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Nonlinear Adaptive Speed Tracking Control for Sensorless Permanent Magnet Step Motors

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Abstract— **Assuming that only stator currents and voltages are available for feedback, a nonlinear output feedback adaptive speed tracking control strategy is proposed for a permanent magnet step motor with unknown load torque. It relies on three new theoretical results: the "s-alignment" and "c-alignment" procedures, in which the motor is forced to reach certain known equilibrium points, and an output feedback controller which guarantees uniform asymptotic speed tracking for every initial condition belonging to an explicitly computed domain of attraction. Global exponential convergence is achieved in the case of known load torque. Numerical simulation results show the effectiveness of the proposed solution.**

I. INTRODUCTION

Due to their excellent serviceability and durability, high efficiency and power density, as well as high torque to inertia ratio and absence of external rotor excitation and rotor windings, the permanent magnet motors are used in practical applications such as printers, tape drives, hard drives in PCs, process control systems, home appliances, and have been gradually replacing DC motors in a wide range of drive applications such as machine tools and industrial robots. The high complexity in the control of these motors is the price to pay for the above advantages. The speed or the position tracking control of permanent magnet synchronous motors requires the knowledge of rotor shaft position and speed signals in order to control the stator current vector suitably and then achieve an high-performance drive ([6], [9], [14], [15]). Recently, a lot of attention has been paid and many research efforts have been spent in developing the machine drive system without rotational transducers: shaft sensors present several drawbacks such as drive cost, machine size, reliability and noise immunity as well as performance degradation owing to the vibration or humidity. Various control strategies for the permanent magnet synchronous machine drives without rotational sensors have been investigated and described by many authors ([1], [3], [4], [7], [12]).

In this paper we propose a novel nonlinear adaptive speed tracking control strategy for a permanent magnet step motor, on the basis of stator currents and voltages measurements only: it relies on three new theoretical results (not based on motor friction) which are summarized in three theorems. The first two theoretical results, concerning the asymptotic properties of the "s-alignment" and "c-alignment" procedures, are based on LaSalle theorem, while the third one presents an output feedback controller which guarantees: i) global exponential rotor speed tracking when the load torque is known; ii) uniform asymptotic rotor speed tracking for every initial condition belonging to an explicitly computed region of attraction when the load torque is unknown. Numerical simulation results with reference to a six pole pairs permanent magnet step motor are included which illustrate the effectiveness of the proposed solution.

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II. DYNAMIC MODEL AND NONLINEAR ADAPTIVE DESIGN

Assuming linear magnetic materials, symmetry of the rotor and between the two phases, nonlinear flux density distribution due to the air gap geometry only and negligible magnetic hysteresis and Foucault currents, the dynamics of a permanent magnet step motor with no saliency and a sinusoidal flux density distribution in a fixed reference frame attached to the stator are given by the well known fourth-order model (see for instance [9] for its derivation and modelling assumptions)

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= -\frac{F}{J}\omega + \frac{k_{\mathrm{M}}}{J}\Big[-i_a \sin(p\theta) + i_b \cos(p\theta)\Big] - \frac{T_L}{J} \\ \frac{\mathrm{d}i_a}{\mathrm{d}t} &= -\frac{R}{L}i_a + \frac{k_{\mathrm{M}}}{L}\omega \sin(p\theta) + \frac{u_a}{L} \\ \frac{\mathrm{d}i_b}{\mathrm{d}t} &= -\frac{R}{L}i_b - \frac{k_{\mathrm{M}}}{L}\omega \cos(p\theta) + \frac{u_b}{L} \end{aligned} \qquad (1)$$

in which: θ is the rotor angle, ω is the rotor speed, (i_a, i_b) are the stator currents [$(\theta, \omega, i_a, i_b)$ constitute the state variables], (u_a, u_b) are the stator voltages (which constitute the control inputs) in a fixed reference attached to the stator; the output to be controlled is the rotor speed ω. The control problem is called sensorless when only stator currents and voltages are measured. The load torque $T_L(t)$ is typically a step disturbance. The motor parameters are: viscous friction coefficient F, number of pole pairs p, rotor moment of inertia J, stator windings resistance R and inductance L, motor torque constant k_{M}. While J, R, L, k_{M} are strictly positive parameters, F is assumed to be non-negative, so that our stability results are not based on friction. If we introduce, as in [8], [13], [15], the Park's transformation

$$\begin{bmatrix} w_d(t) \\ w_q(t) \end{bmatrix} = \begin{bmatrix} \cos(p\theta(t)) & \sin(p\theta(t)) \\ -\sin(p\theta(t)) & \cos(p\theta(t)) \end{bmatrix} \begin{bmatrix} w_a(t) \\ w_b(t) \end{bmatrix}$$

the dynamics (1) expressed in terms of currents and voltages in rotating (d, q) coordinates, become

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= -\frac{F}{J}\omega + \frac{k_{\mathrm{M}}}{J}i_q - \frac{T_L}{J} \end{aligned}$$

$$\frac{\mathrm{d}i_d}{\mathrm{d}t} = -\frac{R}{L}i_d + p\omega i_q + \frac{u_d}{L} \qquad (2)$$
$$\frac{\mathrm{d}i_q}{\mathrm{d}t} = -\frac{R}{L}i_q - p\omega i_d - \frac{k_{\mathrm{M}}}{L}\omega + \frac{u_q}{L}$$

In the following we will denote by $\omega^*(t)$ the arbitrary smooth bounded reference signal with bounded time derivatives $\dot{\omega}^*(t)$ and $\ddot{\omega}^*(t)$ for the rotor speed $\omega(t)$ and by $i_d^*(t)$ the smooth bounded reference signal with bounded time derivative $\frac{\mathrm{d}i_d^*(t)}{\mathrm{d}t}$ for the current $i_d(t)$, which is related to the reference velocity $\omega^*(t)$ to comply with voltage saturation at high speed (field weakening, see [8]). Let us summarize the first two results of this paper.

Theorem 1 ("s-alignment" procedure): *Let $u_{cs} \in \Re^+$ be a positive real scalar and suppose that no load torque T_L is applied to the motor. Then, the static linear controller*

$$u_a = 0, \; u_b = u_{cs} \qquad (3)$$

substituted in model (1), guarantees boundedness of $(\omega,\ i_a,\ i_b)$ and

$$\lim_{t\to\infty}\cos(p\theta(t)) = \lim_{t\to\infty}\omega(t) = \lim_{t\to\infty} i_a(t) = 0$$
$$\lim_{t\to\infty} i_b(t) = \frac{u_{cs}}{R}.$$

Proof: Denoting by $\tilde{i}_b = i_b - \frac{u_{cs}}{R}$ the stator current vector b-component tracking error, the equilibrium points of the closed loop system are $(\theta, \omega, i_a, \tilde{i}_b) = (\theta_{es}, 0, 0, 0)$ with $\theta_{es} = \frac{\pi}{2p}, \frac{3\pi}{2p}, ..., \left(2p - \frac{1}{2}\right)\frac{\pi}{p}$. Consider the function

$$U_s = \frac{1}{2}\Big(\frac{J}{L}\omega^2 + {i_a}^2 + \tilde{i}_b^2\Big) - \frac{k_{\mathrm{M}}u_{cs}}{pLR}\Big(\sin(p\theta) - 2\Big)$$

which is always positive. Its time derivative along the trajectories of the closed loop system is

$$\dot{U}_s = -\frac{F}{L}\omega^2 - \frac{R}{L}\Big({i_a}^2 + \tilde{i}_b^2\Big).$$

Since $\dot{U}_s(t) \le 0$ [recall that $F \ge 0$], by virtue of LaSalle theorem [26.1, [5]], we can establish that every solution is attracted into the largest invariant subset $\mathcal{M}_s$ of the set $\dot{U}_s = 0$, consisting of the equilibrium points $(\theta, \omega, i_a, \tilde{i}_b) = (\theta_{es}, 0, 0, 0)$.

Theorem 2 ("c-alignment" procedure): *Let $u_{cc} \in \Re^+$ be a positive real scalar and assume that: the motor is initially "s-aligned", i.e, $\cos(p\theta(t_0)) = \omega(t_0) = i_a(t_0) = 0$, $i_b(t_0) = \frac{u_{cs}}{R}$ ($u_{cs} \in \Re^+$); no load torque T_L is applied; the positive real scalars u_{cs} and u_{cc} satisfy the condition*

$$\frac{k_{\mathrm{M}}u_{cc}}{pLR} - \frac{(u_{cs}^2 + u_{cc}^2)}{2R^2} > 0. \qquad (4)$$

Then, the static linear controller

$$u_a = u_{cc}, \; u_b = 0 \qquad (5)$$

substituted in model (1), guarantees boundedness of $(\omega,\ i_a,\ i_b)$ and $\lim_{t\to\infty}\cos(p\theta(t)) = 1$.

Proof: Denoting by $\tilde{i}_a = i_a - \frac{u_{cc}}{R}$ the stator current vector a-component tracking error, the equilibrium points of the closed loop system are $(\theta, \omega, \tilde{i}_a, i_b) = (\theta_{ec}, 0, 0, 0)$ with $\theta_{ec} = 0, \frac{\pi}{p}, \frac{2\pi}{p}, ..., (2p-1)\frac{\pi}{p}$. Consider the function

$$U_c = \frac{1}{2}\Big(\frac{J}{L}\omega^2 + \tilde{i}_a^2 + {i_b}^2\Big) - \frac{k_{\mathrm{M}}u_{cc}}{pLR}\Big(\cos(p\theta) - 2\Big)$$

which is always positive. Its time derivative along the trajectories of the closed loop system is

$$\dot{U}_c = -\frac{F}{L}\omega^2 - \frac{R}{L}\Big(\tilde{i}_a^2 + {i_b}^2\Big).$$

Since $\dot{U}_c(t) \le 0$ [recall that $F \ge 0$], by virtue of LaSalle theorem [26.1, [5]], we can establish that every solution is attracted into the largest invariant subset $\mathcal{M}_c$ of the set $\dot{U}_c = 0$, consisting of the equilibrium points $(\theta, \omega, \tilde{i}_a, i_b) = (\theta_{ec}, 0, 0, 0)$. Since $\dot{U}_c(t) \le 0$, we have $U_c(t) \le U_c(t_0)$ for all $t \ge t_0$ and in particular, if condition (4) is satisfied, $\lim_{t\to\infty}\cos(p\theta(t)) = 1$.

Let us summarize the third result of this paper.

Theorem 3: *Assume that $\cos(p\theta(t_0)) = 1$. Then, the sixth order nonlinear output feedback adaptive controller*

$$\begin{bmatrix} u_a(t) \\ u_b(t) \end{bmatrix} = \begin{bmatrix} \cos[p\theta(t)] & -\sin[p\theta(t)] \\ \sin[p\theta(t)] & \cos[p\theta(t)] \end{bmatrix} \begin{bmatrix} u_d(t) \\ u_q(t) \end{bmatrix}$$
$$\cos[p\theta(t)] = -\frac{Lp}{k_{\mathrm{M}}}\Big[i_a(t) - i_a(t_0) - \varphi_1(t)\Big] + 1$$
$$\dot{\varphi}_1(t) = -\frac{R}{L}i_a(t) + \frac{u_a(t)}{L}, \quad \varphi_1(t_0) = 0$$
$$\sin[p\theta(t)] = -\frac{Lp}{k_{\mathrm{M}}}\Big[i_b(t) - i_b(t_0) - \varphi_2(t)\Big]$$
$$\dot{\varphi}_2(t) = -\frac{R}{L}i_b(t) + \frac{u_b(t)}{L}, \quad \varphi_2(t_0) = 0$$
$$u_d(t) = L\Big[-\phi_d(t) - K_i(i_d(t) - i_d^*(t))\Big] \qquad (6)$$
$$u_q(t) = L\Big[-\phi_q(t) - K_i(i_q(t) - i_q^*(t))\Big]$$
$$\begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} = \begin{bmatrix} \cos[p\theta(t)] & \sin[p\theta(t)] \\ -\sin[p\theta(t)] & \cos[p\theta(t)] \end{bmatrix} \begin{bmatrix} i_a(t) \\ i_b(t) \end{bmatrix}$$

$$\phi_d(t) = -\frac{R}{L}i_d^*(t) + p\hat{\omega}(t)i_q(t) - \frac{\mathrm{d}i_d^*(t)}{\mathrm{d}t}$$
$$\phi_q(t) = -\frac{R}{L}i_q^*(t) - p\hat{\omega}(t)i_d(t) - \frac{k_{\mathrm{M}}}{L}\hat{\omega}(t) - \frac{J}{k_{\mathrm{M}}}\Bigg[\frac{F}{J}\dot{\omega}^*(t) - k_\omega \frac{\mathrm{d}\ \mathrm{sat}_\kappa(\hat{\omega} - \omega^*)}{\mathrm{d}\ (\hat{\omega} - \omega^*)}(t) \cdot\Big(\dot{\hat{\omega}}(t) - \dot{\omega}(t)^*\Big) + \frac{\dot{\hat{T}}_L(t)}{J} + \ddot{\omega}^*(t)\Bigg]$$
$$i_q^*(t) = \frac{J}{k_{\mathrm{M}}}\Bigg[\frac{F}{J}\omega^*(t) - k_\omega \mathrm{sat}_\kappa(\hat{\omega}(t) - \omega^*(t)) + \frac{\hat{T}_L(t)}{J} + \dot{\omega}^*(t)\Bigg]$$
$$\dot{\hat{i}}_a(t) = -\frac{R}{L}\hat{i}_a(t) + \frac{k_{\mathrm{M}}}{L}\hat{\omega}(t)\sin[p\theta(t)] + \frac{u_a(t)}{L} + K_e(i_a(t) - \hat{i}_a(t)) + p\hat{\omega}(t)(i_b(t) - \hat{i}_b(t))$$
$$\dot{\hat{i}}_b(t) = -\frac{R}{L}\hat{i}_b(t) - \frac{k_{\mathrm{M}}}{L}\hat{\omega}(t)\cos[p\theta(t)] + \frac{u_b(t)}{L}$$

$$
\begin{aligned}
&\quad +K_e(i_b(t)-\hat{i}_b(t))-p\hat{\omega}(t)(i_a(t)-\hat{i}_a(t))\\
\dot{\hat{\omega}}(t) &= -\frac{F}{J}\hat{\omega}(t)-\frac{\hat{T}_L(t)}{J}+\frac{k_{\rm M}}{J}\Big[-i_a(t)\sin[p\theta(t)]\\
&\quad +i_b(t)\cos[p\theta(t)]\Big]+\frac{2\gamma^2+rF\lambda J^2}{\gamma^2\lambda+rF\lambda^2J^2}\\
&\quad \cdot\Bigg[\frac{k_{\rm M}}{L}\sin[p\theta(t)](i_a(t)-\hat{i}_a(t))\\
&\quad -\frac{k_{\rm M}}{L}\cos[p\theta(t)](i_b(t)-\hat{i}_b(t))\Bigg]\\
\dot{\hat{T}}_L(t) &= \frac{2\gamma^2F+\lambda J\gamma+\lambda J^2rF^2}{\gamma^2\lambda+rF\lambda^2J^2}\Bigg[-\frac{k_{\rm M}}{L}\sin[p\theta(t)]\\
&\quad \cdot(i_a(t)-\hat{i}_a(t))+\frac{k_{\rm M}}{L}\cos[p\theta(t)](i_b(t)\\
&\quad -\hat{i}_b(t))\Bigg]
\end{aligned}
$$

which depends on: the available signals i_a, i_b; the reference signals ω^*, i_d^*; the known positive machine parameters J, K_M, p, R, L and the known non-negative machine parameter F; the positive control parameters K_i, k_ω, κ, K_e; the positive control parameters γ, λ, r satisfying the conditions ($T_p \in \Re^+$)

$$
\begin{aligned}
\gamma &> \frac{\lambda^2 e^{4T_p}L^4}{p_e k_{\rm M}{}^4 T_p^2}+8p_e\frac{k_{\rm M}{}^4}{L^4}\\
p_e &< \frac{\frac{R}{L}+K_e}{12\left(\frac{4k_{\rm M}{}^6}{\lambda^2L^6}+\frac{k_{\rm M}{}^2(R+LK_e)^2}{L^4}+\frac{k_{\rm M}{}^2}{L^2}\right)+\frac{k_{\rm M}{}^2}{L^2}}\\
r &\geq (J^3k_{\rm M}{}^4T_p^2e^{-4T_p}p_e)^{-1}(2\gamma^2L^4) \qquad (7)
\end{aligned}
$$

substituted in model (1), guarantees boundedness of (ω, i_a, i_b, $\hat{\omega}$, $\hat{i}_a$, $\hat{i}_b$, $\hat{T}_L$) and uniform local asymptotic stability of the closed loop system equilibrium point ($\omega-\omega^*$, $i_d-i_d^*$, $i_q-i_q^*$, $i_a-\hat{i}_a$, $i_b-\hat{i}_b$, $\hat{\omega}-\omega$, $T_L-\hat{T}_L$) $=0$ with domain of attraction $\mathcal{B}=\left\{\xi=[\xi_1,\xi_2,...,\xi_7]^T\in\Re^7:\|[\xi_4,\xi_5,\xi_6,\xi_7]^T\|<\varrho\right\}$ *where* $\varrho=\frac{L}{2k_{\rm M}p}\sqrt{\frac{\min\{s_1,p_e\}q}{p_e\left(s_2+2p_e\left(\frac{k_{\rm M}}{L}\right)^2\max\left\{1,\left(\frac{k_{\rm M}}{L}\right)^2\right\}\right)}}$, $\Re^+\ni q=\frac{R}{L}+K_e-p_e\left[12\left(\frac{4k_{\rm M}{}^6}{\lambda^2L^6}+\frac{k_{\rm M}{}^2(R+LK_e)^2}{L^4}+\frac{k_{\rm M}{}^2}{L^2}\right)+\frac{k_{\rm M}{}^2}{L^2}\right]$ *and* $(s_1,s_2)\in\Re^+$ *are the minimum and the maximum eigenvalues of the symmetric matrix* $P=\left[p_{ij}\right]_{1\le i,j\le 4}$, $p_{11}=p_{22}=\frac{1}{2}$, $p_{33}=\frac{\lambda}{2}$, $p_{44}=\left(\frac{\gamma^2}{\lambda J^2}+\frac{rF}{2}\right)$, $p_{34}=-\frac{\gamma}{2J}$, $p_{12}=p_{13}=p_{14}=p_{23}=p_{24}=0$.

Proof: Assume that $\cos(p\theta(t_0))=1$. Then, according to the stator currents dynamics in (1),

$$
\begin{aligned}
\cos(p\theta(t)) &= -\frac{Lp}{k_{\rm M}}\Big[i_a(t)-i_a(t_0)-\varphi_1(t)\Big]+1\\
\dot{\varphi}_1(t) &= -\frac{R}{L}i_a(t)+\frac{u_a(t)}{L},\quad \varphi_1(t_0)=0\\
\sin(p\theta(t)) &= -\frac{Lp}{k_{\rm M}}\Big[i_b(t)-i_b(t_0)-\varphi_2(t)\Big]\\
\dot{\varphi}_2(t) &= -\frac{R}{L}i_b(t)+\frac{u_b(t)}{L},\quad \varphi_2(t_0)=0.
\end{aligned}
$$

Introducing the tracking and the estimation errors: $\tilde{\omega}(t)=\omega(t)-\omega^*(t)$; $e_\omega(t)=\hat{\omega}(t)-\omega(t)$; $\tilde{i}_d(t)=i_d(t)-i_d^*(t)$; $\tilde{i}_q(t)=i_q(t)-i_q^*(t)$; $\tilde{i}_a(t)=i_a(t)-\hat{i}_a(t)$; $\tilde{i}_b(t)=i_b(t)-\hat{i}_b(t)$; $\tilde{T}_L(t)=T_L-\hat{T}_L(t)-F[\hat{\omega}(t)-\omega(t)]$, from (2) and (6) we obtain the closed loop system dynamics

$$
\begin{aligned}
\dot{\tilde{\omega}} &= -\frac{F}{J}(\tilde{\omega}+e_\omega)-k_\omega{\rm sat}_\kappa(\tilde{\omega}+e_\omega)+\frac{k_{\rm M}}{J}\tilde{i}_q-\frac{\tilde{T}_L}{J}\\
\dot{\tilde{i}}_d &= -\left(\frac{R}{L}+K_i\right)\tilde{i}_d-pe_\omega i_q \qquad (8)\\
\dot{\tilde{i}}_q &= -\left(\frac{R}{L}+K_i\right)\tilde{i}_q+pe_\omega i_d+\frac{k_{\rm M}}{L}e_\omega
\end{aligned}
$$

$$
\begin{aligned}
\dot{\tilde{i}}_a &= -\left(\frac{R}{L}+K_e\right)\tilde{i}_a-\frac{k_{\rm M}}{L}e_\omega\sin(p\theta)-p\hat{\omega}\tilde{i}_b\\
\dot{\tilde{i}}_b &= -\left(\frac{R}{L}+K_e\right)\tilde{i}_b+\frac{k_{\rm M}}{L}e_\omega\cos(p\theta)+p\hat{\omega}\tilde{i}_a\\
\dot{e}_\omega &= \frac{2\gamma^2+rF\lambda J^2}{\gamma^2\lambda+rF\lambda^2J^2}\left[\frac{k_{\rm M}}{L}\sin(p\theta)\tilde{i}_a-\frac{k_{\rm M}}{L}\cos(p\theta)\tilde{i}_b\right]\\
&\quad +\frac{\tilde{T}_L}{J} \qquad (9)\\
\dot{\tilde{T}}_L &= -\frac{F}{J}\tilde{T}_L+\frac{J\gamma}{\gamma^2+rF\lambda J^2}\Bigg[\frac{k_{\rm M}}{L}\sin(p\theta)\tilde{i}_a\\
&\quad -\frac{k_{\rm M}}{L}\cos(p\theta)\tilde{i}_b\Bigg].
\end{aligned}
$$

Consider the positive definite function

$$
\begin{aligned}
V &= \frac{1}{2}\left(\tilde{i}_a^2+\tilde{i}_b^2\right)+\frac{1}{2}\lambda e_\omega^2+\left(\frac{\gamma^2}{\lambda J^2}+\frac{rF}{2}\right)\tilde{T}_L^2\\
&\quad -\frac{\gamma}{J}e_\omega\tilde{T}_L \qquad (10)
\end{aligned}
$$

whose time derivative along the trajectories of subsystem (9) satisfies

$$
\begin{aligned}
\dot{V} &\leq -\left(\frac{R}{L}+K_e\right)\left(\tilde{i}_a^2+\tilde{i}_b^2\right)+\frac{\lambda}{J}e_\omega\tilde{T}_L\\
&\quad -\frac{\gamma}{J^2}\tilde{T}_L^2+\frac{\gamma^2}{4J^3r}e_\omega{}^2. \qquad (11)
\end{aligned}
$$

On the other hand, subsystem (9) may be rewritten as

$$
\begin{aligned}
\dot{x} &= Ax+B(t)z+H(t)x\\
\dot{z} &= D(t)x+\frac{1}{J}w \qquad (12)\\
\dot{w} &= -\frac{F}{J}w+\frac{\lambda J\gamma}{2\gamma^2+rF\lambda J^2}D(t)x
\end{aligned}
$$

with $x=[\tilde{i}_a,\tilde{i}_b]^T$, $z=e_\omega$, $w=\tilde{T}_L$ and, for all $t\geq t_0$ ($T_p>0$), $\|A\|=\frac{R}{L}+K_e$, $\|B(t)\|=\frac{k_{\rm M}}{L}$, $\|D(t)\|\leq\frac{2k_{\rm M}}{\lambda L}$ $\|\dot{B}^T(t)+B^T(t)H(t)\|\leq\frac{k_{\rm M}p}{L}|z|$, $\int_t^{t+T_p}B^T(\tau)B(\tau)d\tau=\frac{k_{\rm M}{}^2T_p}{L^2}$. System (12) complies with the hypotheses of Lemma 1 reported in the Appendix by setting $T=$

T_p and, consequently, the point $(\tilde{i}_a, \tilde{i}_b, e_\omega, \tilde{T}_L) = 0$ is locally exponentially stable with domain of attraction $\mathcal{A} = \left\{ \zeta \in \Re^4 : \|\zeta\| < \varrho \right\}$. From the second and third equations in (8), according to Lemma III.1 in [11], $(\tilde{i}_d, \tilde{i}_q)$ tend exponentially to zero for any initial condition $[\tilde{i}_a(t_0), \tilde{i}_b(t_0), e_\omega(t_0), \tilde{T}_L(t_0)]^T$ in $\mathcal{A}$. Recalling the first equation in (8), we can establish that, since, for any initial condition $[\tilde{i}_a(t_0), \tilde{i}_b(t_0), e_\omega(t_0), \tilde{T}_L(t_0)]^T$ in $\mathcal{A}$, $(\tilde{i}_q, e_\omega, \tilde{T}_L)$ tend exponentially to zero, $\tilde{\omega}$ is bounded and tends uniformly asymptotically to zero for any initial condition $[\tilde{i}_a(t_0), \tilde{i}_b(t_0), e_\omega(t_0), \tilde{T}_L(t_0)]^T$ in $\mathcal{A}$ and for any saturation value κ: in particular, the origin is a uniformly locally asymptotically stable equilibrium point for system (8), (9) with domain of attraction $\mathcal{B}$.

Remark 1: If $T_L(t)$ is known, we may set

$$\begin{aligned} i_q^* &= \frac{J}{k_M}\left[\frac{F}{J}\omega^* - k_\omega(\hat{\omega} - \omega^*) + \frac{T_L^*}{J} + \dot{\omega}^*\right] \\ \dot{\hat{\omega}} &= -\frac{F}{J}\hat{\omega} + \frac{k_M}{J}i_q - \frac{T_L}{J} + \frac{1}{\lambda}\left[p i_q \tilde{i}_d - p i_d \tilde{i}_q - \frac{k_M}{L}\tilde{i}_q\right] \\ \phi_q &= -\frac{R}{L}i_q^* - p\hat{\omega} i_d - \frac{k_M}{L}\hat{\omega} - \frac{J}{k_M}\left[\frac{F}{J}\dot{\omega}^* - k_\omega\left(\dot{\hat{\omega}} - \dot{\omega}^*\right)\right. \\ &\quad \left. + \frac{\dot{T}_L^*}{J} + \ddot{\omega}^*\right] \end{aligned} \tag{13}$$

in which $T_L^*(t)$ is the output of a stable, unitary gain, linear filter whose input is the actual step load torque $T_L(t)$, so that $\dot{T}_L^*(t)$ is bounded and available for feedback. The closed loop system becomes

$$\begin{aligned} \dot{\tilde{\omega}} &= -\frac{F}{J}\tilde{\omega} - k_\omega(\tilde{\omega} + e_\omega) + \frac{k_M}{J}\tilde{i}_q - \frac{\tilde{T}_{Lf}}{J} \\ \dot{\tilde{i}}_d &= -\left(\frac{R}{L} + K_i\right)\tilde{i}_d - p e_\omega i_q \\ \dot{\tilde{i}}_q &= -\left(\frac{R}{L} + K_i\right)\tilde{i}_q + p e_\omega i_d + \frac{k_M}{L}e_\omega \\ \dot{e}_\omega &= -\frac{F}{J}e_\omega + \frac{1}{\lambda}\left[p i_q \tilde{i}_d - p i_d \tilde{i}_q - \frac{k_M}{L}\tilde{i}_q\right] \end{aligned} \tag{14}$$

where $\tilde{T}_{Lf} = T_L - T_L^*$ is an exponentially decaying signal. Consider the positive definite function

$$V_n = \frac{1}{2}\left(\tilde{i}_d^2 + \tilde{i}_q^2\right) + \frac{1}{2}\lambda e_\omega^2 \tag{15}$$

whose time derivative is given by

$$\dot{V}_n = -\left(\frac{R}{L} + K_i\right)\left(\tilde{i}_d^2 + \tilde{i}_q^2\right) - \frac{F}{J}\lambda {e_\omega}^2. \tag{16}$$

The last three equations in (14) may be rewritten as ($x = [\tilde{i}_d, \tilde{i}_q]^T$, $z = e_\omega$)

$$\begin{aligned} \dot{x} &= \bar{A}x + \bar{B}(t)z \\ \dot{z} &= -\frac{F}{J}z + \bar{D}(t)x \end{aligned} \tag{17}$$

with $\bar{A} = \begin{bmatrix} -\frac{R}{L} - K_i & -p e_\omega \\ p e_\omega & -\frac{R}{L} - K_i \end{bmatrix}$, $\bar{B}(t) = \begin{bmatrix} -p i_q^* \\ p i_d^* + \frac{k_M}{L} \end{bmatrix}$, $\bar{D}(t) = \begin{bmatrix} \frac{p}{\lambda} i_q, & -\frac{p}{\lambda} i_d - \frac{k_M}{\lambda L} \end{bmatrix}$. If we choose $i_d^*(t)$ so that there exist two positive reals T_s, k such that, for all $t \geq t_0$, $\int_t^{t+T_s} \bar{B}^T(\tau)\bar{B}(\tau)d\tau \geq kI$, system (17) complies with the hypotheses of Lemma 2 in the Appendix by setting $A = \bar{A}$, $B = \bar{B}$, $g = \frac{F}{J}$, $D = \bar{D}$, $T = T_s$ and the point $(\tilde{i}_d, \tilde{i}_q, e_\omega) = 0$ is uniformly globally asymptotically stable and exponentially stable in compact sets. Recalling the first equation in (14), we can establish that, since $(\tilde{i}_q, e_\omega)$ and $\tilde{T}_{Lf}$ converge exponentially to zero, $\tilde{\omega}$ tends exponentially to zero for any initial condition.

Remark 2: In practice, as confirmed by experiments in [1], the most of the possible bias errors introduced by current sensors and voltage actuators can be removed by using the alignment procedures and the fact that, even in the presence of voltage actuator bias errors, the alignment procedures guarantee the convergence to zero of the quantities $\dot{\varphi}_1$ and $\dot{\varphi}_2$ in (6).

III. SIMULATION RESULTS

A. Nonlinear control strategy

On the basis of the three theorems stated in Section 2, we propose the following nonlinear speed tracking control strategy: *"s-alignment" procedure (3)* $[0 \leq t < t_s]$; *"c-alignment" procedure (5)* $[t_s \leq t < t_c]$; *adaptive controller (6)* $[t \geq t_c]$.

B. Control performance

We tested the proposed control strategy (3), (5), (6) ($t_s = 0.3$ s, $t_c = 0.6$ s) by simulations with control parameters (the values are in SI units) $u_{cs} = u_{cc} = 24$, $k_\omega = 100$, $K_e = 100$, $K_i = 20$, $\gamma = J/9$, $\lambda = 0.2$, $\kappa = 9$ for a permanent magnet step motor with sinusoidal flux distribution, whose parameters are: $F = 0$ Kgm^{2}s^{-1}, $p = 6$, $J = 0.01$ Kgm^2, $R = 3$ Ohm, $L = 0.006$ H, $k_M = 2$ NmA^{-1}. All initial conditions of the motor and the controller (6) are set to zero excepting $\theta(0) = 30$ deg., $\hat{i}_a(t_c) = i_a(t_c)$ and $\hat{i}_b(t_c) = i_b(t_c)$. The reference for current i_d was set to zero, while the reference for speed was generated by using ramp functions which were filtered using the third order linear filter $H(s) = \left(\frac{s}{\varpi_0} + 1\right)^{-1} \cdot \left(\frac{s^2}{\varpi_0^2} + 2\frac{s}{\varpi_0} + 1\right)^{-1}$ (ϖ_0=300 rad/s). The necessary first and second order derivatives were obtained from the state space realization of the filter. The load torque (2 Nm, the rated value) is applied at $t = 0.63$ s. The reference for speed along with the applied torque are reported in Fig. 1. Figure 2, which shows the time histories of the electrical angle $[p\theta]$ and the rotor speed for $t \leq t_c$, illustrates the fast response of the alignment procedures, while Fig. 3 shows the time history of rotor speed tracking error for $t \geq t_c$ and the (a, b) components of stator current and stator voltage vectors.

IV. CONCLUSIONS

On the basis of stator currents and voltages measurements only, an innovative nonlinear adaptive speed tracking control

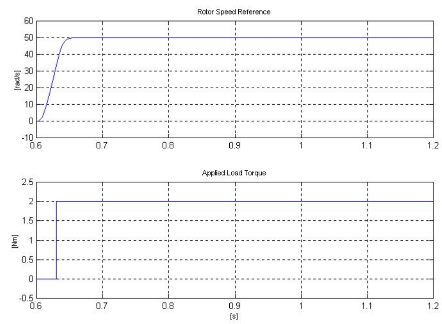

Fig. 1. Rotor speed reference signal and applied load torque.

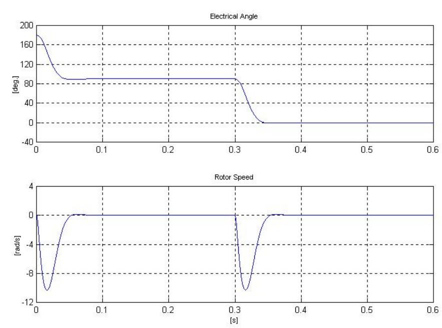

Fig. 2. Electrical angle and rotor speed.

strategy has been proposed for a permanent magnet step motor, relying on the new theoretical results (not based on motor friction) stated in Theorems 1-3. Numerical simulation results illustrate the closed loop performances and show the effectiveness of the proposed solution.

APPENDIX

Lemma 1: Consider the system

$$\begin{aligned} \dot{x} &= f_x(t,x,z,w) = A(t)x + B(t)z + C(t)w + H(t)x \\ \dot{z} &= f_z(t,x,w) = D(t)x + E(t)w \qquad (18) \\ \dot{w} &= f_w(t,x,w) = F(t)x + G(t)w \end{aligned}$$

where $x(t) \in \Re^n$, $z(t) \in \Re^p$, $w(t) \in \Re^m$, $f_x(\cdot,x,z,w)$ is locally Lipschitz continuous, $f_z(t,x,w)$, $f_w(t,x,w)$ are continuous and $B(t)$ is continuous and differentiable with respect to time for all $t \geq t_0$. Assume that, for all $t \geq t_0$,

1) $A(t), B(t), C(t), D(t), E(t), F(t), G(t)$ are bounded, with $\|A(t)\| \leq A_M$, $\|B(t)\| \leq B_M$, $\|C(t)\| \leq C_M$, $\|D(t)\| \leq D_M$, $\|E(t)\| \leq E_M$;
2) there exist two positive reals $T, k \in \Re^+$, such that the *persistency of excitation* condition
$$\int_t^{t+T} B^T(\tau)B(\tau)d\tau \geq kI \qquad (19)$$
is satisfied;
3) there exists a smooth proper function $V(x,z,w,t)$ and suitable positive reals a_1,a_2,a_3,a_4,a_5,$p_e \in \Re^+$, such that
$$\begin{aligned} V(x,z,w,t) &\geq a_1(\|x\|^2 + \|z\|^2 + \|w\|^2) \\ V(x,z,w,t) &\leq a_2(\|x\|^2 + \|z\|^2 + \|w\|^2) \\ \dot{V} &\leq -a_3\|x\|^2 - a_4\|w\|^2 + a_5\|w\|\|z\| \\ &\quad + \frac{p_e}{8}k^2 e^{-4T}\|z\|^2 \qquad (20) \end{aligned}$$
and
$$\begin{aligned} a_4 &> \frac{a_5^2 e^{4T}}{p_e k^2} + 8p_e(B_M^4 E_M^2 + B_M^2 C_M^2) \\ p_e &< \frac{a_3}{12(B_M^4 D_M^2 + B_M^2 A_M^2 + B_M^2) + B_M^2} \end{aligned}$$
where T, k are defined in (19);
4) there exists a suitable positive real $a_6 \in \Re^+$ such that
$$\|\dot{B}^T(t) + B^T(t)H(t)\| \leq a_6\|z\|.$$

Then the origin of system (18) is locally exponentially stable with an explicitly computable domain of attraction.

Proof: As in [10], consider the class of radially unbounded functions

$$W(x,z,w,t) = V(x,z,w,t) + p_e\|Qz - B^T x\|^2 \qquad (21)$$

where $Q(t)$ is generated by the filter $dQ(t)/dt = -Q(t) + B^T(t)B(t)$, with $Q(t_0) = e^{-T}kI$. By virtue of assumption 2, $Q(t+T) \geq e^{-T}\int_t^{t+T} B^T(\tau)B(\tau)d\tau \geq e^{-T}kI > 0$. Since $\|B(t)\| \leq B_M$, it is straightforward to deduce that

$$B_M^2 I \geq Q(t) > ke^{-2T}I. \qquad (22)$$

In view of hypothesis 3,

$$\begin{aligned} W(x,z,w,t) &\geq a_1(\|x\|^2 + \|z\|^2 + \|w\|^2) \\ &\quad + p_e\|Qz - B^T x\|^2 \\ W(x,z,w,t) &\leq a_2(\|x\|^2 + \|z\|^2 + \|w\|^2) \\ &\quad + p_e\|Qz - B^T x\|^2. \qquad (23) \end{aligned}$$

By computing the time derivative of $W(x,z,w,t)$, by virtue of assumption 3, we have

$$\begin{aligned} \dot{W} \leq\; & -a_3\|x\|^2 - a_4\|w\|^2 + a_5\|w\|\|z\| \\ & + 2p_e(Qz - B^T x)^T[QE - B^T C]w \\ & + 2p_e(Qz - B^T x)^T[QDx - Qz - B^T Ax \\ & - \dot{B}^T x - B^T Hx] + \frac{p_e}{8}k^2 e^{-4T}\|z\|^2 \end{aligned}$$

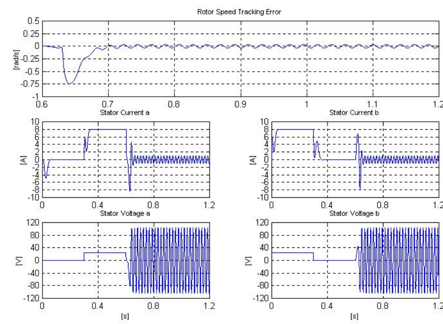

Fig. 3. Rotor speed tracking error and (a, b) components of stator current and stator voltage vectors.

so that by adding and subtracting $2p(Qz - B^T x)^T B^T x$ in the right hand side of the previous inequality, we obtain

$$\begin{aligned}\dot{W} \quad \leq \quad & -a_3\|x\|^2 - a_4\|w\|^2 + a_5\|w\|\|z\| \\ & -2p_e\|Qz - B^T x\|^2 + 2p_e(Qz - B^T x)^T[QE \\ & -B^T C]w + 2p_e(Qz - B^T x)^T[QD - B^T A \\ & -B^T]x - 2p_e(Qz - B^T x)^T[\dot{B}^T \\ & +B^T H]x + \frac{p_e}{8}k^2 e^{-4T}\|z\|^2. \qquad (24)\end{aligned}$$

By hypothesis 1, there exist two positive reals $a_7, a_8 \in \Re^+$, such that $a_7 \geq \|QD - B^T A - B^T\|^2$ and $a_8 \geq \|QE - B^T C\|^2$. By completing the squares and rearranging terms, we obtain

$$\begin{aligned}\dot{W} \quad \leq \quad & -\Big[a_3 - p_e(4a_7 + B_M^2)\Big]\|x\|^2 - \frac{p_e}{8}k^2 e^{-4T}\|z\|^2 \\ & -\left[a_4 - \frac{a_5^2 e^{4T}}{p_e k^2} - 4p_e a_8\right]\|w\|^2 - \frac{p_e}{2}\|Qz - B^T x\|^2 \\ & -2p_e(Qz - B^T x)^T[\dot{B}^T + B^T H]x. \qquad (25)\end{aligned}$$

By virtue of assumptions 3 and 4, there exist three positive scalars $r_1, r_2, a_6 \in \Re^+$ such that

$$\begin{aligned}\dot{W} \quad \leq \quad & -r_1\|x\|^2 - \frac{p_e}{8}k^2 e^{-4T}\|z\|^2 - r_2\|w\|^2 \\ & -\frac{p_e}{2}\|Qz - B^T x\|^2 \\ & +2p_e a_6\|(Qz - B^T x)\|\|z\|\|x\|. \qquad (26)\end{aligned}$$

By completing the squares we obtain

$$\begin{aligned}\dot{W} \quad \leq \quad & -\Big[r_1 - 4p_e a_6^2\|z\|^2\Big]\|x\|^2 - \frac{p_e}{8}k^2 e^{-4T}\|z\|^2 \\ & -r_2\|w\|^2 - \frac{p_e}{4}\|Qz - B^T x\|^2. \qquad (27)\end{aligned}$$

Let $\zeta = [x, z, w, Qz - B^T x]^T$ and $\zeta_s = [x, z, w]^T$; since (23) and (27) imply ($s > 0$)

$$\|\zeta(t)\|^2 \quad \leq \quad \frac{\max\{a_2, p_e\}}{\min\{a_1, p_e\}}\|\zeta(t_0)\|^2 e^{-s(t-t_0)}, \quad \forall t \geq t_0$$

for any initial condition $\zeta(t_0) \in \Re^{n+2p+m}$ such that $\|\zeta(t_0)\| < \frac{1}{2a_6}\sqrt{\frac{\min\{a_1,p_e\}r_1}{\max\{a_2,p_e\}p_e}}$, by virtue of (22) and (23), we can finally establish that for all $t \geq t_0$

$$\|\zeta_s(t)\|^2 \leq \frac{a_2 + 2p_e B_M^2 \max\{1, B_M^2\}}{a_1}\|\zeta_s(t_0)\|^2 e^{-s(t-t_0)}$$

for any initial condition $\zeta_s(t_0)$ belonging to the open ball

$$\mathcal{A} \quad = \quad \left\{\zeta_s(t_0) \in \Re^{n+p+m} : \|\zeta_s(t_0)\| < \frac{1}{2a_6}\sqrt{\frac{\min\{a_1, p_e\}r_1}{p_e\Big(a_2 + 2p_e B_M^2 \max\{1, B_M^2\}\Big)}}\right\}.$$

Lemma 2: Consider the system

$$\begin{aligned}\dot{x} \quad &= \quad f_x(t, x, z) \quad = \quad A(t)x + B(t)z \\ \dot{z} \quad &= \quad f_z(t, x) \quad = \quad -gz + D(t)x \qquad (28)\end{aligned}$$

where $x(t) \in \Re^n$, $z(t) \in \Re^p$, $f_x(t, x, z)$, $f_z(t, x)$ are continuous, g is a non-negative scalar and $B(t)$ is continuous and differentiable with respect to time for all $t \geq t_0$. Assume that, for all $t \geq t_0$,

1) $A(t)$, $B(t)$, $D(t)$ are, with $\|A(t)\| \leq A_M$, $\|B(t)\| \leq B_M$, $\|D(t)\| \leq D_M$ and $\dot{B}(t)$ is bounded, with $\|\dot{B}(t)\| \leq \dot{B}_M$;
2) there exist two positive reals $T, k \in \Re^+$, such that the *persistency of excitation* condition
$$\int_t^{t+T} B^T(\tau)B(\tau)d\tau \geq kI \qquad (29)$$
is satisfied;
3) there exists a smooth proper function $V(x, z, t)$ and suitable positive reals a_1,a_2,$a_3 \in \Re^+$, such that
$$\begin{aligned} a_1(\|x\|^2 + \|z\|^2) \leq V(x, z, t) \leq a_2(\|x\|^2 + \|z\|^2) \\ \dot{V} \leq -a_3\|x\|^2. \qquad (30)\end{aligned}$$

Then the origin of system (28) is globally exponentially stable.

Proof: Arguments similar to those adopted in the proof of Lemma 1 can be used.

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