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# LMI formulations for designing controllers according to time response and stability margin constraints

Mohamed Abbas-Turki, Gilles Duc and Benoît Clement

*Abstract*— **Designing a controller with respect to time and frequency-domain objectives remains a difficult problem, although both kinds are generally present in the manufacturer specifications. In general, the temporal objectives are replaced by frequency dependent ones, which in major cases do not fit the actual expectations. In this paper, convex mathematical translations of both kinds of objectives are proposed using Linear Matrix Inequalities (LMI). The application of Youla parameterization allows to restore the linearity in the compensator parameters, but a huge state space representation of the system is induced. Thus the Cutting Plane Algorithm (CPA) is efficiently used to overcome the problem of having a huge number of added variables, which often occurs in Semi-Definite Programming (SDP) particulary when used in conjunction with the Youla parameterization.**

## I. INTRODUCTION

The commun way to solve a multiobjective control problem is to reformulate the design specifications into more convenient forms such as $\mathcal{H}_\infty$ or $\mathcal{H}_2$ constraints. Unfortunately most of the manufacturer specifications cannot be exactly translated into such formulations, so that this approach leads either to more restrictive constraints or to approximate results. For instance in [1], a LMI specification is proposed to translate a template on a time response, which derives a hard constraint. The time domain specifications can be indirectly handled by $\mathcal{H}_2$ constraints or frequency shaping, but the overshoot and the settling time remain difficult to be adjusted.

The purpose of this work is to design a controller according to time-domain specifications together with gain and phase margins requirements. The case of $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms constraints has been presented in [2], [3]. By using the Youla parameterization, which defines a convex set describing all stabilizing controllers [4], all these specifications are expressed as matrix inequalities which are linear in the decision variables (LMI), provided a particular base is chosen for the Youla parameter. The obtained problem is therefore convex, so that it can be solved using convex optimization techniques. Furthermore, it allows to conclude on the feasibility or non-feasibility of the control problem, provided the basis chosen for the Youla parameter allows to cover appropriately the set of stable transfer functions.

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M. Abbas-Turki G. Duc are with the Service Automatique, École Supérieure d'Electricité, 3 rue Joliot-Curie, 91192 Gif-sur-Yvette cedex, France; first name.last name@supelec.fr

B. Clement is with the Direction des Lanceurs, CNES, Rond Point de l'Espace, 91023 Every cedex, France; benoit.clement@cnes.fr

As a disadvantage, using the Youla parameterization induces a huge state-space representation. The most commonly used technique for solving LMI problems is the semi-definite programming (SDP): however the frequency-dependent contraints generally require introducing a symmetric matrix of the same order as the state-space matrix. Thus this technique should be avoided when the Youla parameterization is used.

In order to avoid the additional variables, Kao [5] presents an alternative based on the eigenvalues of some Hamiltonian matrix, and the application of a Cutting Plane Algorithm (CPA) instead of SDP. Although this method is more sensitive to numerical conditioning, it is less affected by the order of the plant.

In this paper, the efficiency of using CPA in this context will be shown: the time-domain specifications will be directly expressed as LMI constraints, without any restriction nor approximation. The stability margins requirements will be considered as real uncertainties. Contrary to the approach proposed in [6], no decomposition of the Youla parameter is needed and no additional variable has to be introduced. On the other hand, the proposed condition is only sufficient but it has been verified that it is not too conservative in most practical cases.

The paper is organized as follows: section 2 contains a brief presentation of the Youla parameterization; section 3 introduces the CPA. The main contributions appear in sections 4 and 5, where a time-domain template and stability margins constraints are respectively formulated on a suitable form to be used by the CPA. An illustrative example is finally presented in section 5.

## II. YOULA PARAMETERIZATION

### A. Parameterization of the set of stabilizing controllers

The Youla parameterization allows describing all stabilizing controllers by only one stable transfer $Q$, called the Youla parameter [4]. Consider a continuous or discrete-time plant $G$, with $z$ the output to be controlled despite disturbance $w$, using control input $u$ and measurement $y$. A state space realization of $G$ can be written as:

$$G = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} : \quad \begin{array}{c} \\ \\ z \\ y \end{array} \begin{array}{c} \begin{array}{ccc} \phantom{A} & w & u \end{array} \\ \left( \begin{array}{c|cc} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{array} \right) \end{array} \qquad (1)$$

All stabilizing controllers are described by the Redheffer product $K = J * Q$ (see the interconnection structure of figure 1), where the Youla parameter $Q$ is any stable transfer function. System $J$ depends both on coprime factorizations of $G_{22}$ (the transfer between $u$ and $y$) and an initial compensator $K_0$:

$$J = \begin{pmatrix} K_0 & \tilde{V}_0^{-1} \\ V_0^{-1} & -V_0^{-1}N \end{pmatrix}$$

with $G_{22} = NM^{-1} = \tilde{M}^{-1}\tilde{N}$, $K_0 = U_0V_0^{-1} = \tilde{V}_0^{-1}\tilde{U}_0$.

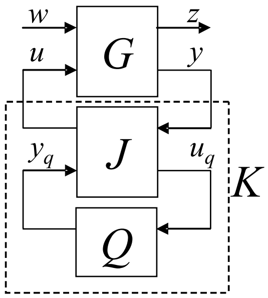


Fig. 1. Closed-loop structure using Youla parameterization

The main result of such an approach is that the Redheffer product $G * J$ (figure 1) exhibits a transfer identically equal to 0 between $u_q$ and $y_q$. Hence the closed-loop transfer $G_{zw}$ depends linearly on $Q$:

$$\begin{aligned} G_{zw} &= \left(G_{11} + G_{12}U_0\tilde{M}G_{21}\right) + \left(G_{12}M\right)Q\left(\tilde{M}G_{21}\right) \\ &= H_{11} + H_{12}QH_{21} \end{aligned} \tag{2}$$

From state space realizations of $H_{11}$, $H_{12}$, $H_{21}$ and $Q$, a non minimal realization of $G_{zw}$ is therefore as follows:

$$G_{zw} = \left(\begin{array}{c|c} A_{zw} & B_{zw} \\ \hline C_{zw} & D_{zw} \end{array}\right) =$$

$$\left(\begin{array}{cccc|c} A_{11} & 0 & 0 & 0 & B_{11} \\ 0 & A_{21} & 0 & 0 & B_{21} \\ 0 & B_QC_{21} & A_Q & 0 & B_QD_{21} \\ 0 & B_{12}D_QC_{21} & B_{12}C_Q & A_{12} & B_{12}D_QD_{21} \\ \hline C_{11} & D_{12}D_QC_{21} & D_{12}C_Q & C_{12} & D_{11} + D_{12}D_QD_{21} \end{array}\right) \tag{3}$$

Unfortunately, the state-space matrices of the Youla parameter enter in matrices $A_{zw}$ and $B_{zw}$: expressing most constraints (using e.g. $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms,...) will generally provide matrix inequalities which are bilinear in the decision variables. However the projection of the Youla parameter on a chosen basis allows restoring the linearity: this will be shown in the second part of this section.

### B. *Finite dimensional approximation of the Youla parameter*

In all the literature concerning the Youla parameterization and convex optimization problems, it is a usual way to approximate the Youla parameter by a truncated projection. Such an approximation can be written:

$$Q(\upsilon) = \sum_{j=1}^{m \times p} \sum_{k=0}^{n_q} q_{k,j} Q_{k,j}(\upsilon) \tag{4}$$

where $m$ and $p$ are the numbers of columns of $B_2$ and the number of lines of $C_2$ respectively, $\upsilon$ is either the discrete-time or the Laplace operator, $\{Q_{k,j}\}$ is the chosen basis of stable transfers and $q_{k,j}$ are the design parameters. Using (4), matrices $A_Q$ and $B_Q$ are fixed, so that all the design parameters enter in $C_Q$ and $D_Q$ only.

As it can be noticed, the order of the Youla parameter rises significantly for systems with large numbers of inputs and outputs. Furthermore the representation (3) of the closed-loop plant is a non minimal one. For these reasons, one has to search for a synthesis method which is the less sensitive to the state-space order.

It remains now to put the design variables only in $C_{zw}$ and $D_{zw}$, which is not the case in (3), for guaranteing in most cases the linearity of the matrix inequalities constraints with respect to the design parameters. A suitable technique has been proposed by [7], which consists in increasing the representation of $G_{zw}$ using the Kronecker product. This representation leads to the state space representation of $G_{zw}$ having a high order (that is $n + 2\,n\,m\,p + 2\,m\,p\,n_Q\,m_i$, where $n$, $n_Q$ and $m_i$ are respectively the dimensions of matrices $A$, $A_Q$ and the number of lines of $C_1$). This means one's again that for avoiding numerical infeasibility, all methods based on introducing a matrix having the same order as $A_{zw}$ should be avoided.

## III. The Cutting Plane Algorithm

This section presents a variant of the Cutting Plane Algorithm (CPA) presented in [5]. Only the case of a feasibility problem is presented.

The presentation of the method is divided into two parts: the first one gives the general principle of the algorithm. The second one brings some details on the operations happening at each step.

### A. *Algorithm*

Consider the following feasibility problem:

$$\text{Find } x \quad \text{subj to} \quad \mathcal{S}_x > 0 \tag{5}$$

where $x$ is the vector of decision variables, and $\mathcal{S}_x$ is a real symmetric matrix expressing a set of constraints on matrix form. The problem (5) can be reformulated into an equivalent eigenvalue maximization problem:

$$\sup_{x,y}\ y \quad \text{subj to} \quad \begin{cases} \mathcal{S}_x - yI > 0 \\ y < 1 \end{cases} \tag{6}$$

The problem (6) is feasible if $y > 0$. From (6) a concave function is defined:

$$q(x) := \sup\{y : \mathcal{S}_x - yI > 0, y < 1\} \tag{7}$$

Using $q(x)$, problem (7) can be replaced by the equivalent optimization problem:

$$y_{opt} = \sup_x q(x) \tag{8}$$

For solving problem (8), the method of Kelly [8] is commonly used. This method needs to compute the values of $q(x)$ and its sub-gradient. In [5], a technique has been presented by Kao, which avoids such a harsh calculation, by solving a Linear Programming Problem (LPP). The function

$q(x)$ is bounded iteratively by a set of hyperplanes, leading to a piecewise linear function $p_k(x)$:

$$q(x) \leq p_k(x) := \min_{1\leq i\leq k} \{a_i x - b_i\} \tag{9}$$

In the following, it is assumed that there exists a mechanism which checks the constraints and generates the hyperplanes (such a mechanism will be introduced in the next subsection). The algorithm begins with an initial value $y_l$ belonging to the feasible set. At iteration $k$ the following LPP is solved:

$$\max_{x_{\min}\leq x\leq x_{\max}} p_k(x) \tag{10}$$

with $x_{\min}$ and $x_{\max}$ defining some numerical limits of the components of vector $x$. Let $y^{(k)}$ be the solution of this problem. A linear interpolation involving a parameter $\alpha \in [0, 1]$ derives a new value of $y$:

$$\hat{y}^{(k)} = \alpha y^{(k)} + (1 - \alpha) y_l \tag{11}$$

If the set of constraints $\mathcal{S}_x - \hat{y}^{(k)} I > 0$ is verified (figure 2(a)), the value of $y_l$ is replaced by $\hat{y}^{(k)}$ else, new hyperplanes are added (figure 2(b)), so that a new LPP can be solved at iteration $k+1$. The principle of the CPA is very simple, but the main task is to verify the constraints and to generate the hyperplanes.

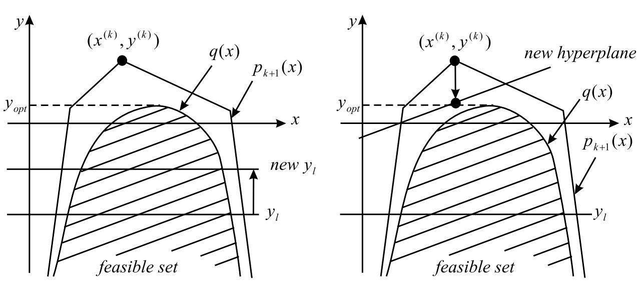


Fig. 2. The CPA in the scalar case

### *B. Mechanism for verifying the constraints and generating the hyperplanes*

The verification of the constraints and the generation of the hyperplanes are linked, so that there are considered in the same mechanism. Some general ideas are given here: the application to different constraints will be detailed in the next section.

Two types of constraints have to be considered: in the first case, the constraint is an explicit translation of the specification onto some matrix inequality, so that the verification is done by directly computing the eigenvalues of the corresponding symmetric matrix. A second case arises when for instance frequency dependent constraints are translated using some equivalent proposition as provided by the Kalman-Yakubovich-Popov (KYP) lemma [9]: such a lemma allows to replace an infinite number of frequency dependent constraints by a unique one, by introducing a Hamiltonian matrix $\mathbf{H}$ which is required to have no eigenvalue on the imaginary axis; if it has one, its value can be reported in the constraint as a frequency where it is not satisfied.

The generation of the hyperplanes is done using the eigenvectors associated to the negative eigenvalues of the matrix $\mathcal{S}_x - \hat{y}^{(k)} I$. For each negative eigenvalue $\lambda_i$, an hyperplane is generated from the associated eigenvector $v_i$, which verifies:

$$v_i^T (\mathcal{S}_x - \hat{y}^{(k)} I) v_i < 0 \tag{12}$$

Since $\mathcal{S}_x$ is affine in $x$, the quadratic product $v_i^T (\mathcal{S}_x) v_i$ has the form:

$$v_i^T (\mathcal{S}_x) v_i = a_i^T x + b_i \tag{13}$$

and an hyperplane corresponding to the new added constraint is described by:

$$a_i^T x + b_i - \left(v_i^T v_i\right) y > 0 \tag{14}$$

The next sections shows how different contraints can be translated into a suitable form for applying the CPA.

## IV. Time response template

To impose a particular template to a time response, most of the works resort to non convex optimization methods or try to translate the time domain constraints to the frequency domain. The first approach induces a huge calculation time, whereas in the second one, informations are lost and the constraint becomes harsh in most cases. In this section a time domain constraint is considered using a LMI formulation. Although this formulation is appropriate to discrete-time problems, it can also be extended to continuous-time ones, as will be explained all along this section.

Given a test input sequence, the aim of time response shaping of discrete time systems can be formulated as follows:

$$\begin{array}{ll} (z_i(nT) - \delta(0))^2 < \tau(0), & n = 0, ..., n_0 \\ (z_i(nT) - \delta(1))^2 < \tau(1), & n = n_0 + 1, ..., n_1 \\ \vdots & \vdots \\ (z_i(nT) - \delta(r))^2 < \tau(r), & n = n_{r-1} + 1, ..., n_r \end{array} \tag{15}$$

where $z_i$ is the $i^{th}$ output; $\delta(j), j = 0, ...r$ is the centre of the allowable interval; $\sqrt{\tau(j)}$ is the maximal tolerated deviation; $T$ is the sample time; $r$ is the number of constraints domains ; $n_r$ is the maximal value of time for which constraints are considered. Figure 3 shows an example of time response shaping for a unit step response, with $r = 3$.

For continuous-time systems $y_i$ is simply obtained by defining a particular sample-time $T$ according to the Shannon condition and computing the corresponding values of the time response.

Each set of contraints in (15) can be treated separately, so only one set is considered in the following. Consider the closed-loop discrete-time system $G_{zw}$ defined in section II. If input $w$ is given[1], the value of the output $z$ at each instant

[1]If $w$ represents an unknown disturbance, a worst case signal should be considered.

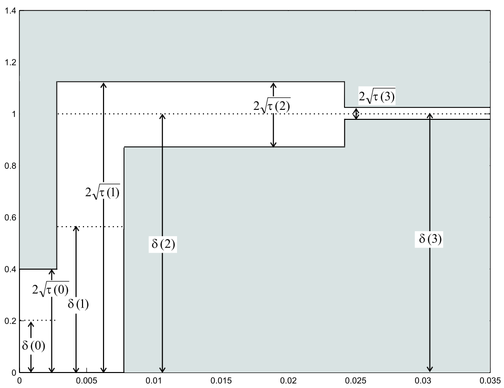


Fig. 3. Example of time response constraints

$n$ can be found using the algebraic formulation:

$$z(nT) = C_{zw}\left(\sum_{k=1}^{n} A_{zw}^{k-1} B_{zw} w_{n-k}\right) + D_{zw} w_n \tag{16}$$

where $A_{zw}$, $B_{zw}$, $C_{zw}$ and $D_{zw}$ are the state-space matrices of $G_{zw}$ and $w_{n-k}$ is the value of the input at time $n-k$; $z(nT)$ is affine on $C_{zw}$ and $D_{zw}$ (which contain the matrices $C_Q$, $D_Q$ of the Youla parameter we are looked for). Each constraint of (15) can be written:

$$(*)\left(C_{zw}\left(\sum_{k=1}^{n} A_{zw}^{k-1} B_{zw} w_{n-k}\right) + D_{zw} w_n - \delta(j)\right) < \tau(j) \tag{17}$$

(where (*) stands for the symmetric term). Inequality (17) is not affine in $C_{zw}$ and $D_{zw}$, but an equivalent LMI formulation is obtained by applying the Schur lemma:

$$\begin{pmatrix} 1 & * \\ \left(C_{zw}\left(\sum_{k=1}^{n} A_{zw}^{k-1} B_{zw} \hat{w}_{n-k}\right) + D_{zw}\hat{w}_n - \hat{\delta}(j)\right) & 1 \end{pmatrix} > 0 \tag{18}$$

with $\hat{w}_{n-k} = \dfrac{w_{n-k}}{\sqrt{\tau(j)}}$, $\hat{w}_n = \dfrac{w_n}{\sqrt{\tau(j)}}$ and $\hat{\delta}(j) = \dfrac{\delta(j)}{\sqrt{\tau(j)}}$.

For continuous systems the term $\sum_{k=1}^{n} A_{zw}^{k-1} B_{zw} \hat{w}_{n-k}$ is simply replaced by $\int_0^{nT} e^{A_{zw}(nT-t)} B_{zw} \hat{w}(t) dt$.

Constraint (18) is duplicated as much as necessary. As an example, for a step input, only constraints corresponding to the transient response and a small part of the permanent response have to be introduced, because the closed-loop plant is guaranteed to be stable.

The verification of the constraint is done directly by computing the eigenvalues of the matrix in (18). Note that since the constraint to be checked in the CPA is actually $\mathcal{S}_x - \hat{y}^{(k)} I > 0$, the first element in matrix (18) has to be replaced by $1 - \hat{y}^{(k)}$.

The new hyperplanes are generated by considering the eigenvectors associated to the negative eigenvalues of (18) (with again the first element in the matrix replaced by $1 - \hat{y}^{(k)}$). Only the worst overshoot for each value of $j$ is considered in order to reduce the number of new hyperplanes.

## V. Stability margins

In this section both gain and phase margins constraints for MISO or SIMO plants will be considered as LMI problems. Continuous-time plants will be considered, but the case of discrete-time ones can be equivalently handled by applying Tustin transforms.

A suitable LFT form (which will be defined below for each margin) enables to consider the margin as a scalar uncertainty $\delta \in [0, 1]$, whereas the nominal closed-loop plant $G_{zw}$ is looped by $-\delta$ (fig. 4): $G_{zw}$ being stable, the stability is guaranteed for all $\delta \in [0, 1]$ if and only if the Nyquist diagram of $G_{zw}$ does not cut the half line $(-\infty, -1]$ of the real axis.

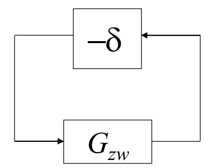


Fig. 4. The stability margin formulated as an uncertainty

To derive a convex formulation, the preceding constraint is substituted by a harsher one, where the Nyquist diagram of $G_{zw}$ must not go into the half-plane to the left from $-1$. This later constraint directly becomes a passivity condition by replacing $G_{zw}$ by $G_{zw} + 1$:

$$(G_{zw}(j\omega) + 1) + (G_{zw}^*(j\omega) + 1) > 0 \qquad \forall\omega \in [0, \infty) \tag{19}$$

According to the KYP lemma [9], two equivalent constraints are:

$$H(\omega) = \begin{pmatrix} G(j\omega)^* & I \end{pmatrix} \begin{pmatrix} 0 & * \\ C_{zw} & R \end{pmatrix} \begin{pmatrix} G(j\omega) \\ I \end{pmatrix} > 0 \tag{20}$$

$$\mathbf{H} = \begin{pmatrix} A_{zw} - B_{zw}R^{-1}C_{zw}^T & B_{zw}R^{-1}B_{zw}^T \\ -C_{zw}^T R^{-1} C_{zw} & -A^T + C_{zw}^T R^{-1} B_{zw}^T \end{pmatrix} > 0 \tag{21}$$

with $G(j\omega) = (j\omega - A_{zw})^{-1} B_{zw}$ and $R = D_{zw} + D_{zw}^T + 2$. The frequency-dependent constraint (20) is affine in $C_{zw}$ and $D_{zw}$, and thus in the matrices $C_Q$ and $D_Q$ we are looked for. The constraint being checked by computing the eigenvalues of the associated Hamiltonian matrix $\mathbf{H}$, if some of them belongs to the imaginary axis, they are reported in $H(\omega)$ which in that case is scalar. The corresponding hyperplane is therefore directly deduced (since in the scalar case no eigenvector has to be computed).

The rest of the section will formulate the gain and phase margins on the form given in figure 4.

### A. Gain margin

In order to put the gain margin constraint as shown in figure 4, one can consider either the Reduction Gain Margin (RGM), which guarantees the stability for gains less then one, or the Increasing Gain Margin (IMG), which concerns gains higher than one. In both cases, the closed-loop plant of figure 4 can be represented as on figure 5, with $g = 1 - 10^{\frac{GM}{20}}$, where $GM$ equals either the RGM or IGM with dB unit. The corresponding state-space representation of $G_{zw}$ can be easily deduced [10].

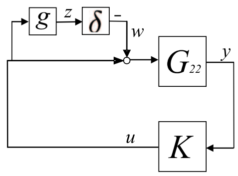


Fig. 5. Closed-loop structure for gain margin analysis

### B. Phase margin

The phase margin is considered by introducing $e^{j\theta}$ in the feedback loop and replacing this perturbation by a rational function also describing the unit circle:

$$e^{j\theta} = \frac{1 + j\hat{\theta}}{1 - j\hat{\theta}} \tag{22}$$

Note that for $\theta \in [0, \theta_e]$, $\hat{\theta}$ is real and belongs to $\left[0, \frac{e^{j\theta_e}-1}{j(e^{j\theta_e}+1)}\right]$. The open-loop plant of figure 4 can be represented as on figure 6) [10], with $\mathcal{N} = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$. Elementary manipulations give again the state-space representation of $G_{zw}$ [10].

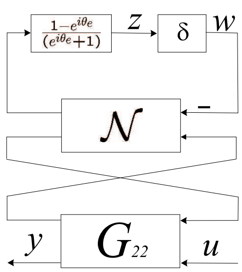


Fig. 6. Open-loop structure for phase margin analysis

## VI. Illustrative example

Let consider an overhead travelling crane system (figure 7). The plant is modelled by a SIMO system where the input is the tension $e$ driving the DC motor and the outputs are the position $x$ of the crane and the deviation from the vertical line $\phi$ of the pendulum. The order of the state space representation is 5 including three rigid modes and one resonance mode. The time-constant of the DC motor is neglected in the synthesis model, which contains only two rigid modes. The data are given in Table I.

The challenge is to move the crane from 0 to 0.4 m in only 1.2 s with no overshoot, the position remaining above 98% of this value after this time. The control value must not overtake $\pm 10$ V, and the oscillation of the pendulum must not exceed $\pm 0.25$ rad. The gain and phase margins of the system should be respectively more than 10 dB and $35°$.

The initial compensator is taken as a stabilizing static one:

$$K_{init} = \begin{pmatrix} -1 & 1 \end{pmatrix} \tag{23}$$

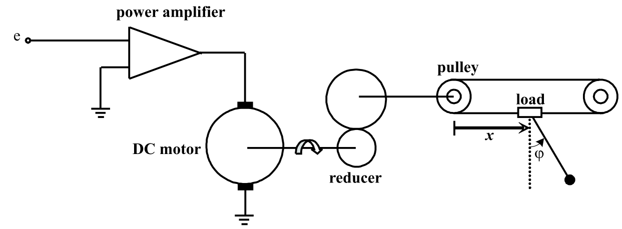


Fig. 7. Overhead travelling crane system

TABLE I
NUMERICAL VALUES

| Parameters | Values |
|---|---|
| Amplifier gain | 1 |
| Rotor inductance | 0.2 mH |
| Total resistance | 2.74 ohm |
| Torque constant | 16.2 mNm/A |
| Total inertia on the motor axis | 3.06 $10^{-6}$ kgm$^2$ |
| Coefficient of friction | 3.2 $10^{-5}$ Nms |
| Reduction ratio | 17 |
| Pulley radius | 22 mm |
| Bar length | 269 mm |
| Viscous damping coefficient | 0.26 m/s |

To describe the Youla parameter, the following orthogonal basis [11] is chosen:

$$Q_i(s) = \frac{\sqrt{2Re(a_i)}}{s + a_i} \prod_{k=1}^{i-1} \frac{s - \bar{a}_k}{s + a_k} \tag{24}$$

The poles of the Youla parameter are therefore $-a_i$. They have to be chosen according to the dynamics imposed to the response, and to make sure that the Shannon condition is verified when choosing the sample-time $T$ of the time response. To this end, we choose the $a_i$ as random numbers distributed between 0 and 30, whereas $T = 0.005$ s.

With these dynamics, a 10-th order of the Youla parameter is sufficient to bring the output $x$ into the template (figure 8) while satisfying the control and oscillation limitations (figure 9 and 10). The gain and the phase margins are equal to 10.4 dB and $40.5°$ as shown in figure 11.

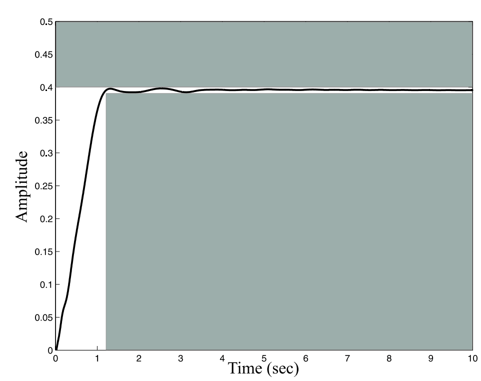


Fig. 8. Output $x$ response

The resulting controller has 14 state variables. Although it is not the objective of this work, it can be mentioned that reducing the order of the controller using the Hankel singular

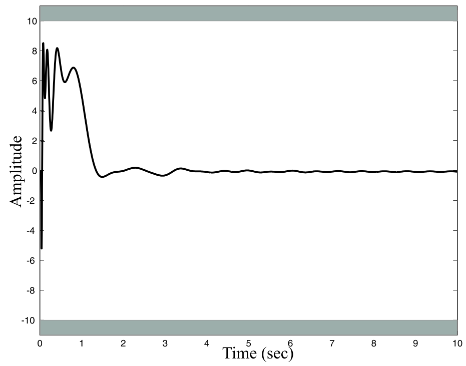


Fig. 9. Input $e$ response

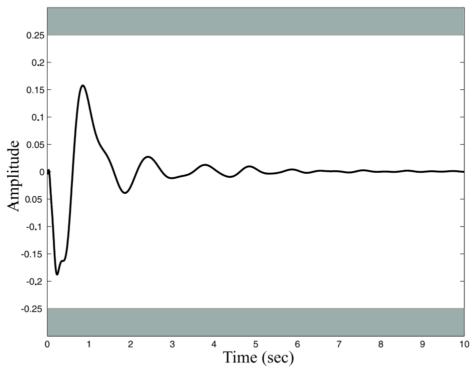


Fig. 10. Output $\phi$ response

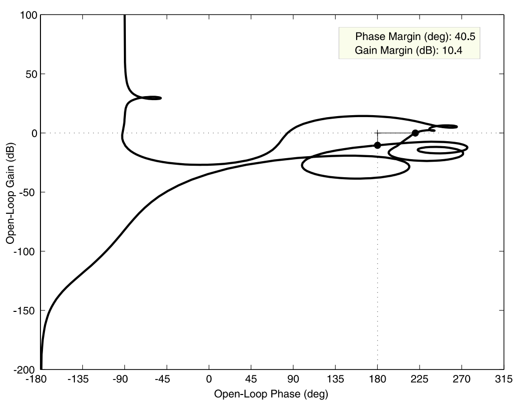


Fig. 11. Open-loop Black diagram

values truncation leads to a controller with 6 state variables, which has an acceptable time response for $x$ (figure 12), while $\phi$ and $e$ also remain in the template. The gain and phase margins are now equal to 11 dB and $58.1°$.

## VII. CONCLUSION

Designing a controller according to time-domain specifications and stability margins requirements can be done using the Youla parameterization: the particular LMI reformulations of the constraints brought in this paper allow to preserve the convexity of the problem.

The application of the CPA leads to prevent the introduction of additional decision variables, which implies that a high order of the Youla parameter can be considered without numerical difficulties. So the feasibility of the problem can be

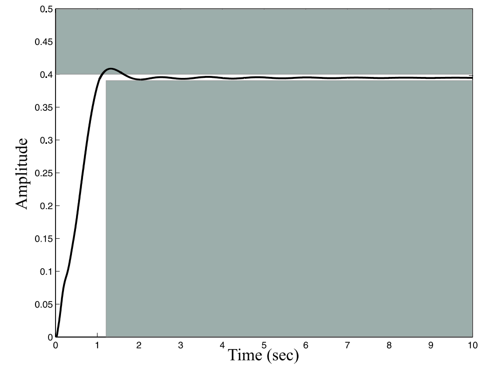


Fig. 12. Output $x$ response for reduced order controller

easily checked by increasing gradually the order of the Youla parameter to be determined. The simplicity of using the CPA makes it attractive, although some numerical improvements can be a subject of forthcoming works.

The numerical efficiency of the proposed developments has been shown by considering an example where a template on a time response has been satisfied, while guaranteeing required stability margins.

The stability margins constraints have been considered for MISO or SIMO plants, the extension to the MIMO case being under investigation. Note also that $\mathcal{H}_\infty$ and $\mathcal{H}_2$ norms constraints can be added to the specifications: the convenient LMI reformulations to be used are given in [2], [3].

Finally, developing a suitable reduction method to approximate the Youla parameter by a rational transfer function while still satisfying the constraints will be the subject of forthcoming studies.

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