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Active control of a tensegrity plane grid

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Abstract— **Tensegrity systems are selfstressed reticulate space structures. As lightweight frames, they are subject to deformation and vibration issues when faced to natural stimulations such as temperature gradients or wind. Classical passive solutions impose to rigidify components or to add damping in the structure using heavy devices. Active systems, mainly developed in space and seismic fields, are controlled using external energy brought by activators. We describe in this paper a mixed geometric and dynamic active control of tensegrity structures using a robust control design technique. An experiment is carried out on a six selfstress states plane tensegrity grid.**

I. INTRODUCTION

TENSEGRITY systems appeared in the Fifties [1][2] as a new class of reticulate space systems. They are defined [3] as systems in a stable selfstressed equilibrium state composed of a discontinuous set of compressed components inside a continuous set of tense components. We can compare selfstress to the pressure of the air inside an inflatable object. This equilibrium between tensioned (usually cables) and compressed components is required to keep them stressed in accordance with unilateral rigidity, thus stabilizing the system and taking advantage of materials. Tensegrity systems present great interest for artists and architects, who design from this principle lightweight and transparent structures (Fig. 1).

Fig. 1. (a) Needle Tower (Keneth Snelson, 1968), (b) Tensarch project (LMGC/SLA, 2002)

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A. Context

At the present time, only a few structures of this kind have yet been built. The necessity to sustain an internal stress state certainly slows down their development because it requires specific design and setting procedures that are often costly to implement. We have developed recently methods to build tensegrity grids. The first step is to define the target selfstress in order to resist the external loads [4][5]. Then we propose a process of adjustment which makes it possible to reach the target selfstress [6].

Tensegrity structures, like any others, are subject to external loads that modify their geometry and selfstress state. In some cases, these forces can be a threat to their stability and integrity, particularly when they are used to support fragile materials such as in the case of a glass covering. This is why we focus here on the control of their behavior under static and dynamic loading.

B. Control of structures

The control of structures is originally motivated by the need to lower vibration effects due to climatic and seismic perturbations. Rather than reinforcing and rigidifying so as to face those rare situations, one wishes to cancel these effects by producing controlled reaction forces. We focus here on an active control scheme. When passive and semi-active controls use the self-oscillatory energy of the structure, active systems inject directly the needed control forces through activators. Costly in the case of heavy buildings, this kind of control is very well adapted for lightweight structures. It allows regulating different kind of variables such as displacement or acceleration, with the best efficiency permitted by the activators.

We study here the active control of a plane tensegrity grid. This kind of system is an attractive inspiration for smart structures that make use of active components so as to actively control the geometry [7][8] or cancel vibrations in modular structures [9]. In our study, we use a tensegrity grid comprising an important number of elements. There are two complementary objectives : first, a geometrical control in which perturbations results from static service loading or evolving characteristics (tension loss, relaxation…). Secondly, a dynamic vibration control which is an active damping of the firsts eigenmodes, in torsion and flexion.

II. MODEL AND ACTIVATORS

The studied structure is a plane double layer tensegrity grid (Fig. 2). It represents a quarter of the Tensarch project structure [10] (Fig. 1b). It is composed of 64 nodes and 177 components : 52 compressed bars and 125 tensioned

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elements. Its initial internal forces state is a combination of six fundamental selfstress states that are determined from the static equilibrium [11][12]. This state is chosen for its regularity and such that the forces allow to face the expected loading. We follow a precise procedure for the implementation of this selfstress state after the assembly phase [13].

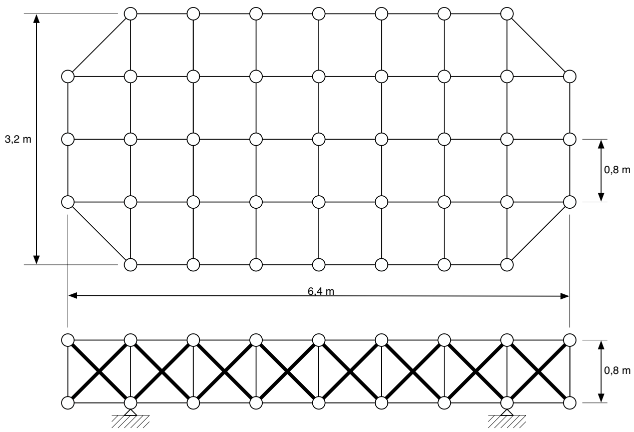

Fig. 2. The studied plane grid

A. Static and dynamic behavior

This plane grid is loaded uniformly in the vertical direction and supported so that finite mechanisms are no longer present. We study numerically its static behavior using the displacement method but taking into account the unilateral response of cables. We observe a classical linear response in small perturbations but the vertical displacement softens in function of the applied load as an increasing number of cables slacken off. The objective of the static geometric control is to reduce the vertical deformation of the structure while maintaining the cables in tension.

We describe in Fig 3 a significant fraction of the dynamic behavior of the studied structure. It represents the transfer function of the output acceleration over an input force along the vertical axis in node nb. 6, near the centre of the grid. This response is computed in the geometric reference state and reveals the characteristic first resonance peaks of the system in torsion and flexion that are to be attenuated by dynamic control.

B. Activation

We integrated linear activators so as to influence at the same time the vertical displacements and the vibration modes. The idea behind their positioning is to provoke a local tension increase in the lower layer so as to induce a global vertical deformation. To achieve this target, we put them vertically in order to pull away one node from its layer, thus increasing tension in the four adjacent cables (see Fig. 4). This mode of integration is also the less perturbing for the system. In the initial configuration, with a zero lengthening of the activators and no energy, we retrieve the system in its initial selfstressed configuration.

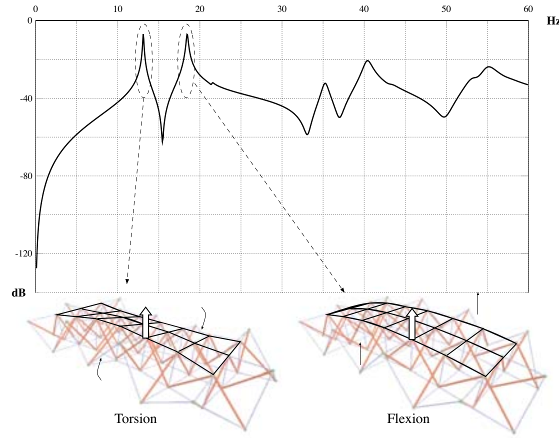

Fig. 3. Transfer function acceleration on force in node 6, vertically (arrow)

The theoretical vertical displacement in C0 and C1 (Fig. 5) when activator Act. 0 is lengthened along its whole course reveals a non-linear influence on the vertical deflection in the static domain. In the dynamic domain, we can observe on the frequency response function between each activator and the vertical acceleration at the node 6 that the main eigenfrequencies of the active system are affected (Fig. 6). Activators are localized there to have the best impact possible on the dynamic behavior of the structure.

As a matter of fact the activators are hydraulic jacks with a course of 100 mm and a maximum effort of 2 kN. We chose this solution for this model of modest proportions because it authorizes dynamic performances that go up to 50 Hz, a domain that covers the most part of the dynamic behavior of the structure. We operate the actuators through two programmable axis cards and from a computer that generates and gathers data running LabView$^{\mathrm{TM}}$Ṁeasures are collected at C0 and C1 (respectively nodes 54 and 56, see Fig. 4) by a laser distance sensor and a piezoelectric accelerometer.

III. CONTROL LOOPS

The control loop represents the architecture of the active system. It takes the usual form presented Fig. 7 where the active system, which is the structure and its activators, is represented by the block noted G. K represents the controller, a z filter generating the order u from the error e between the order r and measures y. On this diagram, we materialize some uncertainties : w_i in input, w_o on the output and n on the measures. In fact, the control is split in two complementary parts, each dedicated to a measurement type and dynamical domain (Fig. 8).

A. Static control loop

We control the geometry from the vertical displacements and an external shape order using a simple PI law with a reaction time of several seconds. In our case, it is sufficient to respect the target geometry exploiting the whole course

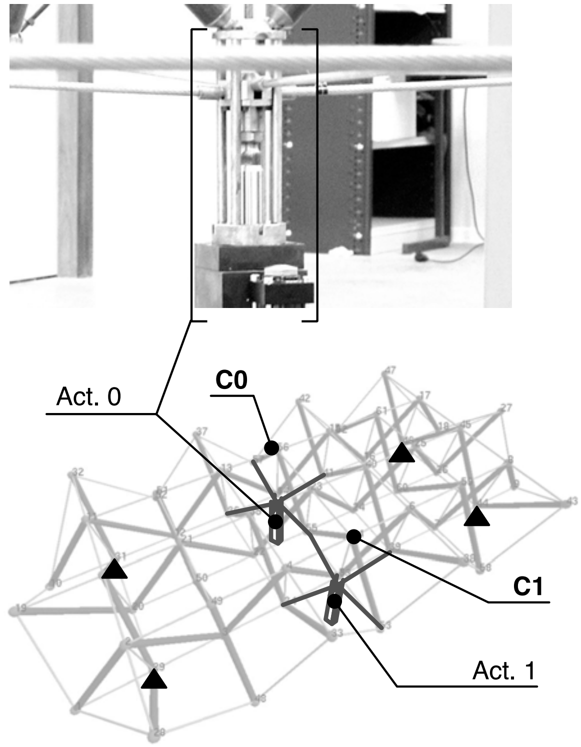

Fig. 4. Integration of activators

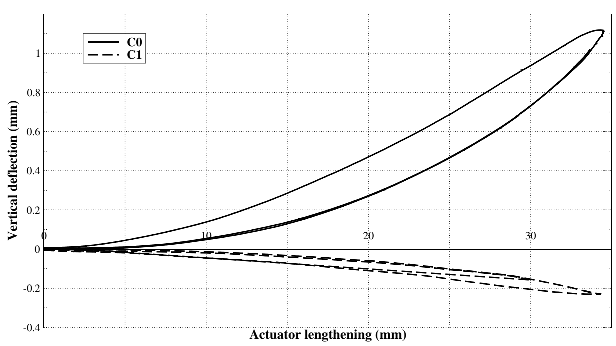

Fig. 5. Static influence of the activators

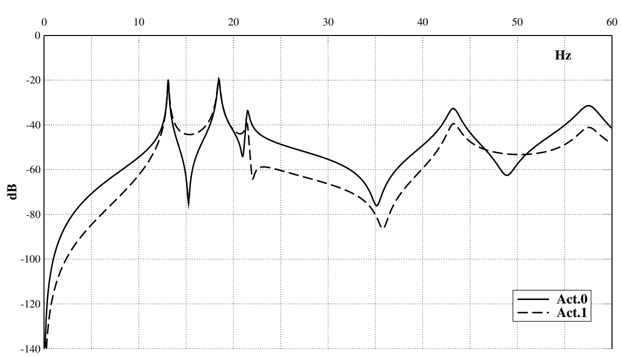

Fig. 6. Dynamic transfer functions between the activators and the output (node 6, see Fig. 3)

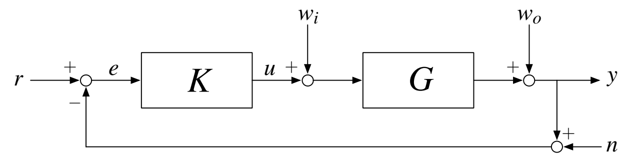

Fig. 7. Closed loop control scheme

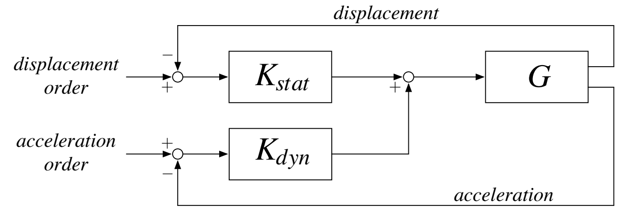

Fig. 8. Mixed control from two measurement types

of the activators. We use the relatively large reaction delay to perform an averaging of the displacement measures that filters the signal noise. It allows also to clearly isolate this control loop from the dynamic domain.

B. Dynamic control loop

Apart from the static control, a dynamic controller is designed to attenuate vibrations using vertical acceleration measures. The control movements will be small around a static equilibrium position, but as seen on Fig. 5, the relation between the activator lengthening and the vertical displacement is non-linear. So the dynamic influence of the activators will depend for the static position. This is why we have to take into account this variability on the active system behavior.

C. Robust control synthesis

Modern control methods [14], and particularly the robust synthesis algorithms (LQG, $PRLQG$, H_∞, μ) consider for the design of an appropriate controller K_{dyn} the uncertainties that affect the model G of the active system and the perturbations on the signals. For example, if an attenuation of the external vibration effects that disturb the output is required, one has to minimize the transfer function between w_o and the output y. In our case, we also require a robust control that must be the less sensible to the variations of the system's characteristics due to unmodeled dynamics, identification errors or any selfstress evolution. So, at the same time, we have to minimize the influence of an input perturbation w_i over y.

With the notations introduced Fig. 7, the output y can be written :

$$\begin{array}{l} y = (1+G.K)^{-1}.w_0 + \\ \quad (1+G.K)^{-1}.G.w_i + (1+G.K)^{-1}.G.K.(r-n) \end{array} \tag{1}$$

We note $S = (1+GK)^{-1}$ and $T = (1+GK)^{-1} = 1-S$. S is the output sensibility, the transfer function between w_o and y. T is the complementary output sensibility, between

$(r-n)$ and y. The transfer between w_i and y is SG. These functions serve to build specifications on the controller. In our case, in order to have good performances, T must be great in the bandwidth of the system and weak beyond in order to eliminate the measurement noise n. To attenuate the external actions requires a small sensibility S. Finally, the uncertainties of the system can be seen as an input perturbation w_i, so robustness requires a small SG transfer function. To these conditions, we add the necessity to limit the intensity of loop signals, for example the orders u, to avoid an inefficient saturation of the activators.

Using the formulation of the H_∞ method, we write this set of conditions under a convenient form, "the standard form" (Fig. 9), centered on the researched controller K. In this

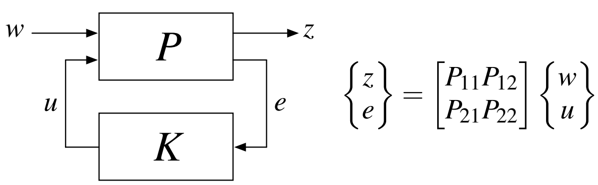

Fig. 9. The "standard" form

diagram, P is an enriched description of the system G, that admits in input a vector of orders u and perturbations w. In output are the error e (input of the controller) and the criteria z. These are signals that are extracted from the loop and pondered by functions W_i which shape the signals (S, $T\ldots$) into normalized criteria z_i to be minimized (Fig. 10).

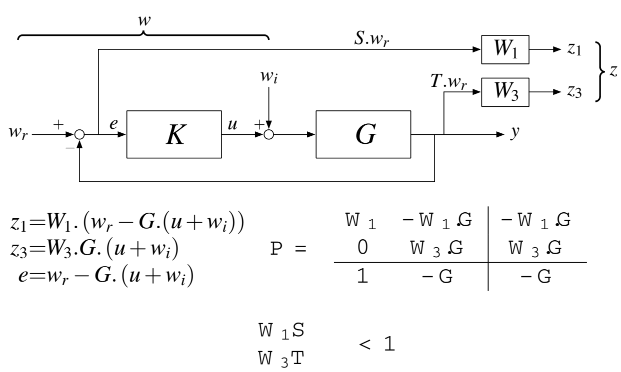

Fig. 10. Loop diagram, constitution of the augmented "standard"" form and normalized criteria

Under this form, the closed-loop transfer $G_{zw}(s)$ between w and z writes:

$$\begin{aligned} G_{zw}(s) &= F_l(P,K) \\ &= P_{11}(s) + \\ &\quad P_{12}(s).K(s)\left(I - P_{22}(s).K(s)\right)^{-1}.P_{21}(s) \end{aligned} \tag{2}$$

The robust synthesis problem consists in determining the controller $K(s)$ that stabilizes and minimizes the H_∞ norm of the signal.

IV. EXPERIMENTAL RESULTS

We present in this paragraph the results of the mixed control. First we show those concerning the static deflection. Then we describe the characteristics of the synthesized dynamic controller for the first flexion mode of the plane grid and its influence on external forced vibrations.

A. Static deflection control

The objective in this figures were to keep the deflections at zero. We applied a uniform load in two progressive steps for each half of the grid. We suspended calibrated weights under each node. This causes temporarily torsion that is visible in Fig. 11 where we see a difference between the displacement measure C0 and C1. This torsion is entireltly cancelled after 15 seconds at the end of the first step. Finally, after the second step, the flexion deflection is reduced to zero. We reach similar results whith other shape order, like an imposed flexion or torsion. This result confirms that a simple PI control is very efficient in following a static geometric order.

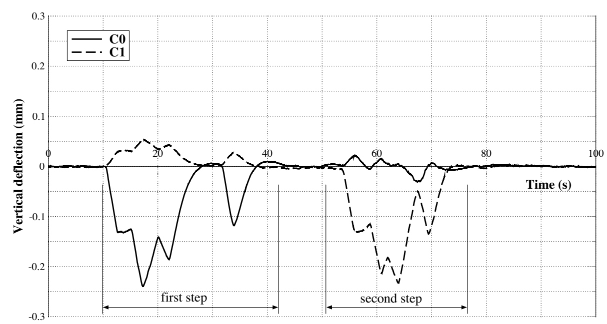

Fig. 11. Time history of the vertical deflection. Conservation of a shape consign

B. Dynamic controller synthesis

In comparison, the design of a dynamic controller is less trivial. In our case, the transfer function of the system G was identified by swept sine analysis between the length consign and the vertical acceleration in C0 and C1 (Fig. 4). Fig. 12 shows the frequency responses of this model G reduced here to the first mode of flexion. GS is the transfer between w_i and output y in the resulting closed loop. We see that GS remains weak around the peak frequency, which shows the robustness of the system in presence of a perturbation w_i on the model.

C. Vibration control results

The controller is implemented as a digital z filter. It is inserted in a software control loop built under LabView^{TM} refreshed at a rate of 200 cycles/s. The performances of the activators are also high above (50 Hz max.) the first controlled mode. In Fig. 13 and Fig. 14, we reveal the impact of the dynamic control on the behavior of the structure

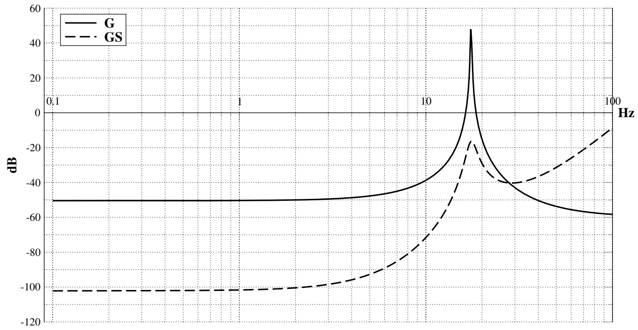

Fig. 12. Transfer function of the nominal (G) and controlled (GS) system

when it is excited in a central point of the lower layer. Fig. 13 is a comparison between two swept sine analyses

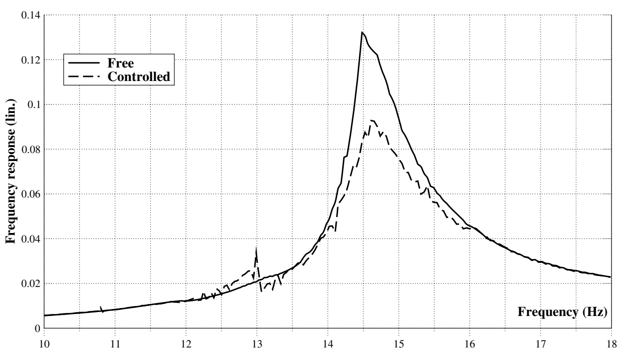

Fig. 13. Frequency response comparison between the free and controlled system, around the first resonance of flexion

around the first mode of flexion. It is the frequency response between the external perturbation in input and the output vertical acceleration in C0. We see that there is a noticeable attenuation when control is switched on. A quantification of the equivalent damping factor for this mode shows that it goes from 2.2 to 3.06 %. The same effect is obvious on the time history plots of the displacement in C0 (see Fig. 14) and reveals an attenuation of nearly 30 %.

V. CONCLUSION

Being lightweight structures, tensegrity systems are sensible to static and dynamic loads coming from their environment. We present a mixed control of the displacements and the vibrations of a plane grid of 20 m^2 using a non-perturbing device. This kind of activator does not interfere with the integrity of the structure since it does not take the place of any element.

We synthetised the dynamic control law using the H_∞ robust control method. It showed theoretically a good efficiency when faced to perturbations in the analytically determined model. The experiment raised on a 1:1 scale model confirms, with encouraging results, the positive influence of this control

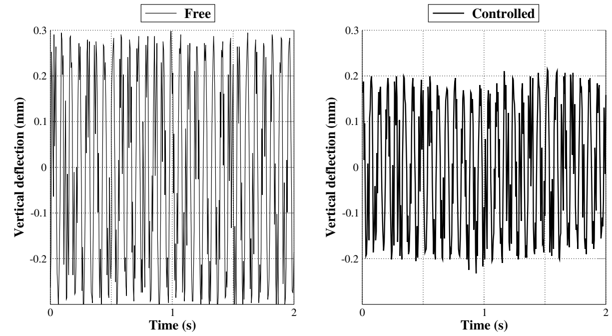

Fig. 14. Time response comparison of the free and controlled system under external dynamic loading

on the behavior of such systems. It revealed robustness and allowed good performances even with a poorly known system or when it's characteristics varied because of an evolution of the selfstress state or a mass loading change. This experiment opens large possibilities and we are look toward other robust approach like sliding mode control. This quality appears necessary for the development of light and efficient smart tensegrity structures.

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