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# Improved Cascade Control Structure and Controller Design

I. Kaya, N. Tan and D. P. Atherton

*Abstract*— **In conventional single feedback control, the corrective action for disturbances does not begin until the controlled variable deviates from the set point. In this case, a cascade control strategy can be used to improve the performance of a control system particularly in the presence of disturbances. In this paper, a new cascade control structure and controller design based on Standard forms are suggested to improve the performance of cascade control. Examples are given to illustrate the use of the proposed method and its superiority over some existing design methods.**

## I. Introduction

Cascade control (CC), which was first introduced many years ago by Franks and Workey [1], is one of the strategies that can be used to improve the system performance particularly in the presence of disturbances. In conventional single feedback control, the corrective action for disturbances does not begin until the controlled variable deviates from the set point. A secondary measurement point and a secondary controller, $G_{c2}$, in cascade to the main controller, $G_{c1}$, as shown in Fig. 1, can be used to improve the response of the system to load changes, $D_2$.

Recent contributions on the tuning of PID controllers in cascade loops include [3]-[5]. In this paper, an improved cascade control structure, where the inner loop incorporates Internal Model Controller (IMC) principles [6]. and the outer loop the Smith predictor scheme, is proposed. It is assumed that the inner loop has a first order plus dead time (FOPDT) process transfer function and the outer loop a FOPDT or a second order plus dead time (SOPDT) plant transfer function.

A cascade control strategy can be used to achieve better disturbance rejections. However, if a long time delay exists in the outer loop the cascade control may not give satisfactory closed loop responses for set point changes. In this case, a Smith predictor scheme can be used for a satisfactory set point response. The proposed improved cascade control structure brings together both the best merits of the cascade control and Smith predictor scheme. Furthermore, a PI-PD structure, which is proved to give better closed loop performances for process transfer functions with large time constants, complex poles, unstable poles or an integrator [7]-[9], is used in the outer loop to improve the performance of the system even better. The outer loop PI-PD controllers' parameters are identified by the use of standard forms, which is a simple algebraic approach to controller design. Another advantage of the standard forms is that one can predict how good will be the performance of closed loop system. The inner loop controller is designed based on IMC principles.

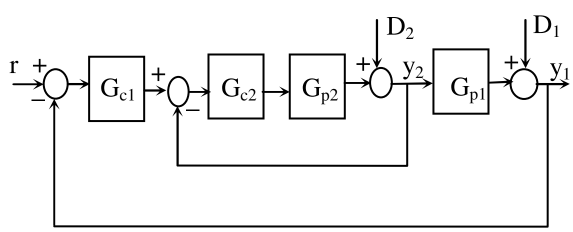


Fig. 1: Cascade Control System

## II. Standard Forms

The use of integral performance indices for control system design is well known. Many text books, such as [10], include short sections devoted to the procedure. For linear systems, the ISE can be evaluated efficiently on digital computers using the s-domain approach with Åström's recursive algorithm [11]. Thus for

$$J_0 = \int_0^\infty e^2(t)dt \tag{1}$$

the s-domain solution is given by

$$J_0 = \frac{1}{2\pi j}\int_0^\infty E(s)E(-s)ds \tag{2}$$

where $E(s)=B(s)/A(s)$, and $A(s)$ and $B(s)$ are polynomials with real coefficients, given by

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$$A(s) = a_0 s^m + a_1 s^{m-1} + ... + a_{m-1} s + a_m$$

$$B(s) = b_1 s^{m-1} + ... + b_{m-1} s + b_m$$

Criteria of the form $J_n = \int_0^\infty [t^n e(t)]^2 \, dt$ can also be evaluated using this approach, since $L[tf(t)]=(-d/ds)F(s)$, where $L$ denotes the Laplace transform and $L[f(t)] = F(s)$. Minimizing a control system using $J_0$, that is the ISE criterion, is well known to result in a response with relatively high overshoot for a step change. However, it is possible to decrease the overshoot by using a higher value of $n$ and responses for $n=1$, that is the ISTE criterion. Therefore, in this paper results for the ISTE criterion only is given.

Another approach to optimization which has been little discussed for many years is the direct synthesis approach where the closed loop transfer function is synthesized to a standard form. Using this approach, it is possible to obtain the optimal parameters of a closed loop transfer function, which will provide a minimum value of the ISE. Tables of such all pole transfer functions were given many years ago [12] but are of little use in design, because even with an all pole plant transfer function the addition of a typical controller produces a closed loop transfer function with a zero. Results with a single zero were also given so that the feedback loop would follow a ramp input with zero steady-state error but these expressions are not appropriate for step response design. For a closed loop transfer function with one zero it is easy to present results for these optimum transfer functions as the position of the zero varies [7]. Briefly, here, assuming a plant transfer function with no zero and a controller with a zero then a closed loop transfer function, $T_{1j}$, of the form

$$T_{1j} = \frac{c_1 s + 1}{s^j + d_{j-1} s^{j-1} + ... + d_1 s + 1} \qquad (3)$$

is obtained, where the subscript '1' in $T_{1j}$ indicates a zero in the numerator of the standard form and the subscript 'j' indicates the order of the denominator. Also, for a unit step set point, the error is obtained as

$$E_{1j} = \frac{s^{j-1} + d_{j-1} s^{j-2} + ... + (d_1 - c_1)}{s^j + d_{j-1} s^{j-1} + ... + d_1 s + 1} \qquad (4)$$

Minimizing $E_{1j}$ for the ISTE, the optimum values of the $d$'s as functions of $c_1$ are shown in Fig. 4 for $T_{13}(s)$ and in Fig. 5 for $T_{14}(s)$. Fig. 6 shows how $J_1$ (the minimum value for the ISTE criterion) varies as $c_1$ increases for both $T_{13}(s)$ and $T_{14}(s)$. The figure illustrates that as $c_1$ increases the step response of the closed loop improves, as the slope of the curves in Fig. 6 has reduced considerably. However, it is also seen from the figure that any further increase in $c_1$ above the value of 4 or 5 has a negligible improvement in the response. Also, the step responses for the $J_1$ criterion for a few different $c_1$ values are shown in Fig. 7 for $T_{14}(s)$. It is seen that as $c_1$ increases the step responses are faster. It should be noted that a similar result can be obtained for $T_{13}(s)$ as well.

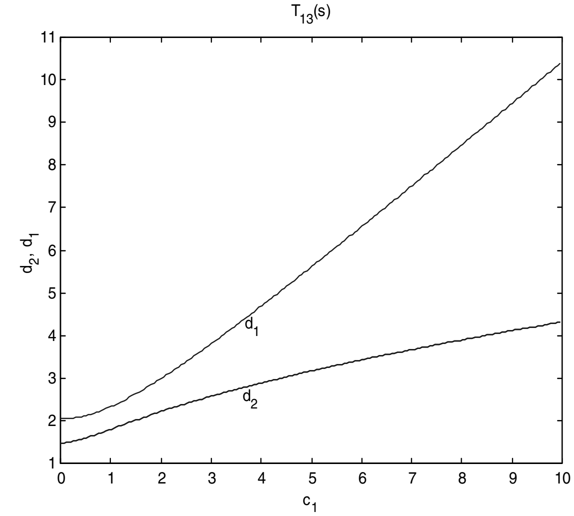


Fig. 4: Optimum values of d1 and d2 for varying c1 values.

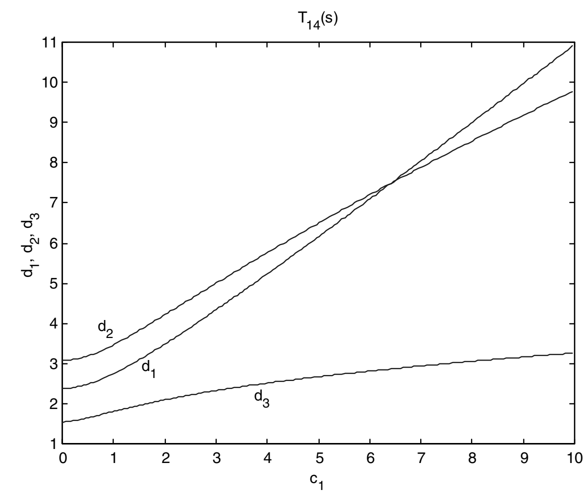


Fig. 5: Optimum values of $d_1$, $d_2$ and $d_3$ for varying $c_1$ values.

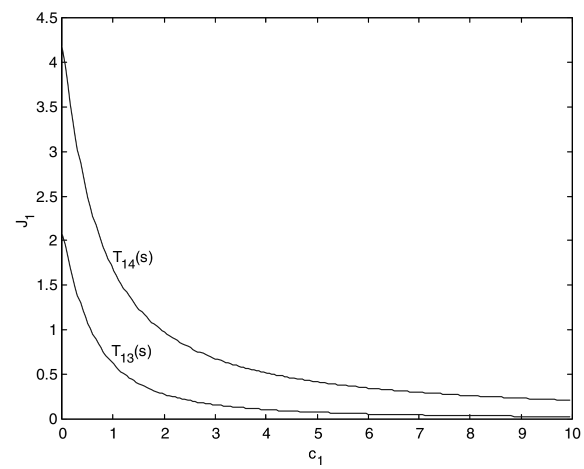


Fig. 6: Optimum values of $J_1$ for varying $c_1$ values and standard forms

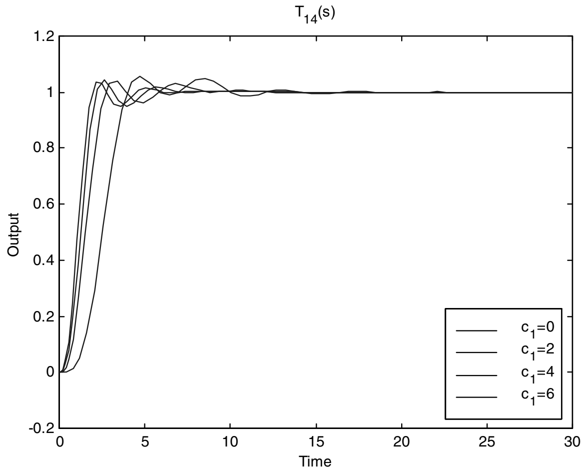


Fig. 7: Step responses for $T_{14}(s)$ and $J_1$ criterion

## III. The New Cascade Control Structure and Design Method

The proposed cascade control structure is shown in Fig. 8. $G_{c2}$ is used for stabilization of the inner loop while $G_{c1}$ and $G_{c3}$ are used for the outer loop stabilization. $G_{p2m}$ and $G_{pm}$ are the model transfer functions of the inner and outer loops respectively. Assuming that the plant transfer functions are known, then two loops can be tuned simultaneously. In next subsections, tuning rules for both loops are derived.

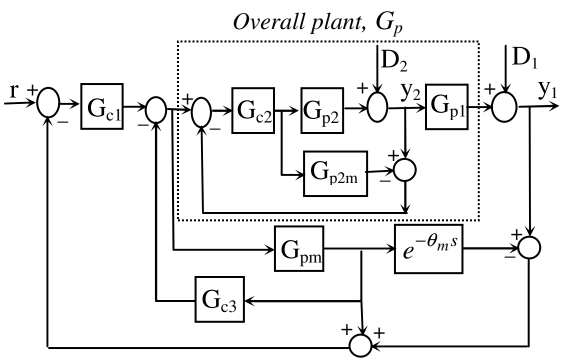


Fig. 8: Improved Cascade Control Structure

### 3.1 Designing inner loop controller ($G_{c2}$):

As stated in the introduction, the inner loop controller is designed based on IMC principles [6]. The details of design procedure are not given here, since; one can easily obtain them from the abovementioned references.The outer and inner loop plant transfer functions are assumed to be FOPDTs with

$$G_{p1}(s) = K_1 e^{-\theta_1 s} / (T_1 s + 1) \qquad (5)$$

$$G_{p1}(s) = K_2 e^{-\theta_2 s} / (T_2 s + 1) \qquad (6)$$

Note that in real case the plant can have higher order transfer functions. However, it is assumed that they can satisfactorily be approximated by the above FOPDT, or SOPDT used in case 2, model transfer functions. For this, any modeling approach existing in the literature, such as [13], can be used.

It can easily be shown that the inner loop controller, using IMC principles, is given by

$$G_{c2}(s) = (T_2 s + 1) / K_2 (\lambda s + 1) \qquad (7)$$

where, $\lambda$ is the only tuning parameter to be found. The faster the inner loop than the outer loop, the better the performance of a cascade control system in the sense of a faster response. That is, the smaller the values of $\lambda$ the better the performance of the cascade control system. Hence, as a rule of thumb $\lambda$ can be chosen equal to inner loop time delay. If a faster response is requested, $\lambda$ can be chosen as half the time delay of the inner loop, namely, $\lambda = \theta_2 / 2$, which is the value used throughout the paper.

### 3.2 Designing outer loop controllers ($G_{c1}$ and $G_{c3}$):

It is easy to show that the inner closed loop transfer function is now given by

$$G_{pi}(s) = e^{-\theta_2 s} / (\lambda s + 1) \qquad (8)$$

Then, the overall plant transfer function for the outer loop is

$$G_p(s) = G_{pi}(s) G_{p1}(s) = G_{pm}(s) e^{-\theta_m s} \qquad (9)$$

where, $G_{pm}(s)$ is the delay free part of the overall plant transfer function and $\theta_m = \theta_1 + \theta_2$.

Since the Smith predictor scheme is used in the outer loop, it can easily be shown that the closed loop transfer function between $y_1$ and r, assuming a perfect matching, that is $G_p = G_{pm}(s) e^{-\theta_m s}$, is given by

$$T(s) = \frac{G_{pm}(s) G_{c1}(s) e^{-\theta_m s}}{1 + G_{pm}(s)[G_{c1}(s) + G_{c3}(s)]} \qquad (10)$$

Eqn. (10) reveals that the parameters of the two controllers $G_{c1}(s)$ and $G_{c3}(s)$ can be determined using the delay free part of the overall plant transfer function.

The outer loop controllers, $G_{c1}(s)$ and $G_{c3}(s)$, are assumed to have the forms

$$G_{c1}(s) = K_c (1 + 1/T_i s) \qquad (11)$$

$$G_{c3}(s) = K_f + T_f s \qquad (12)$$

The next sections consider controller designs for two different cases.

Case 1 (Design for a FOPDT): It is assumed that the outer loop plant transfer function is stable and can be modeled by eqn. (5). Therefore, from eqns. (5) and (8), the overall plant transfer function for the outer loop is

$$G_p(s) = \frac{K_1 e^{-(\theta_1 + \theta_2)s}}{(\lambda s + 1)(T_1 s + 1)} \qquad (13)$$

Eqn. (13) can be rearranged as

$$G_p(s) = \frac{ke^{-(\theta_1+\theta_2)s}}{s^2+as+b} \quad (14)$$

where,

$$k = K_1 / T_1\lambda \quad (15a)$$

$$a = 1/T_1 + 1/\lambda \quad (15b)$$

$$b = 1/T_1\lambda \quad (15c)$$

Taking the delay free part of eqn. (14) equal to $G_{pm}(s)$ and using the delay free part eqn. (10) gives the closed loop transfer function

$$T_{13}(s) = \frac{kK_c(T_i s+1)}{T_i s^3 + (a+kT_f)T_i s^2 + \ldots + (b+kK_f+kK_c)T_i s + kK_c} \quad (16)$$

Normalization of eqn. (16), assuming

$$s_n = s(T_i / kK_c)^{1/3} = s/\alpha \quad (17)$$

which means the response of the system will be faster than the normalized response by a factor of $\alpha$, results in the standard closed loop transfer function

$$T_{13}(s_n) = \frac{c_1 s_n + 1}{s_n^3 + d_2 s_n^2 + d_1 s_n + 1} \quad (18)$$

where,

$$c_1 = \alpha T_i \quad (19a)$$

$$d_2 = (a+kT_f)/\alpha \quad (19b)$$

$$d_1 = (b+kK_f+kK_c)/\alpha^2 \quad (19c)$$

In principle $\alpha$ can be selected by the choice of $K_c$ and $c_1$ by the choice of $T_i$. Based on the value of $c_1$, the coefficients $d_2$ and $d_1$ can be found from Fig. 4 and then the values of $T_f$ and $K_f$ can be computed from eqns. (19b) and (19c) respectively. Note that for a selected $T_i$, choosing larger $K_c$ values results in larger $\alpha$ and $c_1$ values. This implies a faster closed loop system response. In practice, $K_c$ will be constrained, possibly to limit the initial value of the control effort, so that the choice of $K_c$ and $T_i$ may involve a trade off between the values chosen for $\alpha$ and $c_1$.

<u>Case 2(Design for a SOPDT):</u> In this case, it is assumed that the outer loop plant transfer function is stable and can be modeled by

$$G_{p1}(s) = \frac{K_1 e^{-\theta_1 s}}{(T_0 s+1)(T_1 s+1)} \quad (20)$$

Hence, the overall plant transfer function for the outer loop is

$$G_p(s) = \frac{K_1 e^{-(\theta_1+\theta_2)s}}{(\lambda s+1)(T_0 s+1)(T_1 s+1)} \quad (21)$$

Rearranging eqn. (21) gives

$$G_p(s) = \frac{ke^{-(\theta_1+\theta_2)s}}{s^3+as^2+bs+c} \quad (22)$$

where,

$$k = K_1 / T_0 T_1 \lambda \quad (23a)$$

$$a = 1/T_0 + 1/T_1 + 1/\lambda \quad (23b)$$

$$b = 1/T_0\lambda + 1/T_0 T_1 + 1/T_1\lambda \quad (23c)$$

$$c = 1/T_0 T_1 \lambda \quad (23d)$$

Taking the delay free part of eqn. (22) equal to $G_{pm}(s)$ and using the delay free part of eqn. (10) results in a closed loop transfer function

$$T_{14}(s) = \frac{kK_c(T_i s+1)}{T_i s^4 + T_i a s^3 + (b+kT_f)T_i s^2 + \ldots + (c+kK_f+kK_c)T_i s + kK_c} \quad (24)$$

Normalization of eqn. (24), assuming

$$s_n = s(T_i / kK_c)^{1/4} = s/\alpha \quad (25)$$

results in

$$T_{14}(s_n) = \frac{c_1 s_n + 1}{s_n^4 + d_3 s_n^3 + d_2 s_n^2 + d_1 s_n + 1} \quad (26)$$

where,

$$c_1 = \alpha T_i \quad (27a)$$

$$d_3 = a/\alpha \quad (27b)$$

$$d_2 = (b+kT_f)/\alpha^2 \quad (27c)$$

$$d_1 = (c+kK_f+kK_c)/\alpha^3 \quad (27d)$$

In this case, the time scale $\alpha$ and the four coefficients cannot be selected independently using the four controller parameters, namely, $K_c$, $T_i$, $K_f$ and $T_f$. Achieving independency would require feedback of an additional state but often a satisfactory rerponse, provided that $a$ has a resonable value, is possible. The compomise is between $\alpha$ $c_1$and $d_3$ as $d_2$ and $d_1$ can be chosen independently using $K_f$ and $T_f$. As in case 1, the larger values of $\alpha$ implies a faster closed loop system response for a fixed value of $T_i$. However, the choice of the value of $\alpha$ depends on the value of $a$ if a suitable value of $d_3$ is to be obtained. Therefore the larger the value of $a$ the faster the possible response satisfying the ISTE criterion which can be obtained. The procedure for calculating controller parameters can thus be summarised as: For a chosen value of $\alpha$, $d_3$ is determined from eqn. (27b). Once $d_3$ is calculated, $c_1$, $d_2$ and $d_1$ coefficients can be found from Fig. 5 for the ISTE standard form corresponding to the calculated value of $d_3$ to obtain an optimum overall closed loop performance. Note that $a$ must be positive, as seen from Fig. 5, in order to use standard forms in this case.

## IV. Simulation Examples

Two examples are given to illustrate the use of the proposed cascade control structure and design procedure. The first example assumes FOPDT plant transfer functions in both loops. The second example assumes a FOPDT plant transfer function in the inner loop and a SOPDT plant transfer function with poorly located poles in the outer loop.

***Example 1:*** Considered $G_{p1}(s) = e^{-10s}/(100s+1)$, $G_{p2}(s) = 2e^{-2s}/(20s+1)$ which was studied by Lee *et al.* [4]. Taking $\lambda = 1$, which is the half the inner loop time delay, gives $G_{c2}(s) = (20s+1)/(2s+2)$. The resulting overall plant transfer function is given by eqn. (13). Hence, using the above $\lambda$ value in eqns. (15a)-(15c), results in $k = 0.01$, $a = 1.01$ and $b = 0.01$. Limiting $K_c$ to *1.00* and choosing $T_i = 0.25$ gives $\alpha = 0.34$ and $c_1 = 0.086$. The standard form $T_{13}(s)$ to minimise $J_1$ for $c_1 = 0.086$ has $d_2 = 1.484$ and $d_1 = 2.045$, which gives $T_f = -50.24$ and $K_f = 21.92$. The performance of the proposed design method with calculated controller parameters is shown in Fig. 9 for a unit magnitude of set-point change. Alternatively, choosing $K_c = 0.50$ results in $\alpha = 0.27$, $c_1 = 0.068$, $d_2 = 1.482$ and $d_1 = 2.044$. The response for this case is also given in Fig. 9. For comparison, results for the design method proposed by Lee *et al.* [4] are also given in the same figure. They have controller parameters of $K_{pi} = 3.44$, $T_{ii} = 20.66$ and $T_{di} = 0.64$ for the inner loop and $K_{po} = 5.83$, $T_{io} = 105$ and $T_{do} = 4.80$ for the outer loop. Although, both design methods gives similar overshoots, the design method of Lee *et al.* [4] is very sluggish with the response having a much longer settling time. To illustrate the robustness to parameter variations, a $\pm 10\%$ change in the outer loop time delay is assumed, as this normally has the most deteriorating effect on the system step response, and results for this case are given in Fig. 10. Fig. 11 illustrates responses for both the proposed design method and design method of Lee *et al.* [4] to a step disturbance $D_2$, with unit magnitude.

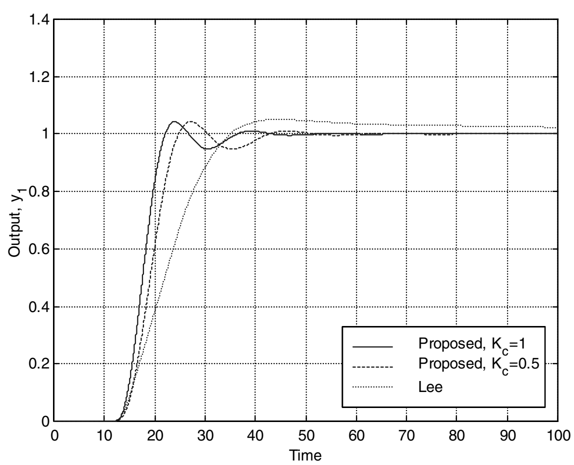


Fig. 9: Responses to a unity step set point change for example 1

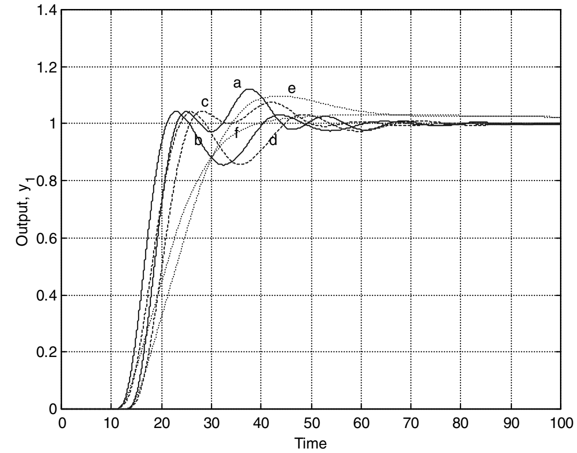


Fig. 10: Responses for example 1, a, c, e) +10% change in the time delay b, d, f) -10% change in the time delay (legends are as shown in Fig. 8)

***Example 2:*** The following plant transfer functions $G_{p1}(s) = e^{-4s}/(s^2+0.2s+1)$, $G_{p2}(s) = e^{-2s}/(s+0.1)$ are considered. Note that the process transfer function in the outer loop has complex poles. Taking $\lambda$ equal to the half the inner loop time delay, i.e. $\lambda = 1$, gives $G_{c2}(s) = (s+0.1)/(s+1)$. Following the procedure given in section 3.2, $k = 1.0$, $a = 1.2$, $b = 1.2$ and $c = 1.0$. In order to obtain a suitable value of $d_3$ to give a standard form for $J_1$ as shown in Fig. 5 requires $\alpha < 1$. Selecting $\alpha = 0.5$, gives $d_3 = 2.4$ from eqn. (27b). The standard form from Fig. 5 requires $c_1 = 3.37$, $d_2 = 5.28$ and $d_1 = 4.67$. These values can be obtained with $T_i = 6.74$ from eqn. (27a), $K_c = 0.42$ from eqn. (25), $T_f = 0.12$ from eqn. (27c) and $K_f = -0.83$ from eqn. (27d). With these calculated controller parameters the performance of the proposed controller design method, together with design method of Lee et al. [4], are shown in Fig. 12 for a unit magnitude of set-point change. The method proposed by Lee et al. [4] have controller parameters of $K_{pi} = 0.36$, $T_{ii} = 10.67$ and $T_{di} = 0.63$ for the inner loop and $K_{po} = 0.36$, $T_{io} = 3.20$ and $T_{do} = 1.13$ for the outer loop. The proposed design method clearly gives better performance in the sense of overshoot and settling time. Responses for both the proposed design method and design method of Lee et al. [4] to disturbance $D_2$, with magnitude of 1, are shown in Fig. 13.

## V. Conclusion

An improved cascade control structure and controller design method has been introduced. A PI-PD Smith predictor scheme is used in the outer loop of the cascade control. This gained two advantages: First, the best merits of the cascade control and a Smith predictor scheme were combined in one structure. Second, the use of PI-PD controller accomplished to obtain improved performance when the process has large time constants or poorly located poles, i.e. lightly damped. Several procedures for obtaining the parameters of the PI-PD controllers are possible, but one of the simplest approaches is to employ standard forms as this enables the design to be completed using simple algebra.

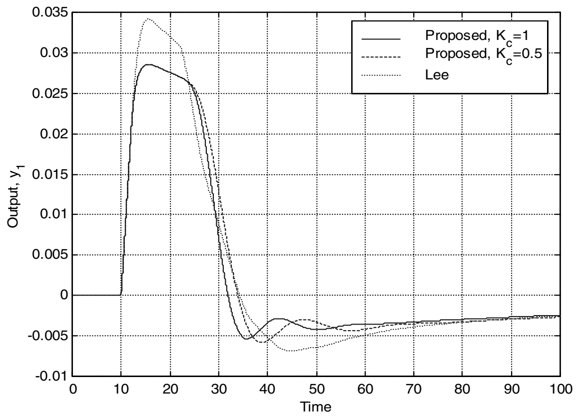


Fig. 11: Responses to disturbance $D_2$ with unity magnitude for example 1

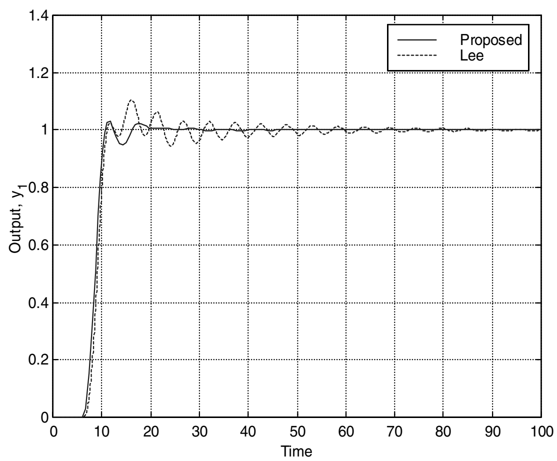


Fig. 12: Responses to a unity step set point change for example 2.

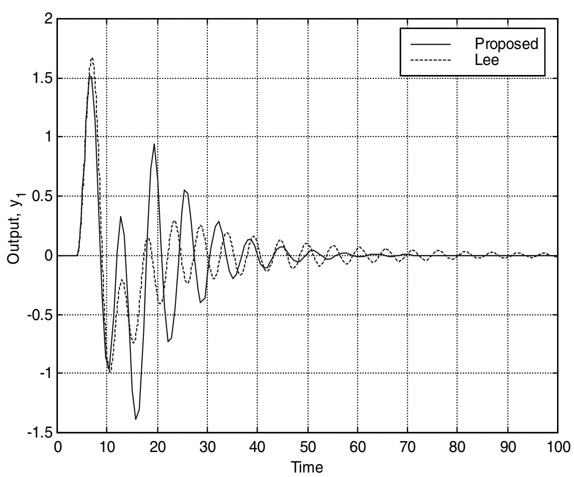


Fig.13: Responses to disturbance $D_2$ with unity magnitude for example 2.

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