_2}}{A(s)}u(s) \quad (9)$$

$$\left[m_1 s^2 + (\bar{c}_1 + \bar{c}_2)s + (\bar{k}_1 + \bar{k}_2 + \alpha k_0)\right] y_1(s) - \left[\bar{c}_2 s + \bar{k}_2\right] y_2(s) = u(s) + (1-\alpha) D k_0 z(s) - m_1 s^2 d(s) \tag{7}$$

$$\left[m_2 s^2 + \bar{c}_2 s + \bar{k}_2\right] y_2(s) - \left[\bar{c}_2 s + \bar{k}_2\right] y_1(s) = -m_2 s^2 d(s) \tag{8}$$

Fig. 3. Laplace transform of the equations (1)-(2).

where

$$\begin{aligned} A(s) =& s^4 + \frac{m_1\bar{c}_2 + \bar{c}_2 m_2 + \bar{c}_1 m_2}{m_1 m_2} s^3 \\ &+ \frac{m_1\bar{k}_2 + \bar{k}_1 m_2 + \bar{k}_2 m_2 + \alpha k_0 m_2 + \bar{c}_1\bar{c}_2}{m_1 m_2} s^2 \\ &+ \frac{\bar{k}_1\bar{c}_2 + \bar{c}_1\bar{k}_2 + \alpha k_0\bar{c}_2}{m_1 m_2} s + \frac{\alpha k_0\bar{k}_2 + \bar{k}_1\bar{k}_2}{m_1 m_2} \end{aligned} \tag{10}$$

The stability of the transfer function in equation (9) implies the stability of the denominator $A(s)$, that is, the roots of $A(s)$ have negative real parts. The stability of this polynomial expression will be used in the following lines.

In the general case ($d(t) \neq 0, z(t) \neq 0$), after some manipulations and eliminating the variables y_2 and x_2, the models (1)-(2) and (3)-(4), respectively, along with equations (5)-(6), can be written as

$$\begin{aligned} y_1(t) &= \frac{B(s)}{A(s)} u(t) + \overbrace{\frac{B_d^r(s)}{A(s)} d(t) + \frac{B_z^r(s)}{A(s)} z(t)}^{p_r(t)} \\ &= \frac{B(s)}{A(s)} u(t) + p_r(t) \end{aligned} \tag{11}$$

$$\begin{aligned} x_1(t) &= \frac{B(s)}{A(s)} u(t) + \underbrace{\frac{B_d^a(s)}{A(s)} d(t) + \frac{B_z^a(s)}{A(s)} z(t)}_{p_a(t)} \\ &= \frac{B(s)}{A(s)} u(t) + p_a(t) \end{aligned} \tag{12}$$

where $A(s)$ coincides with the polynomial expression in equation (10) and

$$\begin{aligned} B(s) =& \frac{1}{m_1} s^2 + \frac{\bar{c}_2}{m_1 m_2} s + \frac{\bar{k}_2}{m_1 m_2} \\ B_d^r(s) =& -s^4 - \frac{m_1\bar{c}_2 + m_2\bar{c}_2}{m_1 m_2} s^3 - \frac{m_2\bar{k}_2 + m_1\bar{k}_2}{m_1 m_2} s^2 \\ B_z^r(s) =& \frac{-(1-\alpha) D k_0}{m_1} s^2 + \frac{-\bar{c}_2(1-\alpha) D k_0}{m_1 m_2} s \\ &+ \frac{-\bar{k}_2(1-\alpha) D k_0}{m_1 m_2} \\ B_d^a(s) =& \frac{\bar{c}_1}{m_1} s^3 + \frac{\bar{k}_1 m_2 + \alpha k_0 m_2 + \bar{c}_1\bar{c}_2}{m_1 m_2} s^2 \\ &+ \frac{\bar{c}_2\alpha k_0 + \bar{k}_2\bar{c}_1 + \bar{c}_2\bar{k}_1}{m_1 m_2} s + \frac{\alpha k_0\bar{k}_2 + \bar{k}_1\bar{k}_2}{m_1 m_2} \\ B_z^a(s) =& \frac{-(1-\alpha) D k_0}{m_1} s^2 + \frac{-\bar{c}_2(1-\alpha) D k_0}{m_1 m_2} s \\ &+ \frac{-\bar{k}_2(1-\alpha) D k_0}{m_1 m_2} \end{aligned}$$

In models (11) and (12), we consider both the earthquake motion $d(t)$ and the hysteretic variable $z(t)$ as unknown disturbances. This is why we define the signals $p_r(t)$ and $p_a(t)$. The direct transfer function between the control force u and the controlled output is the same for both models:

$$\frac{B(s)}{A(s)} = \frac{b_2 s^2 + b_1 s + b_0}{s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{13}$$

where the coefficients are

$$\begin{aligned} b_2 &= \frac{1}{m_1}, \quad b_1 = \frac{\bar{c}_2}{m_1 m_2}, \quad b_0 = \frac{\bar{k}_2}{m_1 m_2} \\ a_3 &= \frac{m_1\bar{c}_2 + \bar{c}_2 m_2 + \bar{c}_1 m_2}{m_1 m_2} \\ a_2 &= \frac{\bar{k}_2 m_2 + \bar{c}_1\bar{c}_2 + \alpha k_0 m_1 + \bar{k}_1 m_2 + m_1\bar{k}_2}{m_1 m_2} \\ a_1 &= \frac{\bar{k}_1\bar{c}_2 + \alpha k_0\bar{c}_2 + \bar{c}_1\bar{k}_2}{m_1 m_2} \\ a_0 &= \frac{\alpha k_0\bar{k}_2 + \bar{k}_1\bar{k}_2}{m_1 m_2}. \end{aligned}$$

In order to use the adaptive backstepping controller developed by [4], we need to show that the signals $p_a(t)$ and $p_r(t)$ are bounded disturbances.

On one hand, we assume that the earthquake motion $d(t)$ is bounded. On the other hand, it has been shown in a previous work ([3]), that the hysteretic component $z(t)$ is always bounded under a particular choice of the parameters $\bar{A}, \beta$ and $\bar{\gamma}$ ($\bar{A} > 0,\ \ \beta + \bar{\gamma} > 0,\ \beta - \bar{\gamma} \geq 0$ or $\bar{A} > 0,\ \beta - \bar{\gamma} < 0,\ \beta \geq 0$).

The boundedness of the signals $d(t)$ and $z(t)$ and the stability of the polynomial expression $A(s)$ allow us to consider $p_r(t)$ and $p_a(t)$ as *bounded disturbances*.

B. Adaptive backstepping control

Since we consider that the parameters of the models are uncertain, we use adaptive control to stabilize the control loop. Denoting anyone of the parameters $m_i, \bar{k}_i, \bar{c}_i, (i = 1,2), k_0$ and α by p, we assume that $p \in [p_{\min}, p_{\max}]$, $p_{\min}$ and $p_{\max}$ being known, i.e. we assume the knowledge of an interval for each parameter.

To have computable bounds on the transient behavior, we use the backstepping approach in the presence of bounded disturbances described in [4].

Our objective is to design an adaptive controller such that

(i) all the closed-loop signals should be globally bounded;
(ii) the output tracking error $y_1(t) - y_r(t)$ (resp. $x_1(t) - x_r(t)$) should be proportional, in the mean, to the size of the unmodelled effects. Furthermore, the transient

behaviour of $y_1(t) - y_r(t)$ (resp. $x_1(t) - x_r(t)$) should be explicitly quantified;

(iii) in the ideal case, i.e., $p_r(t) \equiv 0$ (resp. $p_a(t) \equiv 0$), the error should converge to zero.

In our implementation, the reference signals $y_r(t)$ and $x_r(t)$ are chosen as $y_r(t) = x_r(t) \equiv 0$.

The control law and the parameter update law are designed in $\rho = 2$ steps. Figure 7 summarizes these steps and serves as an algorithm for the control implementation. In this figure, the quantities c_1,c_2,d_1,d_2,g_0,γ,$\sigma_{s\varrho}$,$\sigma_{s\theta}$ and Γ are positive design parameters and M_θ is defined as

$$M_\theta = \sqrt{\sum_{i=0}^{3} a_i^2 + \sum_{i=0}^{2} b_i^2}.$$

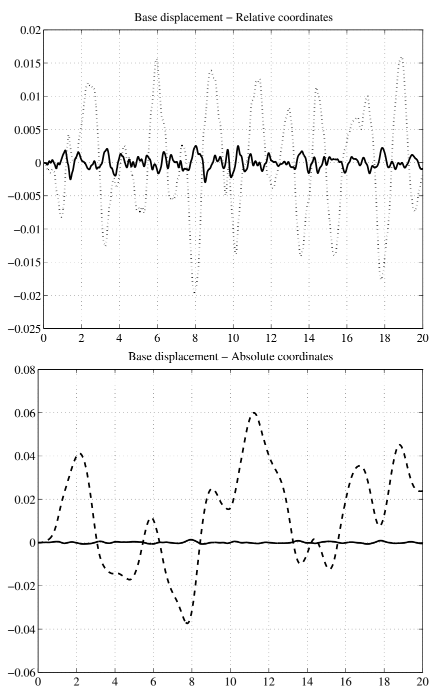

Fig. 4. 1952 Taft earthquake. Base displacement (solid) is clearly reduced with respect to the purely passive control (dashed) in both relative (up) and absolute (down) coordinates.Time (s) in x-axis; length (m) in y-axis.

C. Robustness analysis

In this section we present the robustness and asymptotic performance results which are derived following the reference by [4, Theorem 3.1].

If we consider the system (11) (resp. 12) and the adaptive controller composed of the control law and the parameter update law described in Figure 7, then there exists positive constants c and g independent of p_r and $\dot{p}_r$ (resp. p_a and $\dot{p}_a$) such that

(i) all the signals of the closed loop are globally bounded

(ii) the magnitude of the output is proportional to the size of perturbations according to:

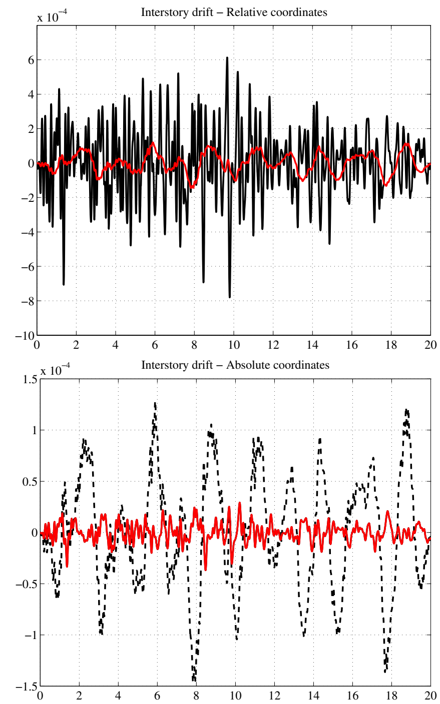

Fig. 5. 1952 Taft earthquake. Interstory drift in the controlled and purely passive cases in both relative (up) and absolute (down) frames of reference. Time (s) in x-axis; length (m) in y-axis.

$$\int_t^{t+T} y_1^2(\tau)d\tau \leq g + c \int_t^{t+T} \left(p_r(\tau)^2 + \dot{p}_r(\tau)^2\right) d\tau \tag{14}$$

$$\int_t^{t+T} x_1^2(\tau)d\tau \leq g + c \int_t^{t+T} \left(p_a(\tau)^2 + \dot{p}_a(\tau)^2\right) d\tau \tag{15}$$

D. Transient bounds

In this section we give explicit $\mathcal{L}_\infty$ bounds on the magnitude of the outputs, as a direct application of [4, Theorem 4.1].

Consider the system (11) (resp. 12) and the adaptive controller composed of the control law and the parameter update law described in Figure 7. Then, from [4] we have:

$$\|y_1(t)\|_\infty \leq \frac{1}{\sqrt{c_0 d_0}} f_r + \frac{1}{\sqrt{c_0 g_0}} g_r \tag{16}$$

$$\|x_1(t)\|_\infty \leq \frac{1}{\sqrt{c_0 d_0}} f_a + \frac{1}{\sqrt{c_0 g_0}} g_a \tag{17}$$

where f_r, g_r, f_a and g_a are scalar functions that depend only on the process parameters a_i, b_i, the observer parameters k_1, k_2, k_3 in the control law (Figure 7), $\|p_r\|_\infty$ and $\|\dot{p}_r\|_\infty$ (resp. $\|p_a\|_\infty$ and $\|\dot{p}_a\|_\infty$). The design parameters c_0 and d_0 are defined as

$$c_0 = \min_{i=1,2} c_i, \quad d_0 = \left(\sum_{i=1}^{2} \frac{1}{d_i}\right)^{-1}$$

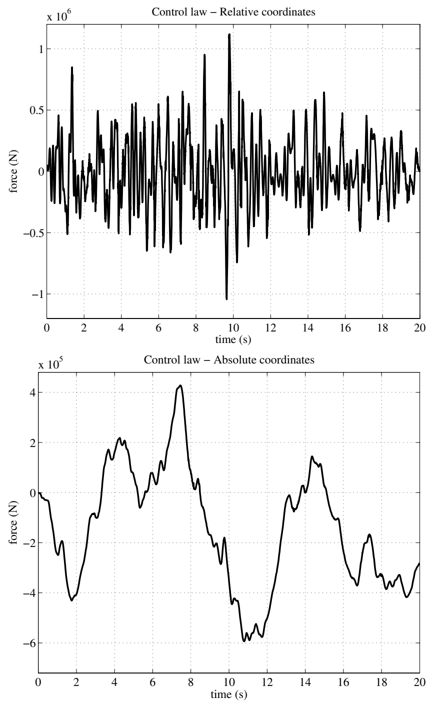

Fig. 6. 1952 Taft earthquake. Control signal force (N) in both the relative (up) and the absolute (down) coordinates cases.

V. NUMERICAL SIMULATIONS

In order to investigate the efficiency of the proposed control and to establish a comparison between the relative and absolute coordinates strategies, we consider the 1952 Taft and the 1989 Loma Prieta earthquakes.

In our simulation, the structure is modeled as a single-degree-of-freedom system, whose parameters are listed in Table I. The hysteretic parameters are also described in Table II. We remark that this particular choice of the hysteretic parameters satisfies $A > 0, \beta + \bar{\gamma} > 0$ and $\beta - \bar{\gamma} \geq 0$, that is, the hysteretic component $z(t)$ will be always bounded, as can be seen in [3].

In general, base isolated buildings tend to behave as rigid body systems concentrating the maximum displacements at the isolators. Consequently, single degree of freedom approximations are useful to simulate their response for the purpose of comparing the performance of different control strategies.

TABLE I

MODEL COEFFICIENTS OF THE SINGLE-DEGREE-OF-FREEDOM SYSTEM.

	base	**structure**
mass	$m_1 = 6 \times 10^5$ kg	$m_2 = 6 \times 10^5$ kg
stiffness	$k_1 = 0.1185 \times 10^8$ N/m	$k_2 = 9 \times 10^8$ N/m
damping	$\bar{c}_1 = 0.1067 \times 10^7$ Ns/m	$\bar{c}_2 = 0.2324 \times 10^7$ Ns/m

A first observation of Figure 4 is that the displacements are significantly reduced both in the case of relative and absolute coordinates. Nevertheless, the interstory drift in the relative case is larger than in the uncontrolled (passive) scheme, as can be seen in Figure 5.

TABLE II

PARAMETERS OF THE HYSTERESIS MODEL.

$\alpha = 0.5$	$A = 1$
$k_0 = 61224.49$ N/m	$\beta = 0.5$
$D = 0.0245$ m	$\bar{\gamma} = 0.5$

From the Laplace transform of equations (1)-(2) in Figure 3, we can derive that any improvement in the absolute base displacement x_1 leads to a direct improvement of the absolute structure displacement x_2. Nevertheless, the improvement in the relative base displacement y_1 does not imply necessarily any improvement in the relative structure displacement, due to the perturbation earthquake term which is not zero during the ground motion.

On one hand and recalling equation (1), the control signal u has to counteract the earthquake excitation which is in the form of ground acceleration. On the other hand, the control signal in equation (3) is needed to overcome only the forces generated by ground displacement and velocity. Then, the control effort in the relative case is larger than in the absolute case, and the control signal in the absolute case is smoother than in the relative case, as can be seen in Figure 6.

VI. CONCLUSIONS

From a structural engineering point of view, the objective of an active control component, as part of a hybrid seismic control system for building structures, is to keep the base displacement relative to the ground and the relative displacement between the structure and the base (interstory drift) within a reasonable range according to the design of the base isolator. In this work, we have proposed a backstepping adaptive control formulated using two alternative coordinates: relative to the ground and relative to an inertial frame (absolute). In order to establish a comparison between both alternatives two control strategies have been considered: (a) measure y_1 and regulate y_1 (in the relative case); and (b) measure x_1 and regulate x_1 (absolute coordinates). In order to investigate the efficiency of the proposed control and to establish a comparison between the relative and the absolute coordinates strategies, the 1952 Taft and the 1989 Loma Prieta earthquakes have been considered. If we consider the following items for our analysis:

(i) base displacement relative to the ground
(ii) interstory drift
(iii) absolute acceleration
(iv) control effort and smoothness of the control signal

we may summarize the following from the results discussed in Section V:

(i) the base displacement y_1 in the relative case is clearly reduced as compared to the passive control. This has been the objective for the backstepping control.
(ii) in the absolute coordinates case, the backstepping control objective has been stated in reducing the absolute base displacement x_1 and this has been achieved. The

relative base displacement $y_1 = x_1 - d$ is augmented but it is still within an acceptable range.

(iii) the interstory drift $x_2 - x_1$ in the absolute case is significantly reduced whereas in the relative case, $y_2 - y_1$ is augmented as compared to the uncontrolled situation. Anyhow, both interstories are considered acceptable.

(iv) the base (absolute) acceleration $\ddot{x}_1$ in the absolute case is significantly reduced whereas in the relative case, $\ddot{x}_1 = \ddot{y}_1 + \ddot{d}$ is augmented with respect to the uncontrolled situation. Anyhow, both absolute accelerations are considered acceptable.

(v) the control signal in the absolute case is relatively smooth. The control signal in the relative case contains high-frequencies. The control effort is larger in the relative coordinates strategy.

From these issues we recommend the use of the absolute coordinates strategy. Besides, the choice of a backstepping adaptive control has allowed to consider that the parameters of the models are uncertain and to have computable bounds on the transient behavior. Finally, the control error can be made arbitrarily small by sufficiently increasing the design gains. When using the absolute coordinates, it has been shown that this increase does not lead to an increase of the peak and the root-mean-square norm of the control forces.

VII. ACKNOWLEDGMENTS

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Error variables

$$z_1 = y - y_r, \quad z_2 = v_{2,2} - \hat{\varrho}\dot{y}_r - \alpha_1$$

Stabilizing functions

$$\begin{aligned}
\alpha_1 &= \hat{\varrho}\bar{\alpha}_1 + \Upsilon_1 \\
\bar{\alpha}_1 &= -(c_1 + d_1)z_1 - \xi_2 - \bar{\omega}^{\mathrm{T}}\hat{\theta} \\
\alpha_2 &= -\hat{b}_2 z_1 - \left[c_2 + d_2\left(\frac{\partial\alpha_1}{\partial y}\right)^2\right] z_2 + \beta_2 \\
&\quad + \frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\tau_2 + \Upsilon_2 \\
\beta_2 &= \frac{\partial\alpha_1}{\partial y}(\xi_2 + \omega^{\mathrm{T}}\hat{\theta}) + \frac{\partial\alpha_1}{\partial\eta}(A_0\eta + e_4 y) \\
&\quad + \frac{\partial\alpha_1}{\partial\lambda_1}(-k_1\lambda_1 + \lambda_2) + \frac{\partial\alpha_1}{\partial\lambda_2}(-k_2\lambda_1 + \lambda_3) \\
&\quad + \frac{\partial\alpha_1}{\partial\lambda_3}(-k_3\lambda_1 + \lambda_4) + \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r + k_2 v_{2,1} \\
&\quad - \left(\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\varrho}}\right)\gamma\mathrm{sgn}(b_2)(\dot{y}_r + \bar{\alpha}_1)z_1 \\
\Upsilon_1 &= -g_0\mathrm{sgn}(b_2)(\dot{y}_r + \bar{\alpha}_1)^2 z_1 \\
\Upsilon_2 &= -\frac{\partial\alpha_1}{\partial\hat{\theta}}\Gamma\sigma_\theta\hat{\theta} - \left(\dot{y}_r + \frac{\partial\alpha_1}{\partial\hat{\varrho}}\right)\gamma\sigma_\varrho\hat{\varrho}
\end{aligned}$$

Tuning functions

$$\tau_1 = (\omega - \hat{\varrho}(\dot{y}_r + \bar{\alpha}_1)e_1)z_1, \quad \tau_2 = \tau_1 - \frac{\partial\alpha_1}{\partial y}\omega z_2$$

Parameter update laws

$$\dot{\hat{\theta}} = \Gamma\tau_2 - \Gamma\sigma_\theta\hat{\theta}, \quad \dot{\hat{\varrho}} = -\gamma\mathrm{sgn}(b_2)\left(\dot{y}_r + \bar{\alpha}_1\right)z_1 - \gamma\sigma_\varrho\hat{\varrho}$$

Switching σ-modification

$$\sigma_\theta = \begin{cases} 0, & \|\hat{\theta}\| \le M_\theta \\ \sigma_{s\theta}, & \|\hat{\theta}\| \ge 2M_\theta \\ -\frac{2\sigma_{s\theta}}{M_\theta^3}\|\hat{\theta}\|^3 + \frac{9\sigma_{s\theta}}{M_\theta^2}\|\hat{\theta}\|^2 & \\ -\frac{12\sigma_{s\theta}}{M_\theta}\|\hat{\theta}\| + 5\sigma_{s\theta}, & \text{otherwise} \end{cases}$$

$$\sigma_\varrho = \begin{cases} 0, & |\hat{\varrho}| \le M_\varrho \\ \sigma_{s\varrho}, & |\hat{\varrho}| \ge 2M_\varrho \\ -\frac{2\sigma_{s\varrho}}{M_\varrho^3}|\hat{\varrho}|^3 + \frac{9\sigma_{s\varrho}}{M_\varrho^2}|\hat{\varrho}|^2 & \\ -\frac{12\sigma_{s\varrho}}{M_\varrho}|\hat{\varrho}| + 5\sigma_{s\varrho}, & \text{otherwise} \end{cases}$$

Adaptive control law

$$u = \alpha_2 - v_{2,3} + \hat{\varrho}\ddot{y}_r$$

Fig. 7. The control law and the parameter update laws are designed in $\rho = 2$ steps (see [7]).