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Friction Compensation in Robotics: an Overview

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Abstract— **Friction effects are particularly critical for industrial robots, since they can induce large positioning errors, stick-slip motions, and limit cycles. This paper offers a reasoned overview of the main friction compensation techniques that have been developed in the last years, regrouping them according to the adopted kind of control strategy. Some experimental results are reported, to show how the control performances can be affected not only by the chosen method, but also by the characteristics of the available robotic architecture and of the executed task.**

I. Introduction

Each time precise motion control must be achieved by a robotic system, friction compensation represents a crucial step for the designer, who must solve various theoretical and practical problems. Friction effects are particularly critical for industrial robots: it has been observed [1] that "friction can cause 50% error in some heavy industrial manipulators". A poor (or absent) friction compensation action in the control scheme may lead to significant tracking errors (especially at low velocities), stick-slip motions, hunting in the stopping phase of the robot movement, and limit cycles when velocity reversals occur in the assigned trajectory.

Several friction models have been proposed [2]-[12], having different levels of accuracy, and a wide variety of control solutions can be found in literature [1], [3]-[6], [11], [13]-[15], [19]-[25], [27]-[43], but no strategy can be definitely considered more effective than the others, since many factors can significantly affect the practical implementation and the performances of each scheme.

This paper proposes an overview of the most used and recent control strategies for friction compensation in robotics, with the purpose to offer a guideline to the reader among the various solutions that have been developed in literature. A classification of the various control schemes can be made following different criteria: in this paper the *control strategy* is considered instead of the assumed friction model (as it is often done), to help the controller designer to choose the most suitable solution for the task to be executed, given the available hardware and software control architecture. We have considered only the most recent papers (as the interested reader can find previous results among their references) and the control schemes that have been experimentally tested.

The paper is organized as follows. In Section II, the most common friction models are briefly pointed out, while Section III deals with the different control strategies developed for friction compensation, distinguishing four main types of solutions, which are discussed in the four corresponding subsections. Section IV draws some conclusions concerning the reviewed methods, and discusses some additional issues, on the basis of experimental tests carried out in our laboratory for a planar manipulator with standard-resolution resolvers.

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II. Friction models

Friction forces between two surfaces in contact arise as a consequence of the irregularities and asperities at microscopical level, and their effects depend on many factors, such as displacement and relative velocity of the bodies, properties of the surface materials, presence of lubrication, temperature, etc. The experimental observation of friction phenomena has led to various, deeply different models, which capture the friction components in a more or less accurate way. Interesting reviews of the main friction characteristics and classical models can be found in [3], [4], [6], starting from the basic concept of friction as a force that opposes motion, captured by the pure Coulomb model, up to complex static and dynamic models.

The simplest static friction model that can be adopted is the *Coulomb + viscous* one:

$$F(v) = F_c \mathrm{sgn}(v) + \beta v \tag{1}$$

where F is the friction force, v is the relative velocity of the contact surfaces, β is the viscous friction coefficient, and F_c is the Coulomb friction.

Model (1) can be easily upgraded taking into account the presence of *stiction*, i.e. a higher friction force F_s acting while the system is at rest: motion can start only when the applied external forces (i.e. the command torque applied to the joint in the robotic case) are greater than F_s [6]. From experimental observation it is evident that the passage from stiction to friction during motion is not discontinuous: on the basis of such a consideration, different mathematical models have been formulated to represent friction vs. velocity by continuous functions that account for the so-called *Stribeck effect*, i.e. the decrease of the friction amount as velocity increases within the low-velocity Stribeck region. The most used nonlinear expression leads to a model of the following type:

$$F(v) = \left[F_c + (F_s - F_c)e^{-|\frac{v}{v_s}|^{\delta_s}}\right] \mathrm{sgn}(v) + \beta v \tag{2}$$

where v_s is the *Stribeck velocity* that defines the region in which such an effect is present, and δ_s can be equal to 2 (but not necessarily).

Other models have been used for control purposes to represent friction, including Stribeck, Coulomb and

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viscous effects, such as a simple polynomial function of a proper order as in [13], [16], or sigmoid-functions as in [7], [17]. All the models analyzed up to this point, and many others that have been developed from the Karnopp model (see Section III-B and references [2], [6]) are *static* models, as friction depends only on the *current* velocity value. The friction phenomena related to non-stationary velocities, variations in the break-away force, small displacements occurring during the stiction phase, and hysteretic effects can be captured only by *dynamic* models, which describe the pre-sliding behavior representing the microscopic asperities of the contact surfaces by means of elastic bristles, whose deflections give rise to the friction forces. The well-known Dahl model (illustrated and discussed in many papers, such as [4], [6], [14]) is the simplest possible description of the dynamic friction behavior, and it can be expressed introducing an internal state variable z (representing the average bristle deflection at the contact points of the moving surfaces) and defining the friction force accordingly as:

$$\dot{z} = v - \frac{|v|}{F_c}\sigma z \tag{3a}$$

$$F = \sigma z \tag{3b}$$

where σ represents the bristle stiffness parameter. The Dahl model neither includes stiction, nor the Stribeck effect, since it simply adds a lag in the changes of the friction forces as velocity changes.

The widely used *LuGre* friction model, proposed for the first time in [5], is *complete* from this point of view, since it includes also these effects, describing the behavior of the friction force as:

$$\dot{z} = v - \frac{|v|}{g(v)}\sigma_0 z \tag{4a}$$

$$F = \sigma_0 z + \sigma_1 \dot{z} + f(v) \tag{4b}$$

where z is an internal state variable as in (3), σ_0 and σ_1 are model parameters assumed to be constant, and functions $g(v)$ and $f(v)$ model the Stribeck effect and the viscous friction, respectively. For constant velocity, the steady-state friction force is then given by:

$$F_{ss} = g(v)\mathrm{sgn}(v) + f(v) \tag{5}$$

Different parameterizations are possible for $g(v)$ and $f(v)$: the most used choices are those leading to an expression for the steady-state friction similar to (2). Details about them and about the identification procedure for the LuGre model can be found in [15]. It must be underlined that the identification of a friction dynamic model like the LuGre one always presents some peculiar difficulties, related to different aspects, such as the impossibility to directly measure the internal state z, the high sensitivity of the stick-slip motions to the values of the dynamic parameters (i.e. σ_0 and σ_1 for the LuGre model), and the necessity of high precision sensors to correctly capture the phenomena of the pre-sliding phase.

Even if the LuGre model is often used for control, it does not represent the final solution from the modelling point of view, since it does not take into account possible hysteresis effects. Alternative friction models, including also hysteresis effects at low velocity, have been proposed and discussed e.g. in [8], [9], and [11], whereas updates to the LuGre model have been proposed in [10] to avoid nonphysical drift phenomena in the presliding phase. More recently, single and multistate integral friction models have been proposed in [12], to account for the hysteresis behavior with non local memory.

III. Control strategies for friction compensation

Several solutions have been proposed to compensate for friction in robotics, considering different kinds of friction phenomena, and hence different models to represent it, and various strategies to eliminate or satisfactorily reduce its undesirable effects on the robot motion. In practice the choice of a particular solution is strongly influenced by factors like the available actuators, sensors, and hardware/software control architecture, as well as by constraints on the real-time computational burden. It can be useful then to analyze the various friction compensation techniques classifying them according to the adopted *kind of control strategy*, thus recognizing the following four main types of solutions.

A) A 'fixed' friction compensation term is added to a more general control scheme, like a joint independent one or an inverse dynamics control scheme [18], by estimating the friction parameters off-line, on the basis of an assigned (more or less complicated) friction model, following proper, *ad hoc* identification procedures [17], [22], [23]. If a dynamic friction model is considered, an observer is inserted to estimate the friction internal state [24], [25]. A further correction or a robust control action can be possibly added to the a priori estimated compensation term [1], [5], [14], [27], [28].

B) Only some main characteristics of the friction phenomena (e.g. the maximum stick component) are taken into account for compensation within the control scheme. The estimates of the required friction characteristics are determined off-line, whereas the on-line compensation action is tuned on the basis of such estimates [29]-[31].

C) Model-based adaptive algorithms are applied for on-line friction compensation [15], [34]-[39]. The adaptive schemes are based on a particular, static or dynamic, friction model, whose parameters are tuned on-line to obtain a satisfying compensation action also when significant variations are present.

D) Strategies that are not based on a particular friction model can be applied to counteract the friction effects, by properly choosing the control gain parameters or by using non model-based observers [40]-[43].

Another group of friction compensation techniques takes advantage of the so-called *soft computing* approach, using fuzzy, neural, and genetic algorithms to reconstruct the friction torques to be compensated or for a suitable self-tuning of the controller gains (see e.g. [19]-[21]). These approaches will not be reviewed in detail in the present paper for space reasons.

Each type of control solution has advantages and drawbacks that cannot be ignored. Solutions of type **A)**, especially if based on static friction models, are the 'cheapest' to be implemented in practice, since the required on-line computational burden is limited (a feedforward compensation term could be possibly used, if necessary, in the static case using the reference position/velocity values instead of the current ones), but they require an accurate friction identification phase (possibly with high-precision sensors), and they cannot, obviously, account for friction variations. Solutions **B)** could offer some more robustness features, but a correct tuning of the compensation action is crucial to avoid limit cycles, and to obtain an accurate final positioning. Solutions **C)** often require a significant on-line computational burden, and high-precision sensors when dynamic friction models are considered. Besides, only some of the friction parameters can be easily updated on-line, as it will be discussed in Section III-C. In the control solutions of type **D)**, the friction effects are often estimated and compensated only together with other disturbances acting on the robot motion, so that the actual control performances could significantly vary in practice for different trajectories and/or be satisfying only if high control values can be sustained by the considered robotic system.

It is then evident that none of these types of control solutions is superior to the others, in every case. Some control schemes proposed in literature, and regrouped according to the above classification, are discussed in the following, to analyze with more details their main characteristics.

A. *Fixed friction compensation techniques*

Satisfactory results can be obtained by adding a 'fixed' friction compensation term to standard control algorithms, if a good off-line estimate of the friction parameters has been performed, provided that friction variations with time, temperature, etc. are negligible.

It is quite obvious that the friction model used to define the compensation term must be sufficiently accurate: compensation terms based on the pure Coulomb + viscous friction model (1) cannot provide accurate tracking and positioning results, as shown e.g. in [22], where this kind of solution is compared with friction compensation performed on the basis of the nonlinear, static model (2) in the case of a simple servomechanism. As expected, the best results are obtained when the second kind of compensation term is added to a model-based control algorithm, also thanks to a scrupulous identification of the friction model parameters, aimed at the avoidance of limit cycles generation.

More recently, another compensation solution based on the same kind of discontinuous, static friction model has been proposed in [23], where an observer-based compensation scheme is developed by estimating again the parameters of the friction model off-line. A linear observer reconstructs the system states (i.e., joint position and velocity in the robotic case), whereas a nonlinear friction compensation term, defined by using the identified friction model and the estimated velocity, is added to a standard observer-based state feedback control. Good performances can be obtained by a proper tuning of the controller and the observer gains, if the presliding friction dynamics is actually negligible.

A different kind of static, a priori estimated friction compensation solution is proposed in [17] for a three revolute joints robot, on the basis of the following three-sigmoid-function friction model:

$$\tau_i^f = \sum_{k=1}^{3} f_{i,k} \left(1 - \frac{2}{\mathrm{e}^{2 w_{i,k} \dot{q}_i} + 1} \right) + b_i \dot{q}_i, \quad i = 1, 2, 3 \quad (6)$$

where τ_i^f is the friction torque acting on the i-th joint, $\dot{q}_i$ is the i-th joint velocity, and $f_{i,k}$, $w_{i,k}$ and b_i are the parameters that define the friction model. It is important to note that model (6) does not capture the static friction component, since $\tau_i^f = 0$ at $\dot{q}_i = 0$: its compensation then cannot be performed by the model-based term but only via an integral control action. The results obtained in a hand-writing task by a complete inverse dynamics control scheme with friction compensation are extremely good.

If the dynamic friction effects in the presliding phase cannot be neglected, the compensation term must be defined on the basis of a dynamic model, with the insertion of an observer for the friction internal state estimation, as in [24], [25], where the Dahl model (3) is considered, with the addition of a viscous friction term, to compensate friction within velocity control schemes. In particular, in [24] an inverse dynamics control scheme and a PD-like algorithm (given by a nonadaptive version of the motion controller proposed in [26]) are applied to a two dof direct-drive robot arm, comparing the results obtained by a pure Coulomb + viscous friction compensation with those provided by the Dahl model-based compensation. As expected, the second friction compensation solution gives the best results with both controllers, while the PD algorithm provides better performances with respect to the inverse dynamics scheme, when the same friction compensation term is inserted, perhaps because of a higher sensitivity of the last controller to model and parameter estimation errors.

Since in practice it is not always possible to obtain a sufficiently accurate friction description to be used for a pure fixed compensation, some kind of on-line correction action is added to the compensation provided by the off-line estimated friction model to obtain better control performances, thus getting to schemes that lie between types **A)** and **C)**. In [27], the Coulomb and viscous friction parameters are estimated off-line, whereas the nonlinear, Stribeck effects are compensated within a sliding model control scheme by means of a disturbance observer; only an upper bound of the Stribeck term is required for the implementation of such a controller. It is important to note that the estimated disturbance term is not necessarily given by friction only, even if its effects are certainly dominant in the low velocity region. A nonlinear $\mathcal{H}_\infty$-controller is proposed in [14], where friction is supposed to be described by the Dahl friction model, assuming that its parameters are known (i.e. identified off-line), while the discrepancies between the actual friction and the estimated one are overcome by the robustness properties of the approach.

The control solution proposed in [5] is constituted by a linear position or velocity controller + a friction compensation term, estimated according to the LuGre friction dynamic model (4) with the insertion of an 'observer' term, proportional to the tracking error in (4a), thus obtaining an estimate of the friction force $\hat{F}$ as:

$$\begin{aligned} \dot{\hat{z}} &= v - \frac{|v|}{g(v)}\sigma_0 \hat{z} - Ke && \text{(7a)} \\ \hat{F} &= \sigma_0 \hat{z} + \sigma_1 \dot{\hat{z}} + \beta v && \text{(7b)} \end{aligned}$$

where e is the tracking error, K is the observer gain, and $f(v) = \beta v$ has been considered in (4b). A similar approach for friction compensation on robot joints has been considered in [1], together with a PD algorithm with gravity compensation; an interesting discussion is developed by the authors about the friction identification problems for a 2 dof micro-manipulator and a 4 dof macro-manipulator, showing the importance of such a phase for the control performances, together with the inability of a pure static, steady-state model to describe friction beyond a certain accuracy, at least for the considered manipulators. A similar compensation term is added also in [28] to an output feedback controller, by introducing in (7a) a properly designed output function $K(y)$ instead of Ke, but no experimental results are reported to test the actual performances of such a scheme.

B. Techniques based on a partial knowledge of the friction characteristics

These techniques are based on the addition of a friction compensation term to standard control algorithms, like in the previous cases, but without requiring the knowledge of the exact friction model.

In particular, this kind of approach, originally developed in [29], introduces a specific nonlinear compensation term that supplements a standard PD control algorithm; asymptotic stability is guaranteed for a stick-slip friction system, provided that the upper bounds of the static friction levels are known; robustness is also assured with respect to the characteristics of the slipping force, assumed to lie within a piecewise linear band. The friction torque τ_f acting on each joint of a robot is described according to the Karnopp model [2], as the sum of the static torque τ_{stick} and the slipping torque τ_{slip}, with:

$$\tau_{stick} = \begin{cases} \tau_s^+ & 0 < \tau_s^+ < \tau_c \\ \tau_c & \tau_s^- \le \tau_c \le \tau_s^+ \\ \tau_s^- & \tau_c < \tau_s^- < 0 \end{cases} \qquad \text{(8a)}$$

$$\tau_{slip}(\dot{q}) = \tau_d^+(\dot{q})\mu(\dot{q}) + \tau_d^-(\dot{q})\mu(\dot{q}) \qquad \text{(8b)}$$

where τ_c is the command torque, τ_s^+ and τ_s^- are the positive and negative limits of the static friction torque, $\tau_d^+(\dot{q})$ and $\tau_d^-(\dot{q})$ are the slipping torque functions for positive and negative velocities, respectively, supposed to be bounded within the first and third quadrants, and $\mu(\cdot)$ is the right-continuous Heaviside step function.

The application of a traditional PD control leads to a steady-state position error, since all the trajectories end up within an equilibrium region in which $q_L \le q \le q_H$, with $q_L = -\tau_s^+/K_P < 0$, $q_H = -\tau_s^-/K_P > 0$, K_P being the position control gain, and assuming $q_r = 0$ as reference position, for the sake of simplicity. In the nonlinear solution proposed in [29], the steady-state position error is eliminated by defining the command torque as $\tau_c = -K_D\dot{q} - \tau_n$, with

$$\tau_n = \begin{cases} -\tilde{\tau}_s^- & 0 < q \le \tilde{q}_H \\ 0 & q = 0 \\ -\tilde{\tau}_s^+ & \tilde{q}_L \le q \le 0 \\ K_P q & \text{otherwise} \end{cases} \qquad \text{(9a)}$$

where

$$\begin{aligned} &\tilde{q}_H = q_H + \varepsilon, \;\; \tilde{q}_L = q_L - \varepsilon, \;\; \varepsilon > 0 \\ &\tilde{\tau}_s^+ = -K_P\tilde{q}_L = \tau_s^+ + K_P\varepsilon \\ &\tilde{\tau}_s^- = -K_P\tilde{q}_H = \tau_s^- - K_P\varepsilon \end{aligned} \qquad \text{(9b)}$$

The nonlinear compensation torque τ_n is active only when the joint position is between the augmented sticking limits $\tilde{q}_L$, $\tilde{q}_H$, while the controller is essentially a PD one outside such a region.

Some modifications and upgrades have been proposed more recently to this technique, extending in particular its application to digital control systems, too, for which a stable limit cycle response would be induced otherwise, due to the time delay introduced by the sample-and-hold operations. In particular, in [30] an hysteresis expression has been proposed for the nonlinear compensation term τ_n, introducing two nonzero constants, δ_L and δ_H, denoting the bounds of the velocity-dependent dead zone (see [30] for details). Their choice is crucial to eliminate the destabilizing effect of the time delay: greater values improve the stability margin, but enlarge the error bounds, since the steady-state position will lie between δ_L and δ_H. Slight modifications to the method, e.g. in [31], have been proposed to try to obtain a smaller final error, but the actual possibility to achieve good performances seems to be strongly related to the characteristics of the considered robotic systems and of the hardware control architecture, which influences the choice of δ_L, δ_H and of the sampling time.

C. Adaptive compensation schemes

As friction characteristics vary with time, temperature and system operating conditions, the adaptive compensation approach seems to be the natural solution to maintain satisfying and constant control performances in the various situations. Even if the researchers interest has been devoting to this kind of approach since the beginning of the 90's (see e.g. [32], [33]), two main problems emerge each time a complete, dynamic friction model is considered: (*i*) some friction parameters enter in a nonlinear way in the model, and (*ii*) part of the system dynamics (i.e. the internal friction state z), that in turn depends on the unknown parameters, is not measurable. There is no global solution to a problem like this, in which system state variables and parameters of a nonlinear model should be simultaneously estimated. Leaving to the interested reader more details about each particular adaptive algorithm that has been proposed in the last years, it is interesting to compare the different approaches that has been followed to overcome the above two problems.

In [15] adaptive versions of the observer-based friction compensation scheme proposed in [5], and briefly recalled in Section III-A, are developed to cope with *structured* friction variations in two cases. In the first one, variations of the normal forces exchanged between the contact surfaces are assumed to mainly affect the friction static parameters, whereas the dynamic parameters are considered as invariant due to the unchanged lubricant characteristics, and possible variations of the viscous coefficient β are directly dealt with the linear controller. In the second case, temperature variations are assumed to uniformly affect both static and dynamic friction parameters. The resulting parameter uncertainties in both cases are captured by a unique variable parameter θ, by rewriting model (4) with $f(v) = \beta v$ as follows:

$$\text{Case 1):} \qquad \left\{ \begin{array}{rcl} \dot{z} & = & v - \theta \dfrac{|v|}{g(v)} \sigma_0 z \\ F & = & \sigma_0 z + \sigma_1 \dot{z} + \beta v \end{array} \right. \tag{10}$$

$$\text{Case 2):} \qquad \left\{ \begin{array}{rcl} \dot{z} & = & v - \dfrac{|v|}{g(v)} \sigma_0 z \\ F & = & \theta \left(\sigma_0 z + \sigma_1 \dot{z} + \beta v \right) \end{array} \right. \tag{11}$$

where parameters σ_0, σ_1, β and function $g(v)$ are supposed to be known, i.e. a priori identified, and θ is updated on-line, according to a proper adaptation law (see [15] for details and for some experimental results). This kind of approach has been upgraded in [34], where three different observers are developed for the estimation of the unmeasurable friction state, in order to relax the conditions required in [5], and hence to facilitate the use of different control loops. On the basis of such observers, two adaptive controllers are proposed: the first one compensates for all the mechanical parameters variations, except for the parameters associated to the Stribeck effect (i.e. the parameters of function $g(v)$ in (4a)); the second one compensates for *only* a single parameter associated with normal force variations in the Stribeck effect function, following the same approach of case 1) in [15], but utilizing a nonlinear filter structure that allows an active compensation for the observer transient. Experimental results are available only in a 1-dof case, given by a switched reluctance motor, with a metal disk attached to the rotor.

An interesting comparison of different static and dynamic friction compensation techniques, including adaptive algorithms, is presented in [35], in the case of a two-dof planar manipulator, showing in particular the different performances that can be obtained by using (*i*) a pure computed torque scheme, (*ii*) the adaptive controller developed in [26] without friction compensation, the same kind of adaptive control scheme with (*iii*) static or (*iv*) dynamic friction compensation. In the static friction case, only the Coulomb and viscous components are considered and directly included in the adaptive law, since they enter linearly in the model, and similarly, in the dynamic friction case, only the parameters that linearly enter in equation (4b) are included in the adaption law, while the parameters defining function $g(v)$ in (4a) are a priori identified, and an observer is inserted for the estimation of z and $\dot{z}$. This last solution gives the best results, as expected, thanks also to the available high-precision resolver.

All the considered adaptive algorithms, based on *dynamic* friction models, result in compromises between the necessity of estimating the unmeasurable part of the friction dynamics and of updating on-line the friction parameters, typically disregarding the nonlinear parameters related to the Stribeck effect in the adaptation law. If a *static* model is considered to represent friction, nonlinear adaptive control schemes can be developed, taking into account also the variations of the Stribeck effect parameters, as in [36], where friction is modelled as:

$$F(v) = \left[F'_c + F'_s \mathrm{e}^{(-F_\tau v^2)} \right] \mathrm{sat}(v) \tag{12}$$

where the sat($\cdot$) function can be defined as the standard, discontinuous signum function (see [36] for details). In particular, one of the two control schemes that are proposed is given by an adaptive set-point controller (i.e. performing only *regulation* tasks), which compensates for the uncertainty associated with *all* the friction parameters. In particular, the stiction and Stribeck parameters F'_s and F_τ are updated by laws of the following types:

$$\begin{aligned} \dot{\widehat{F}}'_s &= -\gamma_0 v \ \mathrm{sat}(v) \ \mathrm{e}^{(-\widehat{F}_\tau v^2)} && \text{(13a)} \\ \dot{\widehat{F}}_\tau &= \gamma_1 v^3 \ \mathrm{sat}(v) \ \mathrm{e}^{(-\widehat{F}_\tau v^2)} && \text{(13b)} \end{aligned}$$

where γ_0 and γ_1 are positive, constant gains. Experimental results are shown for the same setup used in [34].

The tracking problem has been addressed in [37], where a general framework of adaptive control is proposed to compensate for uncertain nonlinear parameters appearing in robot dynamic model, assuming that friction is described by the static, nonlinear model (2) with $\delta_s = 2$. The resulting adaptive controller, applicable under Lipschitzian conditions, incorporates observers of minimum dimension, independently of the dimension of the unknown parameter vector. In particular, the applicability of the proposed controller is guaranteed by the possibility of decomposing the friction force (2) into a linear part $F_L(v)$ and a nonlinear one $F_N(v)$ as $F(v) = F_L(v) + F_N(v)$, with

$$\begin{aligned} F_L(v) &= F_c \mathrm{sgn}(v) + \beta v && \text{(14a)} \\ F_N(v) &= (F_s - F_c) \mathrm{e}^{-\left(v^2/v_s^2\right)} \mathrm{sgn}(v) && \text{(14b)} \end{aligned}$$

where $F_N(v)$ can be rewritten as the product of two Lipschitzian functions in the parameter vector $\theta = [\theta_1 \ \theta_2]^T := \left[(F_s - F_c) \ \ 1/v_s^2 \right]^T$ as:

$$F_N(v) = g(v, \theta) \, h(v, \theta) \tag{15}$$

with $g(v, \theta) = [1 \ \ 0] \, \theta$ and $h(v, \theta) = \mathrm{e}^{-\left(v^2 \theta_2\right)} \mathrm{sgn}(v)$. Details about the adaptive controller can be found directly in [37], together with some experimental results for a two-dof planar manipulator.

In [38], [39], a decomposition-based friction compensation method is proposed, designing a separate compensator for each type of friction, utilizing different control techniques. Friction is described by the static model (12) with the addition of the viscous component and a further term that takes into account the position dependency of friction and other modelling errors. The Stribeck term is linearized at the nominal parameter values $\overline{F}_s^{'}$ and $\overline{F}_\tau$, so that all the friction parameters appear linearly in the linearized model, and adaptive control and robust control techniques can be easily applied. In particular, while the nominal friction is compensated by feedforward (on the basis of off-line estimates), an adaptive compensator is designed to compensate for parametric unmodelled friction with unknown but constant parameters, and a robust compensator is used to deal with friction model parameter variations, as well as non-parametric unmodelled friction.

Finally, it is worth to be noted how an adaptive friction compensation law has been successfully utilized in [13] in some hybrid force/velocity contour tracking tests, by using a polynomial friction model of properly high degree: the advantage of this solution is given by the fact that the friction parameters, i.e. the polynomial coefficients, enter linearly in the model, and hence update laws can be easily designed for their on-line adaptation.

D. *Non model-based compensation schemes and neural-fuzzy techniques*

The friction compensation action of non-model based control schemes is generally accomplished by proper choices of the gains of standard control algorithms. Since the beginning of the 90's, the properness of using integral control actions has been repetitively discussed, since although a conventional integral term eliminates the steady-state positioning error, it could produce limit cycles about a set-point for stick-slip systems. Varying integral actions must then be applied. An interesting experimental comparison of different control schemes, including also some classic integral-based techniques (a rate-varying integral algorithm and a reset-off integral law) can be found in [40], showing that satisfying results can be actually obtained in practice. More recently, the importance of the integral action has been put in evidence in [11], where the precision-limit positioning (PLP) is experimentally obtained for a direct-drive DC motor by using different PI or PID controllers (with different control gains) in the stick and slip phases; in particular a large integral action, that could not be applied in the slip phase (otherwise the system would become unstable) is used in the final positioning stage to achieve PLP.

An integral-based solution has been developed also in [41] for a parallel manipulator with unknown Coulomb friction; the proposed control law is composed of a position PD controller and a reversed position error integral controller, given by a nonlinear control input u_{Irev} defined as:

$$u_{Irev} = K_I \ \mathrm{sgn}(v) \int_0^t e(\tau) \ \mathrm{sgn}(v(\tau)) d\tau \qquad (16)$$

where K_I is the integral control gain and $e(t)$ is the tracking error. A correct compensation action is based on the fact that the sign of the integrated output is reversed each time the sign of the velocity v changes, and the integral controller consequently restarts.

A nonlinear proportional-integral-derivative (NPID) control has been designed and experimentally implemented in [42], showing the possibility to compensate friction effects and improve tracking accuracy by applying a state feedback NPID control law with time-varying state feedback gains, properly switching between higher and lower values according to the system conditions.

All these solutions, which are not based on a particular friction model, obviously lead to compensation actions that include not only friction, but also all the other disturbances acting on the system. This consideration is at the basis of the control scheme proposed in [43], where the problem of friction compensation is solved by means of a nonlinear disturbance observer for robotic manipulators, where friction is considered as a disturbance on the control torque, similar to other unknown torques, without using a specific friction model. Even if the stability properties of the controlled system are analytically proven under the assumption that the disturbance term varies slowly with respect to the observer dynamics (thus assuming it as practically constant), the reported simulated and experimental results show that also some fast varying disturbances can be tracked by the observer.

Finally, various control schemes (that are not discussed in detail in this paper) have been proposed in literature to compensate for friction effects by using genetic, neural and fuzzy techniques. Such methods are used to ensure stability properties of the controlled system by means of a fuzzy self-tuning of the controller gains as in [19], or to generate the shapes of the applied torque pulses, to achieve a high positioning accuracy for stick-slip systems as in [21], or again to approximate the unknown dynamics by fuzzy logic systems, thus obtaining a signal to compensate for both structured and unstructured uncertainties as in [20].

IV. Some experimental results and conclusions

In our opinion, no method among the reviewed ones can be considered as intrinsically superior to the others. The choice of a particular friction compensation technique must be made taking into account the characteristics of the considered robotic systems and of its hardware/software control architecture, since practical implementation issues, as sensors accuracy, actuators characteristics, and real-time constraints, can discourage the application of a certain method, or deeply influence the achieved results. Hence, an experimental comparison of the main friction techniques in the case of a particular robot would be interesting, but it would not lead anyway to a final judgement.

Some experimental results, obtained for a two-dof planar manipulator with standard resolution resolvers, are reported to underline two further important issues, which are not strictly related to a particular friction compensation method: (*i*) model-based techniques are suitable only if there is a good correspondence between the assumed and the actual friction model for the specific task to be performed; (*ii*) significantly different performances can be achieved by a certain friction compensation approach when different tasks or trajectories are executed; in these cases, an average performance method might preferable to those providing an high accuracy only in specific conditions.

The considered two revolute-joint planar manipulator, moving in an horizontal plane, is actuated by direct-drive (i.e. without reduction gears) brushless motors, and it is equipped by resolvers, having a standard resolution of $8 \cdot 10^{-5}$ rad (more details can be found in [44]). Its dynamic model can be expressed as:

$$D_d(q, \dot{q}, \ddot{q})\theta_d + \tau_f(\dot{q}) = \tau_c \tag{17}$$

where $q, \dot{q}$, and $\ddot{q}$ are the vectors of joint angles, angular velocities and angular accelerations, respectively, $\tau_f(\dot{q})$ is the friction torque vector, τ_c is the command torque vector, while the contributions of the inertial, centrifugal and Coriolis torques are regrouped in the term $D_d(q, \dot{q}, \ddot{q})\theta_d$ that is linear with respect to the vector of the identifiable inertial parameters θ_d [45].

The following inverse dynamics control law has been applied:

$$\tau_c = D_d(q, \dot{q}, \ddot{q}_r - v_c)\hat{\theta}_d + \hat{\tau}_f(\dot{q}), \tag{18}$$

where $\ddot{q}_r$ is the reference acceleration vector, $\hat{\theta}_d$ and $\hat{\tau}_f(\dot{q})$ are estimates of the inertial parameter vector θ_d and of the friction torques $\tau_f(\dot{q})$, respectively. The term v_c represents the command vector of the outer loop, obtained by a standard PID control algorithm, identical in all the tests. In the first implemented solution (denoted as C.I), the available nominal values of the inertial parameters have been used to define $\hat{\theta}_d$, while $\hat{\tau}_f(\dot{q})$ has been defined according to the static, steady-state LuGre friction model (5), with:

$$g_i(\dot{q}_i) = \left(\alpha_{0i} + \alpha_{1i}\, \mathrm{e}^{-\left(\frac{\dot{q}_i}{\omega_{s,i}}\right)^2}\right) \mathrm{sgn}(\dot{q}_i) \tag{19}$$

$$f(\dot{q}_i) = \alpha_{2i}\dot{q}_i \tag{20}$$

The friction parameters have been identified off-line, moving the joints at constant velocity values by means of a PD joint-independent control law; a new improved prototyping architecture has allowed a better fitting of these simpler functions $g_i(\dot{q}_i)$ and $f(\dot{q}_i)$ than that obtained in previous papers [16] and [46] with more complex expressions. In the second implemented solution (denoted as C.II), friction on each joint has been described by a third-order polynomial function, whose coefficients have been identified off-line together with the robot inertial parameters by a Least-Squares algorithm, collecting data during the execution of an "optimal" trajectory, according to the method developed in [45] (more details can be found in [16]). The differences between the so-identified inertial parameters and their nominal values are extremely small [16], so that the different control performances of C.I and C.II are mainly due to the different friction compensation solutions.

The same circular trajectory has been tracked twice for each of the two control solutions: in the first case (LV) with *low* joint velocities (less than 2 rad/s), and in the second case (HV) with *high* joint velocities (values greater than 4 rad/s). As shown in Figure 1, while in the LV test the LuGre static compensation gives the best tracking accuracy, in the HV test the best results are achieved by the polynomial friction function. The accu-

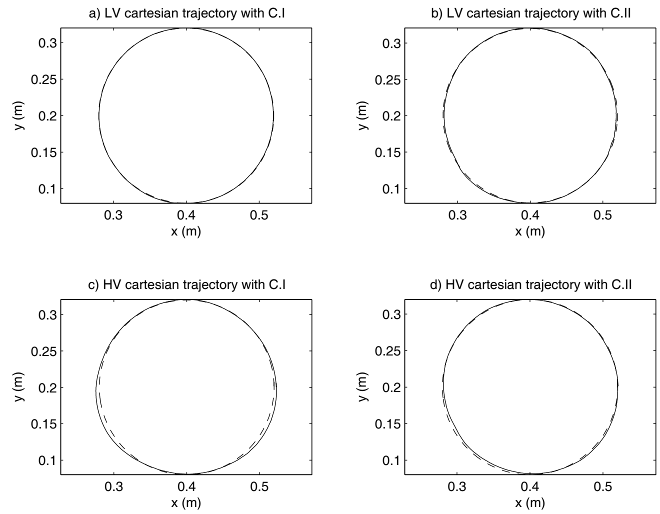

Fig. 1. Comparison of the experimental results: reference (dashed line) and tracked (solid line) trajectory with C.I and C.II in the LV and HV tests.

rate friction model used in C.I has led to the best results at the low velocities range of the LV test, i.e. where friction is the main disturbance acting on the robot, and it is well described by such a model. On the contrary, in the high-velocity range of the HV test, the polynomial model seems to better capture the behavior of the $\hat{\tau}_f(\dot{q})$ term, considered as friction but that can actually contain also other dynamic disturbances. From the control point of view then, the polynomial friction model could be preferable, since the quality of the achieved tracking results are comparable in the two tests: the rms value of the Cartesian tracking error with the polynomial C.II solution is 1.8 mm in the LV test and 2.9 mm in the HV one (while the maximum error values are 3.2 mm and 4.9 mm, respectively), whereas with the C.I solution the rms value passes from 0.5 mm to 3.5 mm, and the maximum one from 1.0 mm to 5.9 mm. Similar results have been obtained also by a *feedforward* compensation, i.e., when the reference joint position and velocity values have been used, instead of the current ones, in the inner loop of the inverse dynamics control scheme and in the friction compensation term, with a significantly reduced on-line computational burden.

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